Macroeconomics of “Large Firms”

Lecture 2: Firm dynamics with frictional labor market

Alessandro Ruggieri
University of Nottingham

CEMFI, PhD short course

This lecture

- Elsby M. and Michaels R. 2013. "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows." American Economic Journal: Macro, Vol. 5, No. 1, pp. 1-48
  - Multi-worker heterogeneous firms $\implies$ well-defined firm-size
  - Idiosyncratic productivity shocks $\implies$ endogenous job destruction
  - Search frictions $\implies$ unemployment
  - Wage bargaining $\implies$ wage dispersion

- Entry/exit dynamics
- Firing costs
  - Bertola G. and Caballero R. 1994. “Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment”. Review of Economic Studies, Vol. 61, No. 3, pp. 435-456
- Heterogeneous agents:
  - Cahuc P., Marque F. and Wasmer E. 2008. “A Theory of Wages and Labor Demand with Intra-Firm Bargaining and Matching Frictions.” International Economic Review, Vol. 49, No. 3, pp. 943-972
- Aggregate shocks

The baseline model

- The labor market is in a perpetual state of flux
  - Workers move across labor market states
  - Firms grow/shrink via job creation/destruction
  - Worker and job flows are large and correlated
- Goal: Synthesis of these ingredients
- How would changes in labor demand, induced by the changes in firm dynamics, affect unemployment?

- Time is discrete
- Exogenous measure of potential workers L
  - infinitely lived
  - risk neutral
  - homogeneous
  - employed/unemployed
- Unitary measure of producers
  - heterogeneous in productivity
    - fixed productivity level, s
    - time-varying productivity level, z
  - no entry/exit dynamics
  - job creation/destruction
  - search frictions and wage bargaining
  - no aggregate shocks

- Decreasing return to scale firm-level production function

$$y = szF(n)$$

  where $F'(n) > 0$ and $F''(n) < 0$
  - the marginal product of labor declines with firm employment, and generates a downward-sloped demand for labor at the firm level
- Time invariant productivity $s \sim \mathcal{H}(s)$
- Idiosyncratic productivity z assumed to follow a markov chain $z' \sim \Gamma(z'|z)$

- Labor market is subject to search and matching functions
  - Workers need to search in order to find jobs
  - Firms need to post vacancies in order to attract workers
- Only unemployed workers can search - no on-the-job search
- The matching process between vacancies V and unemployed workers U is governed by a CRS matching function $m(U, V)$:
  - increasing in both arguments

$$\frac{\partial m(U,V)}{\partial U} > 0 \quad \frac{\partial m(U,V)}{\partial V} > 0$$

  - concave in both arguments

$$\frac{\partial \partial m(U,V)}{\partial U^2} < 0 \quad \frac{\partial \partial m(U,V)}{\partial V^2} < 0$$

  - homogeneous of degree one in both arguments

- Cobb-Douglass function:

$$m(U,V) = m_0 U^{\alpha} V^{1-\alpha} \quad m_0 > 0, \alpha \in [0,1]$$

where

  - m_0 stands for matching efficiency
  - α stands for the elasticity of the matching function with respect to unemployment

- Other functional form (Den Haan et al., 2001)

$$m(U,V) = \frac{UV}{(U^{\eta} + V^{\eta})^{\frac{1}{\eta}}} \quad \eta > 0$$

where the elasticity of matching function equal to:

$$\frac{\partial m(U,V)}{\partial U} \frac{U}{m(U,V)} = \left(1 + \left(\frac{V}{U}\right)^{\eta}\right)^{-\frac{1}{\eta}}$$

- Labor market tightness: $\theta = V/U$
- Search frictions in the labor market limit the rate at which unemployed workers and hiring firms can meet:
  - Job finding probability for workers:

$$\phi_w = \frac{m(U,V)}{U} = m\left(1, \frac{V}{U}\right) = m_0 \theta^{1-\alpha}$$

  - Vacancies posted by firms are filled with probability:

$$\phi_f = \frac{m(U,V)}{V} = m\left(\frac{U}{V}, 1\right) = m_0 \theta^{-\alpha}$$

- Market tightness sufficient statistics for the job filling and job finding probabilities in the model

- $\Pi(s, z, n_{-1})$ denotes the value of a firm entering the period with productivity (s, z) and employment n_{-1}

- Firms choose how many vacancies v to post and current stock of employees, n, by solving the following problem

$$\Pi(s, z, n_{-1}) = \max_{n,v} \quad szF(n) - w(s, z, n)n - c_v v + \frac{1}{1+r}\tilde{\Pi}(s, z, n_{-1})$$

$$\text{and} \quad \tilde{\Pi}(s, z, n) = \sum_{z'} \Pi(s, z', n)\Gamma(z'|z)$$

where

- $w(s, z, n)$ is the wage bargained by the firm with its workers
- c_v is the cost of posting a vacancy

