Macroeconomics of “Large Firms”

Lecture 3: Firm growth with on-the-job search

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CEMFI, PhD short course

- Fajgelbaum P. 2020. "Labour Market Frictions, Firm Growth, and International Trade." Review of Economic Studies, Vol. 87, No. 3, pp. 1213–1260
    - Multi-worker heterogeneous firms
    - Search frictions with on-the-job search
    - Wage settings
        - Nash bargaining between firm and unemployed worker
        - Bertrand competition between firms to poach employer worker
    - Life-cycle wage growth
    - Endogenous *misallocation* of labor across firms
- Extensions:
    - Ex-ante heterogeneous workers
    - Dynamic investment decisions

- Labor market frictions prevent labor reallocation across firms
    - lower firm growth
    - higher resource misallocation
- Firms grow by poaching workers
    - Employees move directly to new employers
- Goal: Combine firm dynamics with on-the-job search
- How would changes in search frictions affect firm growth?

- Time is continuous
- Exogenous measure of workers
  - stochastic life-cycle
    - δ_w: workers retirement rate
  - ex-ante homogeneous
  - employed/unemployed
  - off- and on-the-job search
- Endogenous measure of firms
  - stochastic life-cycle
    - δ_f: exogenous firms' exit rate
  - heterogeneity in productivity
  - entry/exit dynamics
  - linear production function
  - firm size bounded by convex recruiting costs

- Firm-level production technology

$$y = \int_0^{\ell} g(i, z)\psi^e(i|z)di$$

where $\psi^e(i|z)$ denotes the share of worker i in a firm z with total workforce ℓ

- Firm-worker match production:

$$g(i, z) = Az = g(z) \quad \forall i$$

where A is an aggregate shifter

- Given workers homogeneity, technology can be expressed as a linear function of total workforce

$$y = Az\ell$$

- Search frictions with on-the-job search
  - workers search for jobs at different (exogenous) intensity
  - firms search for workers at different (endogenous) intensity
- Total pool of searching workers :

$$\tilde{\lambda}_u u + \tilde{\lambda}_e (1-u)$$

  where $\tilde{\lambda}_i$ is the search intensity (visibility) of group $i = u, e$
- Total pool of searching firms:

$$M\bar{s}$$

  where M is the measure of active firms while $\bar{s}$ is the average search effort
- CRS matching function: $m(M\bar{s}, \tilde{\lambda}_u u + \tilde{\lambda}_e(1-u))$

- Contact rate for workers in group $i = u, e$

$$\lambda_i = \tilde{\lambda}_i \frac{m(M\bar{s}, \tilde{\lambda}_u u + \tilde{\lambda}_e(1-u))}{\tilde{\lambda}_u u + \tilde{\lambda}_e(1-u)}$$

- By homogeneity of degree 1:

$$\lambda_i = \tilde{\lambda}_i \chi\left(\frac{M\bar{s}}{\tilde{\lambda}_u u + \tilde{\lambda}_e(1-u))}\right)$$

  where $\chi(x) = m(1, x)$

- Labor market tightness

$$\theta = \frac{M\bar{s}}{\tilde{\lambda}_u u + \tilde{\lambda}_e(1-u))}$$

- Total number of matches

$$m = \lambda_u u + \lambda_e (1-u)$$

- Total number of matches of a firm exerting $s(z)$ effort

$$m(z) = [\lambda_u u + \lambda_e (1-u)] \frac{s(z)}{M\bar{s}}$$

- Rate at which firms contact workers from unemployment

$$q_u = \frac{m(z)}{s(z)} \frac{\lambda_u u}{\lambda_u u + \lambda_e (1-u)} = \frac{\lambda_u u}{M\bar{s}}$$

- Rate at which firms contact workers from employment

$$q_e = \frac{m(z)}{s(z)} \frac{\lambda_e (1-u)}{\lambda_u u + \lambda_e (1-u)} = \frac{\lambda_e (1-u)}{M\bar{s}}$$

- Postel-Vinay and Robin (2002): firms observe the current status of the worker, tender take-it-or-leave-it wage offers, and commit to the value promised
  - the outcome similar to Bertrand competition
  - the firm offering the job of greater total value obtains the worker, offering in exchange a value equal to what the worker could obtain in the alternative employment
- Cahuc et al. (2006): the worker additionally splits the surplus with the higher-value firm according to the conventional Nash solution rule
  - total value in the lower-value job used as outside option
- In both settings a worker is hired under a flat wage profile until leaving or triggering a renegotiation.

