Chapter 1

Descriptive Statistics

Exercise 1 Since the sample size is $n = 10$, the sample mean is equal to

$$\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{14+28+40+13+25+27+20+29+49+66}{10} = \frac{311}{10} = 31.1$$

To find the median, arrange first the observations in order:

$$13, 14, 20, 25, 27, 28, 29, 40, 49, 66$$

The median is the middle observation. When the number of observations is even, it is the mean of the two middle observations. There are 10 observations in this case, thus the median is 27.5 minutes.

The sample variance is given by

$$s^2 = \frac{\sum_{i=1}^{10}(x_i-\bar{x})^2}{10-1} =$$
$$\frac{(14-31.1)^2+(28-31.1)^2+(40-31.1)^2+(13-31.1)^2+(25-31.1)^2+(27-31.1)^2+(20-31.1)^2+(29-31.1)^2+(49-31.1)^2+(66-31.1)^2}{9}$$
$$= \frac{2428.9}{9} = 269.88$$

The sample standard deviation is given by

$$s = \sqrt{s^2} = \sqrt{269.88} = 16.43$$

The coefficient of variation is given by

$$CV = \frac{s}{\bar{x}}100 = \frac{16.43}{31.1}100 = 52.83\%$$

Exercise 2 Price and quantity move in opposite directions; there appears to be a negative relation between the two variables. Therefore we anticipate that the covariance will be negative. Define prices as X and quantity as Y . The sample covariance between X and Y is given by

$$\text{COV}[X,Y] = \frac{\sum_{i=1}^{n}(x_1-\bar{x})(y_i-\bar{y})}{n-1}$$

Notice that

$$\bar{x} = \frac{\sum_{i=1}^{5} x_i}{5} = \frac{100}{5} = 20 \qquad \bar{y} = \frac{\sum_{i=1}^{5} y_i}{5} = \frac{315}{5} = 63$$

Therefore the sample covariance is

$$\text{COV}[X,Y] = \frac{(1-20)(100-63)+(15-20)(90-63)+(20-20)(75-63)+(25-20)(50-63)+(30-20)(0-63)}{4} = \frac{-1200}{4} = -300$$

The sample correlation coefficient is given by:

$$r_{xy} = \frac{\text{COV}[X,Y]}{s_x s_y}$$

Notice that

$$s_x = \frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{5-1} = \sqrt{\frac{250}{4}} = 7.91 \qquad s_y = \frac{\sum_{i=1}^{5}(y_i - \bar{y})^2}{5-1} = \sqrt{\frac{6380}{4}} = 39.94$$

Therefore, the sample correlation coefficient is

$$r_{xy} = \frac{-300}{7.91(39.94)} = -0.95$$

There is a strong negative linear relationship between price and quantity.

Exercise 3

- The sample size n is equal to 16.
- The sample average is equal to:

$$\bar{x} = \frac{\sum_{i=1}^{16} x_i}{16}$$
$$= \frac{1.0+1.0+1.1+1.1+1.1+1.2+1.2+1.4+1.5+1.6+1.6+2.2+3.4+7.6+10.6+26.4}{16}$$
$$= \frac{64}{16} = 4$$

- The sample median is equal to:

$$\frac{1.4+1.5}{2} = \frac{2.9}{2} = 1.45$$

- The sample mode is 1.1.
- The range is equal to $26.4 - 1 = 25.4$

- The sample variance is equal to:

$$s^2 = \frac{\sum_{i=1}^{16}(x_i-\bar{x})^2}{16-1}$$
$$= \frac{(1.0-4)^2+(1.0-4)^2+(1.1-4)^2+(1.1-4)^2+(1.1-4)^2+(1.2-4)^2+(1.2-4)^2+(1.4-4)^2}{15} +$$
$$\frac{(1.5-4)^2+(1.6-4)^2+(1.6-4)^2+(2.2-4)^2+(3.4-4)^2+(7.6-4)^2+(10.6-4)^2+(26.4-4)^2}{15}$$
$$= \frac{(-3)^2+(-3)^2+(-2.9)^2+(-2.9)^2+(-2.9)^2+(-2.8)^2+(-2.8)^2+(-2.6)^2}{15} +$$
$$\frac{(-2.5)^2+(-2.4)^2+(-2.4)^2+(-1.8)^2+(-0.6)^2+(3.6)^2+(6.6)^2+(22.4)^2}{15}$$
$$= \frac{9+9+8.41+8.41+8.41+7.84+7.84+6.76}{15} +$$
$$\frac{6.25+5.76+5.76+3.24+0.36+12.96+43.56+501.76}{15} = 43.0213$$