- Firms seek a level of employment that maximizes profits subject to a dynamic constraint on the evolution of firm's employment
- Firms face frictions that limit the rate at which vacancies may be filled.
- Since vacancy posted in a given period will be filled with probability $\phi_f < 1$ prior to production, then:

$$\underbrace{n}_{\substack{\text{new stock}\\ \text{of employees}}} = \underbrace{\phi_f v}_{\text{new hires}} + \underbrace{n_{-1}}_{\substack{\text{old stock}\\ \text{of employees}}} \quad \text{if} \quad \mathbf{1}^+[n > n_{-1}] = 1$$

- Notice:
  - No downward adjustment costs (e.g. firing costs) for firm-initiated separation
  - No worker-initiated separation

- Using the law of motion for firm-level employment, the problem of the firms becomes:

$$\Pi(s,z,n_{-1}) = \max_{n} \quad szF(n) - w(s,z,n)n - \frac{c_v}{\phi_f} \times \max\{0, n - n_{-1}\} + \frac{1}{1+r}\sum_{z'} \Pi(s,z',n)\Gamma(z'|z)$$

where $\frac{c_v}{\phi_f}$ is the (endogenous) cost of scaling employment up.

- Recall firm problem in Hopenhayn and Rogerson (1993)

$$\Pi(s,z,n_{-1}) = \max_{n} \quad szF(n) - wn - \tau \times \max\{0, n_{-1} - n\} + \frac{1}{1+r}\sum_{z'} \Pi(s,z',n)\Gamma(z'|z)$$

- Necessary (not sufficient) condition for a solution to the firm problem, conditional on $\Delta n \neq 0$

$$sz\frac{\partial F(n)}{\partial n} + \frac{1}{1+r}\sum_{z'}\frac{\partial \Pi(s,z',n)}{\partial n}\Gamma(z'|z) = \frac{\partial w(s,z,n)n}{\partial n} + \frac{c_v}{\phi_f}\mathbf{1}^+$$

- There is a kink in the value function around $n = n_{-1}$
  - partial irreversibility of separation decisions in the model
  - while separation is costless, it is costly to reverse such a decision because of hiring (posting vacancies) costs
  - as in Bentolila and Bertola (1990) but hiring costs endogenous

- Optimal employment policy characterized by two reservation values for firm's productivity z, $z_L(s, n_{-1})$ and $z_H(s, n_{-1})$ such that $\forall z \in (z_L(s, n_{-1}), z_H(s, n_{-1})) \implies n = n_{-1}$ (employment inaction region)

The problem of the employed workers

- Value of worker currently employed in a firm with productivity (s, z) and n_{-1} employees

$$J^e(s, z, n_{-1}) = w(s, z, n(s, z, n_{-1})) + \beta \tilde{J}^e(s, z, n)$$

where

$$\tilde{J}^e(s, z, n) = \sum_{z'} \underbrace{p^f(s, z', n(s, z, n_{-1}))}_{\substack{\text{endogenous} \\ \text{firing probability}}} \Gamma(z'|z) J^u +$$

$$+ \sum_{z'} (1 - p^f(s, z', n(s, z, n_{-1}))) J^e(s, z', n) \Gamma(z'|z)$$

- Value of an unemployed worker

$$J^u = b + \beta \left((1-\phi_w) J^u + \phi_w \sum_{s,z'} \int_n J^e(s,z',n) d \underbrace{\psi(s,z',n)}_{\substack{\text{endogenous} \\ \text{distribution of} \\ \text{hiring firms}}} \right)$$

or equivalently

$$J^u = \frac{b}{1-\beta} + \frac{\beta}{1-\beta} \left[\phi_w \sum_{s,z'} \int_n \underbrace{[J^e(s,z',n) - J^u]}_{\text{gain from being hired}} d\psi(s,z',n) \right]$$

- Frictions in the labor market implies makes costly for firms and workers to find alternative employment relationships.
- Quasi-rents that firm and its workers can bargain over.
- Standard search model with constant marginal product (without large firms):
  - the rents of each employment relationship are independent of the rents of all other employment relationships
  - firms can bargain with each of their workers independently
- Search model with decreasing marginal product:
  - the rents of each individual employment relationship depend on the number of workers employed!
  - the rent from "the" marginal worker lower than the rent from all infra-marginal hires due to diminishing marginal product

- Intra-firm bargaining protocol a là Stole and Zwiebel (1994)
  - generalization of the Nash solution to a setting with diminishing returns
  - Nash bargaining over the marginal surplus of firm-worker relationship
- Intuitions:
  - If the firm has only one worker, the firm and worker simply strike a Nash bargain.
  - If a second worker is added, the firm and the additional worker know that, if their negotiations break down, the firm will agree to a Nash bargain with the remaining worker. In this sense, the second employee regards herself as being on the margin.
  - By induction, then, the firm approaches negotiations with the n^{th} worker as if that worker were marginal too
  - the wage that solves the bargaining problem is that which maximizes the marginal surplus.