- Flinn and Mullins (2016), Flinn et al. (2017): employers are not able to commit to wage offers, or is not able to verify claims that the individual has received an competing offers.
  - the outside option in the wage determination problem remains the value of unemployed search
  - moving to unemployment is the only action available to the employee at any moment in time
  - no wage renegotiation triggered within job
  - wage gains only through job-to-job mobility
- In this last setting a worker is hired under a flat wage profile until leaving

- Bertrand competition + wage bargaining (Cahuc et al., 2006)
  - collapses to Mortensen-Pissarides' bargaining when $\lambda_e = 0$

- $\beta \in (0, 1)$: workers bargaining power

- Negotiations with the unemployed
  - Split the surplus according to Nash bargaining
  - Negotiate wage w_u such that:

$$\underbrace{W(w_u, z)}_{\substack{\text{value accruing} \\ \text{to workers}}} = \beta \underbrace{V(z)}_{\substack{\text{value of} \\ \text{the match}}}$$

- Notice: value of the match $V(z)$ independent of wages w!

- Renegotiation with the employed
  - happens only when either side has an interest to separate if they do not obtain an improved offer
  - on-the-job search generates alternative opportunities for workers triggering either job mobility or responses to the outside offers
- Match values alternative employers: $V(z')$
- Three scenarios:
  - $V(z') > V(z)$: the worker moves to the alternative job
  - $W(w,z) < V(z') < V(z)$: worker uses the outside offer to negotiate up her wage
  - $V(z') < W(w,z)$: the worker has nothing to gain from the competition between z and z'

- Scenario 1: the worker moves to the alternative jobs and uses the previous match value as the outside option when bargaining
  - Let $\mathcal{A}(z) := \{z' \in \mathcal{Z} \quad : V(z') > V(z)\}$
  - Negotiate w_e such that:

$$W(w_e(z, z'), z') = \beta V(z') + (1 - \beta)V(z)$$

- Scenario 2: the worker doesn't move and uses the outside offer as the option value when bargaining
  - Let $\mathcal{B}(z) := \{z' \in \mathcal{Z} : V(z') < V(z) < W(w, z)\}$
  - Negotiate w_e such that:

$$W(w_e(z, z'), z) = \beta V(z) + (1 - \beta)V(z')$$

- Scenario 3: nothing happens

- Value of an unemployed worker:

$$(r+\delta_w)U = b + \lambda_u \int_z \max\{0, \underbrace{W(w_u(z), z) - U}_{\text{gains from UtE movements}}\} dP(z)$$

- Value of a worker employed in a firm z at a wage z

$$\begin{aligned}(r+\delta_w)W(w,z) = w + \delta_f & \underbrace{(U - W(w,z))}_{\text{losses from separation}} \\ & + \lambda_e \int_{z' \in \mathcal{A}(z)} \underbrace{(W(w_e(z,z'), z) - W(w,z))dP(z')}_{\text{gains from re-negotiation}} \\ & + \lambda_e \int_{z' \in \mathcal{B}(z)} \underbrace{(W(w_e(z,z'), z') - W(w,z))}_{\text{gains from EtE movements}} dP(z')\end{aligned}$$

- Value for firm with productivity z with worker employed at wage w

$$
\begin{aligned}
(r+\delta_w)J(w,z) = g(z) - w + \delta_f \quad & \underbrace{(0 - J(w,z))}_{\text{losses from separation}} \\
& + \lambda_e \int_{z' \in \mathcal{A}(z)} \underbrace{(J(w_e(z,z'),z) - J(w,z))}_{\text{losses from re-negotiation}} dP(z') \\
& + \lambda_e \int_{z' \in \mathcal{B}(z)} \underbrace{(0 - J(w,z))}_{\text{losses from JtJ movements}} dP(z')
\end{aligned}
$$

