Chapter 5

Estimators

Exercise 1 We want to compute $P(\bar{x} < 245)$. We are not told that the volume of the bottles is normally distributed, but $n \geq 30$ so the Central Limit Theorem applies here, and we can assume that the sample mean of bottle volume is normally distributed with mean μ and variance $\frac{\sigma^2}{n}$. Therefore:

$$\bar{x} \sim \mathcal{N}\left(250, \frac{400}{30}\right)$$

We need to transform this normal distribution into a standard normal:

$$P(\bar{x} < 245) = P(Z < \frac{245 - 250}{\sqrt{\frac{400}{30}}} =$$

$$P(Z < -1.37) = P(1 > 1.37) = 1 - P(1 < 1.37) = 1 - 0.9147 = 0.0853$$

Exercise 2 To compute the confidence interval for the population mean, we need the formula

$$P\left(\bar{x} - t_{n-1,\frac{\alpha}{2}}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{n-1,\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\right) = 1 - \alpha$$

We know that $n = 16$, $\bar{x} = 20.4$, $s = 6.4$, and $\alpha = 0.05$. The relevant critical value $t_{n-1,\frac{\alpha}{2}} = t_{15,0.025} = 2.131$. Therefore

$$P\left(20.4 - 2.131\frac{6.4}{4} \leq \mu \leq 20.4 + 2.131\frac{6.4}{4}\right) = 0.95$$

$$P\left(16.9904 \leq \mu \leq 23.8096\right) = 0.95$$

Exercise 3 We know that $n = 400$, $\bar{x} = 250$, $s = 64$, and $\alpha = 0.05$. The relevant critical value $t_{n-1,\frac{\alpha}{2}} = t_{399,0.025} = 1.96$. Note that $t_{399,0.025} = z_{0.025}$, i.e. given that n is large, we could use the normal distribution rather than the t distribution. Therefore:

$$P\left(250 - 1.96\frac{64}{\sqrt{400}} \leq \mu \leq 250 + 1.96\frac{64}{\sqrt{400}}\right) = 0.95$$
$$P\left(243.728 \leq \mu \leq 256.272\right) = 0.95$$