- Wages are set after employment has been determined
  - Hiring costs are sunk and labor market is closed setting
- Firm marginal surplus $S^f(s, z, n; w)$

$$
\begin{aligned}
S^f(s,z,n;w) &= \frac{\partial \Pi(s,z,n_{-1})}{\partial n} \\
&= sz\frac{\partial F(n)}{\partial n} - w(s,z,n) - \frac{\partial w(s,z,n)}{\partial n}n + \frac{1}{1+r}\frac{\partial \tilde{\Pi}(s,z,n)}{\partial n}
\end{aligned}
$$

- Worker marginal surplus $S^w(s, z, n; w)$

$$
S^w(s,z,n;w) = J^e(s,z,n) - J^u
$$

- Bargaining problem

$$w(s,z,n) = \arg\max_{w} \quad S^f(s,z,n;w)^{\gamma} S^w(s,z,n;w)^{1-\gamma}$$

where $\gamma \in (0,1)$ is the worker's bargaining power

- Nash splitting rule

$$S^w(s,z,n;w) = \gamma[S^f(s,z,n;w) + S^w(s,z,n;w)]$$
$$S^f(s,z,n;w) = (1-\gamma)[S^f(s,z,n;w) + S^w(s,z,n;w)]$$

- Wage solves the following ODE

$$w(s,z,n) = (1-\gamma)b + \gamma\left[sz\frac{\partial F(n)}{\partial n} - \frac{\partial w(s,z,n)}{\partial n}n + \frac{1}{1+r}c_v\phi_w\right]$$

- Wages are:
  - increasing in the worker's bargaining power
  - increasing in the marginal product of labor
  - increasing in job finding probability and marginal costs of hiring
  - increasing in home production
- Extra term: $\frac{\partial w(s,z,n)}{\partial n} n$
  - If negotiation breaks, the firm will have to pay its remaining workers a higher wage (higher marginal product)
  - Inefficient incentive to over-employ workers
- Specific solution (with $F(n) = n^{\alpha}$)

$$w(s, z, n) = (1 - \gamma)b + \gamma \left[\frac{szn^{\alpha-1}}{1 - \gamma(1 - \alpha)} + \frac{1}{1 + r} c_v \phi_w \right]$$

- Binmore et al. (1986) bargaining solution
  - alternating offers generalized to a setting when marginal returns are diminishing
- The threats are to extend bargaining rather than to terminate it in case of disagreement
  - disagreement payoffs determines the bargaining outcomes, not the outside option payoff
- Breakdown of negotiations generates a surplus to split between parties, which is equal to the marginal flow surplus

- Firm marginal flow surplus $S^f(s, z, n; w)$

$$S^f(s,z,n;w) = sz\frac{\partial F(n)}{\partial n} - w(s,z,n) - \frac{\partial w(s,z,n)}{\partial n}n$$

- Worker marginal flow surplus $S^w(s, z, n; w)$

$$S^w(s,z,n;w) = w(s,z,n) - b$$

- Wage solves the following ODE

$$w(s,z,n) = (1-\gamma)b + \gamma\left[sz\frac{\partial F(n)}{\partial n} - \frac{\partial w(s,z,n)}{\partial n}n\right]$$

- No influence of labor market tightness on wages (Hall and Milgrom 2008)

- Let $\mu_t(s, z, n; \theta_t)$ be the measure of firms over individual state (s, z, n) when the market tightness is θ_t at time t
- Evolution of distribution over time:

$$\mu_{t+1}(s, z, n'; \theta_{t+1}) = T_t(\mu_t(s, z, n; \theta_t), \theta_t)$$

where

$$T_t(\mu_t(s, z, n; \theta_t); \theta_t) = \sum_{s,z} \int_n \psi_t(s, z', n' | z, n; \theta_t) d\mu_t(s, z, n; \theta_t)$$

and

$$\psi_t(s, z', n' | z, n; \theta_t) = \mathbf{1}[n(s, z', n; \theta_t) = n'] \Gamma(z' | z)$$

- Aggregate employment:

$$N = \sum_{s,z,n_{-1}} n(s,z,n_{-1}) d\mu(s,z,n_{-1})$$

- Total separation:

$$S = \sum_{s,z,n_{-1}} \max\{0, n_{-1} - n(s,z,n_{-1})\} d\mu(s,z,n_{-1})$$

- Total hires: $H = \phi_w U$
- Labor market dynamics: $N' = H - S + N$
- Labor resource constrain: $N + U = L$

