Chapter 2

Set and probability theory

Exercise 1 Define the following two events:

- Event A: enjoys cycling
- Event B: enjoys reading

We want to calculate the number of people in the intersection of A and B ($A \cap B$). 2 out of 20 people enjoy neither cycling nor reading. Therefore 18 people must enjoy either cycling or reading or both and the number of people in the union of A and B ($A \cup B$) is 18. Recall that

$$A + B + (A \cap B) = A \cap B$$

By assumption $A = 15$ and $B = 8$. Then

$$15 + 8 + (A \cap B) = 18 \implies (A \cap B) = 15 + 8 - 18 = 5$$

Exercise 2 Define the following events:

- Event A: the first member is male
- Event B: the second member is male

Both the first and second members being male is the intersection of events A and B. We want to calculate $P(A \cap B)$. We can calculate $P(A)$ as there are six possible members, 4 of whom are male. So the probability of selecting a male is

$$P(A) = \frac{4}{6} = \frac{2}{3}$$

We can calculate the probability that the second member is male, given that the first member was male, i.e. $P(B|A)$. If one male member has been selected, there are 5 candidates left of whom 3 are male and 2 are female.

$$P(B|A) = \frac{3}{5}$$

Using the multiplication rule:

$$P(A \cap B) = P(B|A)P(A) = \frac{3}{5}\frac{2}{3} = 0.40$$

Exercise 3 Define the following events:

- Event A: a customer asks for help. $P(A) = 0.3$.
- Event B: a customer makes a purchase before leaving. $P(B) = 0.2$.

We are told that the intersection of these events is $P(A \cap B) = 0.15$. Using the addition rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.2 - 0.15 = 0.35$$

The probability of B occurring conditional on A having already occurred is equal to

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.15}{0.30} = 0.5$$

Are these events mutually exclusive? No, The probability of the intersection is not 0, i.e. $P(A \cap B) = 0$. Are they collectively exhaustive? No. The probability of the union is not 1, i.e. $P(A \cup B) \neq 1$. Are they independent? No. Independence requires: $P(A \cap B) = P(A)P(B)$ but $P(A)P(B) = 0.3(0.2) = 0.06 \neq P(A \cap B)$.

Exercise 4 The sets are $A = \{2, 4, 6\}$ and $B = \{4, 5, 6\}$. The answers are:

- complement of A: $\bar{A} = \{1, 3, 5\}$
- complement of B: $\bar{B} = \{1, 2, 3\}$
- intersection of A and B: $A \cap B = \{4, 6\}$
- intersection of A and B: $\bar{A} \cap B = \{5\}$
- union of A and B: $A \cup B = \{2, 4, 5, 6\}$
- union of A and A: $A \cup \bar{A} = \{1, 2, 3, 4, 5, 6\} = S$

Are A and B mutually exclusive? No. The outcomes 4 and 6 are common to both. Are A and B collectively exhaustive? No. The union of A and B does not contain the outcomes 1 or 3.