Advanced Macroeconomics

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PROBLEM SET 1

1. The geography of firm and job dynamism.
How does firm dynamism differ across local labor markets in the United States? Answer this question using data on firm entry and exit rates, job creation and destruction rates for every US Metropolitan Statistical Areas (MSAs) in the United States. Specifically,

1. Use data on entry and exit rate by MSAs (BDS) and population data by MSAs (Census) to replicate Figure 4 in Rubinton (2020)

2. Use data on job creation and job destruction rates by MSAs (BDS) and GDP by MSAs (BLS) to replicate a version of Figure 7 in Kuhn et al(2021)

You can find the data here:

1. Business Dynamics Statistics (BDS) dataset

2. Census Metropolitan and Micropolitan Statistical Area (Census) dataset

3. Bureau of Labor Statistics (BLS) dataset

You can find the papers in the Moodle.
References

1. Rubinton H. 2020. "The Geography of Business Dynamism and Skill Biased Technical Change". FRB St. Louis Working Paper, N.2020-020.

2. Kuhn, M., Manovskii, I., and Qiu, X. 2021. The geography of job creation and job destruction,. NBER working paper No. w29399

2. Firm dynamics with financial frictions.
Consider the following extension of the model economy in Hopenhayn (1992). Firms produce output y according to the following technology

$$y = (zf(m,n))^{\eta} \quad \eta \in (0,1)$$

where z is an input-augmenting firm-specific productivity, while $f()$ is a Cobb-Douglas composite of two production inputs, i.e. goods used as intermediates m and labor n

$$f(m, n) = m^{1-\alpha} n^{\alpha} \quad \alpha \in (0, 1)$$

Firms takes prices as given but face financial constraints on the total purchase of production inputs, i.e.

$$m + wn \leq \chi y$$

where the price of goods is normalized to 1 (numeraire), and w denotes the wage rate. Input expenditure is constrained to be less than χ of firms' earnings. More generally, χ parameterizes financial constraint. The objective of the firm is to choose inputs and output so as to maximize profits subject to its financial constraint, taking the wage and prices as given.

Everything else remains the same as in the original paper. Firms face idiosyncratic uncertainty in their productivity, which are assumed to follow a markow process $\Gamma(z'|z)$, and have to pay a per-period operating costs c_o. Conditional on their productivity, firm are allowed to choose whether to continue operating or not. Finally, every period there is a large measure of potential firms that enter the industry until the free-entry condition holds.

1. Write the expenditure minimization problem of the firm.

2. Characterize the solution to the expenditure minimization problem and show that labor expenditure can be written as:

   $$wn = \alpha\eta\phi y$$

   where $\phi = \min\{\frac{\chi}{\eta}, 1\}$

3. Show that this version of the model with financial constraint is isomorphic to a model without financial constraint where instead firms face a tax τ on their output, i.e. the firm's problem and solution are equivalent across the two economies when $1 - \tau = \phi$

4. Write the dynamic problem of the firm using a recursive formulation.

5. Define a recursive competitive equilibrium.

References

1. Hopenhayn, Hugo A. 1992. "Entry, exit, and firm dynamics in long run equilibrium." Econometrica: Journal of the Econometric Society. pp. 1127-1150.

3. Span of control model.
Consider a version of the model in Lucas (1978). There is a representative household with L members. Household members are heterogeneous in their managerial ability, $z \in Z$ given by a CDF $F(z)$ and density $f(z)$ and they are all endowed with one unit of labor. Each household member has two options:

- become a worker, i.e. supply their labor on the labor market and earn a wage w_t
- become an entrepreneur (manager), i.e. purchase the following production technology

$$y_t(z,n) = z^{1-\gamma} n_t^{\gamma} \quad \gamma \in (0,1)$$

and earn profits $\pi_t(z)$, equal to

$$\pi_t(z) = \max_n \quad y_t(z,n) - w_t n$$

1. State formally the problem of the manager and obtain the corresponding first order conditions.
2. Show that labor demand $n_t(z)$ and profits $\pi_t(z)$ are linear in z.

The problem of the representative household is to optimally choose consumption and to assign each member to an occupation, i.e. worker or manager. This last problem consists in choosing a productivity threshold, z^*, such that $\forall z \geq z^*$, a household member will become a manager, and $\forall z < z^*$ a household member will become a worker.

Therefore, the representative household choose C_t and z_t^* in order to maximize her aggregate utility:

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t L \log\left(\frac{C_t}{L}\right) \quad \beta \in (0,1)$$

subject to the following budget constraint:

$$C_t \leq L \int_0^{z_t^*} w_t f(z) dz + L \int_{z_t^*}^{\infty} \pi_t(z) f(z) dz$$

Assume that z follows a Pareto distribution, i.e. $f(z) = \alpha z^{-\alpha-1}$.

1. State formally the problem of the household and obtain the corresponding first order conditions.
2. Obtain a value for z_t^* and C_t as a function of the wage, w_t.
3. Define a competitive equilibrium.

In this economy, the labor market must be in equilibrium, i.e. labor supply from household members who choose to become workers must be equal to labor demand of the firms managed by the entrepreneurs, i.e.

$$\int_{z_t^*}^{\infty} n_t(z) f(z) dz = \int_0^{z_t^*} f(z) dz$$

1. Use the labor market condition to solve for the equilibrium wage w.
2. Obtain an expression for z^* in terms of primitive of the model.

Assume now that the government introduces a payroll tax τ, such that firm profits become

$$\pi(z) = \max_n \quad y(z, n) - (1 + \tau) w n$$

1. How does this tax alter the previous equilibrium? Do the comparative statics of z^* and w with respect to τ

References

1. Lucas, R. 1978. "On the size distribution of business firms." The Bell Journal of Economics. pp. 508-523.