A steady-state competitive equilibrium is a market tightness θ, a wage schedule $w(s, z, n)$, an optimal policy function for employment, $n(s, z, n_{-1})$, and a distribution $\mu(s, z, n)$, such that:

- **Firms optimality**: $n(s, z, n_{-1})$ solves the problem of the firm
- **Bargaining**: $w(s, z, n)$ are the solution to the intra-firm Nash bargaining problem
- **Stationarity**:
  - the distribution $\mu(s, z, n)$ replicates itself through productivity shocks and hiring/firing decisions
  - workers hires and separation balance each other, i.e. $\phi_w U = S$

- **Step 1**: guess market tightness θ^0
- **Step 2**: compute job finding and filling probabilities, ϕ_w^0, ϕ_f^0
- **Step 3**: compute wages, $w(s, z, n; \theta^0)$
- **Step 4**: solve the problem of the firm and obtain policy functions for employment $n(s, z, n_{-1})$
- **Step 5**: simulate the economy for a large number of firms and compute the stationary distribution, $\mu(s, z, n)$
- **Step 6**: use $\mu(s, z, n)$ to compute aggregate employment and total separation
- **Step 7**: obtain new value for market tightness, θ^1, using the stationarity condition and labor resource constraint
- **Step 8:** check convergence
  - if not achieved, use θ_1 and go back to step 2 till convergence
  - if achieved, store $\theta^* = \theta_0$

Implication 1: Inaction region

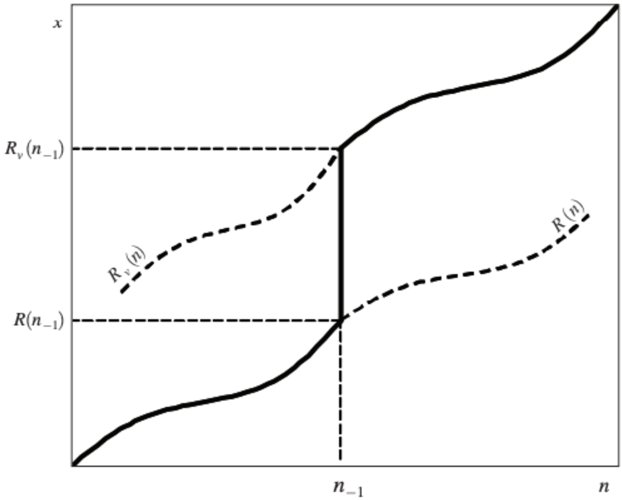

FIGURE 1. OPTIMAL EMPLOYMENT POLICY OF A FIRM

- Vacancy costs create a kink in the policy function

Implication 2: Firm size distribution

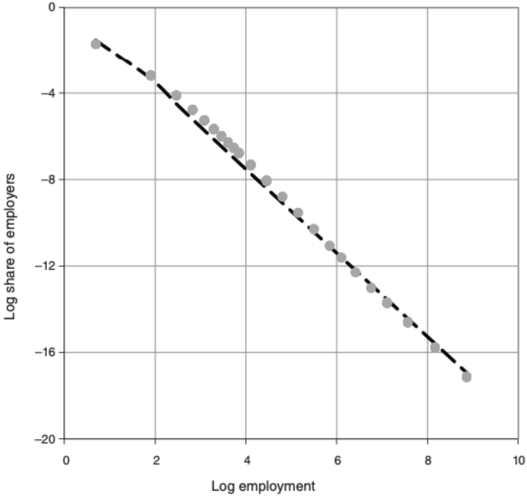

Figure 2. Employer Size Distribution: Model versus Data

Notes: The dots plot data on the shares of firms in successive employment categories for the years 2002 to 2006 based on data on employment by firm-size class from the Small Business Administration. The dashed line plots the steady-state distribution of employment across firms implied by the model using the parameters reported in Table 1.

Implication 3: Firm growth distribution

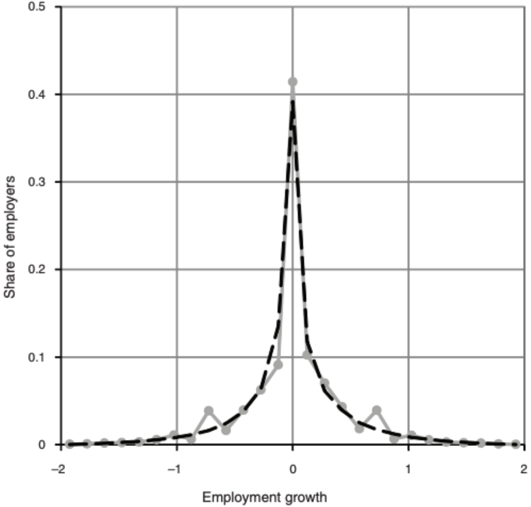

FIGURE 3. EMPLOYMENT GROWTH DISTRIBUTION: MODEL VERSUS DATA

Notes: The dotted line plots the cross-sectional distribution of employment growth based on data for continuing establishments from the Longitudinal Business Database pooled over the years 1992 to 2005. The dashed line plots the steady-state distribution of employment growth in the model using the parameters reported in Table 1.

Implication 4: Wage dispersion

- Wage dispersion between firms
  - usually accounts for 20-30% percent of overall wage dispersion
  - possible fixes:
    - workers heterogeneity
    - on-the-job search
- Counterfactual size-wage premium
  - small firms pay higher wages in the model (for some calibration)
  - two conflicting forces:
    - large firms are those more productive: wages $\uparrow$
    - large firms have lower marginal product of labor: wages $\downarrow$

Extension: Entry and Exit dynamics

- The problem of the incumbent firms now reads as follows:

$$\begin{aligned}\Pi(z, n_{-1}) = \max_{n} \quad & zF(n) - w(z,n)n - c_o \\ & - \frac{c_v}{\phi_f} \max\{0, n - n_{-1}\} \\ & + \frac{1}{1+r} \max\left\{0, \sum_{z'} \Pi(z', n)\Gamma(z'|z)\right\}\end{aligned}$$

  where c_o denotes a cost of operation

- A solution to this problem is a policy function for employment $n(z, n_{-1})$ together with policy function for exit $\mathbf{1}^x(z, n_{-1})$
- There exists a thresholds $z^*(n)$ such that $\Pi(z^*(n), n) = 0$ and $\forall z < z^*(n) \implies \mathbf{1}^x(z, n_{-1}) = 1$.

- Potential entrants are ex-ante identical
- New entrants $M \geq 0$ pay c_e and enter the industry with no employees
- Draw productivity level z from $\Gamma^e(z)$ (ergodic distribution obtained from $\Gamma(z|z_{-1})$)
- Free entry condition:

$$\Pi^e = \frac{1}{1+r} \sum_z \Pi(z, 0)\Gamma^e(z) \leq c_e$$

with equality if $M > 0$.

- **Step 1**: guess market tightness θ^0
- **Step 2**: compute job finding and filling probabilities, ϕ_w^0, ϕ_f^0
- **Step 3**: compute wages, $w(s, z, n; \theta^0)$
- **Step 4**: solve the problem of the incumbent firms
- **Step 5**: compute the value of entry, Π^e and check if free-entry condition is satisfied:
  - if no, make a new guess θ^1 and go back to step 2 till convergence
  - if yes, store $\theta^* = \theta^0$
- **Step 6**: simulate the economy for a large number of firms and compute average employment and job destruction, $\bar{\ell}$ and $\bar{s}$
- **Step 7**: recover unemployment and number of firms M using stationarity condition $M\bar{s} = \phi_w U$ and labor resource constraints, $M\bar{\ell} + U = L$

Extension: Firing costs

- The problem of the firms now reads as follows:

$$\begin{aligned}\Pi(z, n_{-1}) = \max_{n} \quad & zF(n) - w(z,n)n \\ & - \frac{c_v}{\phi_f} \max\{0, n - n_{-1}\} - c_f \max\{0, n_{-1} - n\} \\ & + \frac{1}{1+r} \sum_{z'} \max\{0, \Pi(z', n)\} \Gamma(z'|z)\end{aligned}$$

  where c_f denotes an (exogenous) cost of scaling employment down

- Firing costs have similar implications as fixed costs. Why?

- Conditional on $\Delta n > 0$ and not-exiting

$$z\frac{\partial F(n)}{\partial n} - \frac{\partial w(z,n)n}{\partial n} + \frac{1}{1+r}\sum_{z'}\frac{\partial \Pi(z',n)}{\partial n}\Gamma(z'|z) = \frac{c_v}{\phi_f}$$

- Conditional on $\Delta n < 0$ and not-exiting

$$z\frac{\partial F(n)}{\partial n} - \frac{\partial w(z,n)n}{\partial n} + \frac{1}{1+r}\sum_{z'}\frac{\partial \Pi(z',n)}{\partial n}\Gamma(z'|z) = -c_f$$

- Optimal employment choice characterized by two reservation values for productivity z, $z_L(n_{-1})$ and $z_H(n_{-1})$ such that $\forall z \in (z_L(n_{-1}), z_H(n_{-1})) \implies n = n_{-1}$
- Notice
  - $z_L(n_{-1})$ decreases with firing costs!
  - marginal firm surplus is negative when $c_f > 0$

- Bargaining with a hiring firms

$$w^h(z,n) = \arg\max S^f(z,n;w)^\gamma S^w(z,n;w)^{1-\gamma}$$

- Bargaining with a firing firms:
  - firing firms optimally choose n s.t. $S^f(z,n;w) = -c_f < 0$
  - Nash splitting rule implies negative workers' surplus

$$S^w(z,n;w) = \frac{\gamma}{(1-\gamma)} S^f(z,n;w) < 0$$

    hence workers have incentive to quit!
  - Ensure workers' participation constraint, i.e.

$$S^w(z,n;w) \geq 0$$

  - Wage solution such that $S^w(z,n;w) = 0$, i.e.

$$w^f(z,n) = J^u - \tilde{J}^e(z,n)$$

Output loop

- **Step 1**: guess a wage schedule $w^0(z, n)$
- **Step 2**: solve the **Inner loop**
- **Step 3**: use $n(s, z, n_{-1})$ together with solution to bargaining problem to obtain a new schedule for wages $w^1(z, n)$
- **Step 4**: check convergence
  - if no, use $w^1(z, n)$ as new guess and go back to step 2 till convergence
  - if yes, store $w^*(z, n) = w^0(z, n)$
- **Step 5**: simulate the economy for a large number of firms and compute average employment and job destruction, $\bar{\ell}$ and $\bar{s}$
- **Step 6**: recover unemployment and number of firms M using stationarity condition $M\bar{s} = \phi_w U$ and labor resource constraints, $M\bar{\ell} + U = L$

Inner loop

- **Step 1**: guess market tightness θ^0
- **Step 2**: compute job finding and filling probabilities, ϕ_w^0, ϕ_f^0
- **Step 3**: solve the problem of the firm and obtain policy functions for employment $n(s, z, n_{-1})$
- **Step 4**: compute the value of entry, Π^e and check if free-entry condition is satisfied:
  - if no, make a new guess θ^1 and go back to step 2 till convergence
  - if yes, store $\theta^* = \theta^0$

Extension: Heterogeneous Agents

- Concave firm-level production function with J types of workers

$$y = z \underbrace{\left(\sum_{j=1}^{J} (s_j n_j)^{\rho} \right)^{\frac{\alpha}{\rho}}}_{F(\boldsymbol{n})}$$

where

  - n_j denotes the number of group-j employees
  - $\boldsymbol{n} = (n_1, n_2, ..., n_J)$ is a vector of n_j
  - s_j denotes labor efficiency of group-j employees
  - ρ governs the elasticity of substitutions across workers groups
  - $\alpha \in (0, 1)$ governs the return to scale of the aggregate labor input in production

- Innate productivity differences across firms, $z \sim \Gamma(z)$

- Labor market segmented by skill groups j
- To hire workers, firms need to post separate vacancies for each type-j worker
  - Group specific hiring cost, c_v^j, per vacancy posted
- Unemployed group$-j$ workers can search for job in their market only
- Vacancies are matched to the pool of unemployed workers according to a matching technology $m_j(U_j, V_j)$
  - matching function allowed to differ across groups (e.g. different matching efficiencies, etc...)
- Exogenous group-specific job destruction rate δ_j

- The problem of an incumbent firms read as follows:

$$\Pi(z, \boldsymbol{n_{-1}}) = \max_{\{v_j\}_{j=1}^{J}} \quad zF(\boldsymbol{n}) - \sum_{j=1}^{J} w_j(z, \boldsymbol{n}) n_j - \sum_{j=1}^{J} c_v^j v_j + \frac{1}{1+r} \Pi(z, \boldsymbol{n})$$

subject to a low of motion for each group-j workers:

$$n_j = n_{j-1}(1 - \delta_j) + \phi_f^j v_j$$

- Focus on steady-state: $\boldsymbol{n_{-1}} = \boldsymbol{n}$

- Steady state firm-level employment satisfies:

$$-c_v^j + \frac{1}{1+r}\frac{\partial \Pi(z,\boldsymbol{n})}{\partial n_j}\phi_f^j = 0$$

- Firms' marginal surplus

$$\begin{aligned} S_j^f(z,\boldsymbol{n}) &= \frac{\partial \Pi(z,\boldsymbol{n})}{\partial n_j} \\ &= \frac{1+r}{\delta_j + r}\left[z\frac{\partial F(\boldsymbol{n})}{\partial n_j} - w_j(z,\boldsymbol{n}) - \sum_{i=1}^{J}\frac{\partial w_i(z,\boldsymbol{n})}{\partial n_j} n_i\right] \end{aligned}$$

- Firms' marginal product at the optimal level of employment is equal to the expected recruitment

- A solution to the problem of the incumbent firms is a policy function for employment of each group-j workers such that:

$$\underbrace{z\frac{\partial F(\boldsymbol{n})}{\partial n_j}}_{\text{marginal product}} = \underbrace{w_j(z,\boldsymbol{n})}_{\text{wage}} + \underbrace{\sum_{i=1}^{J}\frac{\partial w_i(z,\boldsymbol{n})}{\partial n_j}n_i}_{\text{employment effect on wages}} + \underbrace{(\delta_j + r)\frac{c_v^j}{\phi_f^j}}_{\text{turnover cost}}$$

- Value of being employed

$$V_j^e(z,\boldsymbol{n}) = w_j(z,\boldsymbol{n}) + \frac{1}{1+r}\left[\delta_j V_j^u + (1-\delta_j)V_j^e(z,\boldsymbol{n})\right]$$

- Value of being unemployed

$$V_j^u = b + \frac{1}{1+r}\left[\phi_w^j \sum_z \int_{\boldsymbol{n}} V_j^e(z,\boldsymbol{n})\psi(z,\boldsymbol{n})d\boldsymbol{n} + (1-\phi_w^j)V_j^u\right]$$

- Workers' marginal surplus

$$\begin{aligned} S_j^w(z,\boldsymbol{n}) &= V_j^e(z,\boldsymbol{n}) - V_j^u = \\ &= \frac{1+r}{\delta_j + r}\left[w_j(z,\boldsymbol{n}) - \frac{r}{1+r}V_j^u\right] \end{aligned}$$

- Intra-firm bargaining problem (Stole and Zwiebel 1994) extended to heterogeneous workers
- Workers bargaining power γ_j allowed to vary across groups
- Nash sharing rule

$$\gamma_j S_j^f(z, \boldsymbol{n}) = (1 - \gamma_j) S^w(z, \boldsymbol{n}) \quad \forall j = 1, 2, ...J$$

- Wage solves the following a non-linear system of ODE

$$w_j(z, \boldsymbol{n}) = \frac{r}{1+r}(1 - \gamma_j) V_j^u + \gamma_j \left[z \frac{\partial F(\boldsymbol{n})}{\partial n_j} - \sum_{i=1}^{J} \frac{\partial w_i(z, \boldsymbol{n})}{\partial n_j} n_i \right] \quad \forall j$$

- Wage solution

$$w_j(z,\boldsymbol{n}) = \underbrace{\frac{r}{1+r}(1-\gamma_j)V_j^u}_{(1-\gamma_j)b+\gamma_j\theta\frac{cv}{1+r}} + \int_0^1 x^{\frac{1-\gamma_j}{\gamma_j}} z \frac{\partial F(\boldsymbol{nA_j})}{\partial n} dx$$

where the vector $\boldsymbol{nA_j}$ is equal to

$$\boldsymbol{nA_j} = \left(n_1 z^{\frac{\beta_1}{1-\beta_1}\frac{1-\beta_j}{\beta_j}}, n_2 z^{\frac{\beta_2}{1-\beta_2}\frac{1-\beta_j}{\beta_j}}, ..., n_J z^{\frac{\beta_J}{1-\beta_J}\frac{1-\beta_j}{\beta_j}} \right)$$

Extension: Aggregate shocks

- Baseline model with aggregate productivity shocks:

$$p' = \begin{cases} p + \sigma_p & \text{with probability} \quad 1/2 \\ p - \sigma_p & \text{with probability} \quad 1/2 \end{cases}$$

- Firms need to forecast future wages, hence labor market tightness, hence entire distribution of employment across firms
- Approximate evolution of aggregate employment and market tightness around their steady-state values (for $\sigma_p \approx 0$)

$$N' \approx N^* + \nu_N(N - N^*) + \nu_p(p' - p)$$
$$\theta' \approx \theta^* + \mu_N(N - N^*) + \mu_p(p' - p)$$

- Krusell-Smith algorithm:
  - iterate numerically over the parameters $(\nu_N, \nu_p, \mu_N, \mu_p)$

- **Step 1**: guess a set of coefficients $\nu_N^0, \nu_p^0, \mu_N^0, \mu_p^0$
- **Step 2**: forecasting rules gives value for (N', θ, θ') given (p, N)
- **Step 3**: solve the problem of the firm
  - state-space expanded to (p, N)
  - discreteness of p effectively implies only one more state variable

  and store policy functions for employment $n(s, z, n_{-1}, p, N)$
- **Step 4**: Simulate the economy for a larger number of firms and compute aggregate number of hires and separation in each period, $H(p, N)$ and $S(p, N)$
  - this requires to compute the distribution of employment across incumbent firms

- **Step 5**: Compute employment using the labor market law of motion:

$$N = N_{-1} + H(p, N) - S(p, N)$$

- **Step 6**: Compute equilibrium market tightness from the identity condition for hires:

$$H(p, N) = \theta(L - N)$$

- **Step 7**: Use time series to estimate the forecasting coefficient via OLS, $\nu_N^1, \nu_p^1, \mu_N^1, \mu_p^1$
- **Step 8**: Check if convergence is satisfied
  - if no, use $\nu_N^1, \nu_p^1, \mu_N^1, \mu_p^1$ as new guess and go to Step 1 until convergence
  - if yes, store $\nu_N^* = \nu_N^1, \nu_p^* = \nu_p^1, \mu_N^* = \mu_N^1, \mu_p^* = \mu_p^1$

Implication 1: Beveridge curve

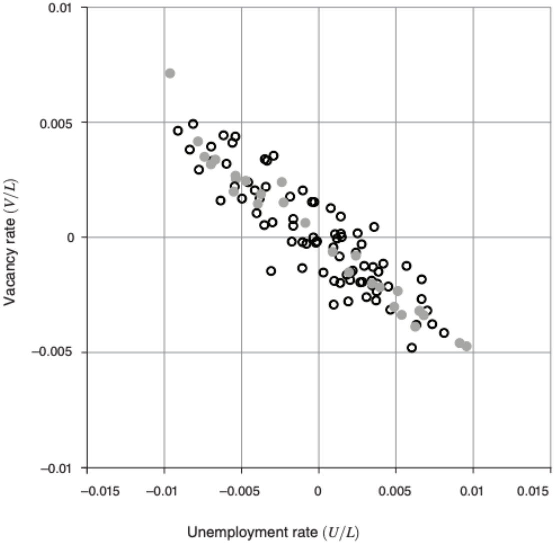

FIGURE 5. BEVERIDGE CURVE: MODEL VERSUS DATA

- Standard search model unable to generate enough cyclicality in job creation jointly with workers flow into unemployment

$$\frac{\partial \theta}{\partial p} \approx \frac{(1-\gamma)p}{m_0[(1-\gamma)(p-b) - \frac{\gamma}{1+r}c_v\theta] + \frac{\gamma}{1+r}c_v\theta}$$

- Large firms induce an amplification mechanism

$$\frac{\partial \theta}{\partial p} \approx \frac{(1-\gamma)p}{\omega m_0[(1-\gamma)(p-b) - \frac{\gamma}{1+r}c_v\theta] + \frac{\gamma}{1+r}c_v\theta}$$

where

- $\omega < 1$ is the steady-state employment share of hiring firms (always one in standard model)
- p is a a weighted average of the average and marginal flow surpluses (only average in standard model)

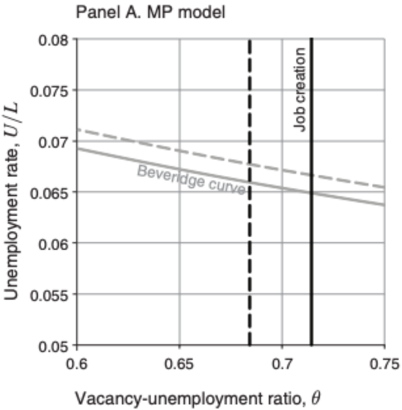

FIGURE 4. JOB CREATION AND BEVERIDGE CURVE CONDITIONS: MP MODEL VERSUS GENERALIZED MODEL

Notes: The figure illustrates the steady-state shifts in the job creation and Beveridge curve conditions in the MP model (panel A) and the generalized model (panel B) detailed in Sections I and II. The calibrations, respectively, are taken from panels B and A of Table 2. As a result, the initial steady states, and the shifts in the Beveridge curve are identical across models.

Implication 3: Propagation

- Standard search model unable to generate sluggish response of equilibrium labor market tightness to aggregate shocks to labor productivity
  - vacancy-unemployment ratio is a jump variable
- Large firms induce a propagation mechanism
  - vacancy-unemployment ratio depends on the distribution of employment across establishments
  - distribution of establishment is not a jump variable
  - sluggish behavior generated by vacancy costs and idiosyncratic shocks

Implication 3: Propagation

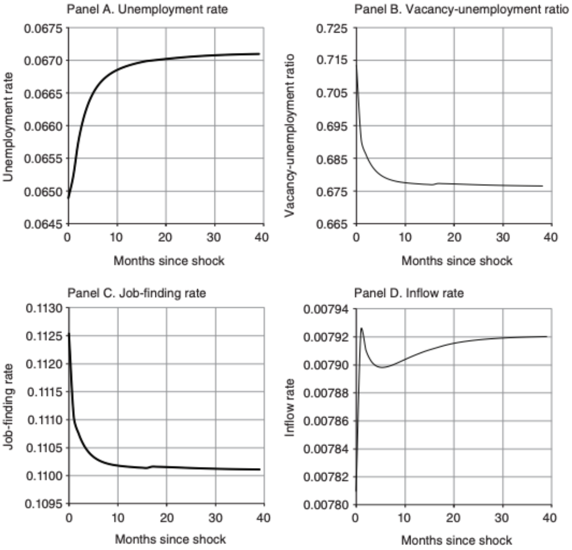

Figure 6. Model Impulse Responses to a Permanent 1 Percent Decline in Aggregate Labor Productivity, p

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