Using the implicit solutions for wages:

$$
\begin{aligned}
(r+\delta_w)U &= b + \beta\lambda_u \int_z \max\{0, V(z) - U\} dP(z) \\
(r+\delta_w+\delta_f)W(w,z) &= w + \delta_f U \\
&\quad + \lambda_e \int_{z'\in\mathcal{A}(z)} ((1-\beta)V(z') - \beta V(z) - W(w,z)) dP(z') \\
&\quad + \lambda_e \int_{z'\in\mathcal{B}(z)} ((1-\beta)V(z) - \beta V(z') - W(w,z)) dP(z') \\
(r+\delta_w+\delta_f)J(w,z) &= g(z) - w \\
&\quad + \lambda_e \int_{z'\in\mathcal{A}(z)} ((1-\beta)(V(z) - V(z')) - J(w,z)) dP(z') \\
&\quad + \lambda_e \int_{z'\in\mathcal{B}(z)} (-J(w,z)) dP(z')
\end{aligned}
$$

- Value of match $V(z) = W(w,z) + J(w,z)$ between firm with productivity z and worker employed at wage w:

$$(r+\delta_w)V(z) = g(z) + \delta_f(U - V(z)) + \lambda_e \beta \int_{z' \in \mathcal{B}(z)} (V(z') - V(z))dP(z')$$

- Value of a match is independent of wages!
  - Re-negotiation within the firm doesn't change the total value of the match, only triggers its redistribution
- When $\lambda_e \beta = 0$, the equilibrium distribution of jobs from which workers sample, $P(z)$, does not impact the value of a match

- Value of a new worker hired from unemployment

$$S_u^f(z) = (1-\beta)(V(z) - U)$$

- Value of a new worker poached from another employer

$$S_e^f(z, z') = (1-\beta)(V(z') - V(z))$$

- Expected value of a new worker

$$S^f(z) = q_u S_u^f(z) + q_e \int_{z' \in \mathcal{C}(z)} S_e^f(z, z') dG(z')$$

where $G(z')$ is the distribution of employment across firms

- On-the-job search expands the rate for firms by a factor of

$$q_e/q_u = \frac{\lambda_e(1-u)}{\lambda_u u}$$

- Present discounted value of profits generated by all workers who are hired by a firm with productivity z

$$\pi(z) = \max_{s} \quad S^f(z)s - c(s)$$

where

  - s denotes search/recruiting effort exerted by the firm
  - $c(s)$ is cost of search, increasing and convex in s

- Optimal search effort: $s(z) = (c')^{-1}(S^f(z))$
- Number of workers arriving at a firm z (new hires)

$$h(z) = (q_u + q_e)s(z)$$

- Firms may speed up growth by increasing recruitment effort

- Discounted sum of per-period aggregate profits

$$\Pi(z) = \int_0^\infty \pi(z) e^{-(r+\delta_f)t} dt = \frac{\pi(z)}{r+\delta_f}$$

- Number of entrants $M^e \geq 0$
- Free entry condition

$$\Pi^e = \int_{\underline{z}}^{\bar{z}} \max\{0, \Pi(z)\} d\Gamma(z) \leq c_e$$

where $\Gamma(z)$ is a CDF for firm-level productivity $z \in [\underline{z}, \bar{z}]$

- Entry decision: $\mathbf{1}^e(z) = \begin{cases} 1 & \text{if} \quad \Pi(z) > 0 \\ 0 & \text{otherwise} \end{cases}$

- Evolution of firm size (conditional on not exiting)

$$N'(z) = \underbrace{h(z)}_{\text{new hired workers}} - \underbrace{N(z)[\delta_w + \lambda_e(1 - P(z))]}_{\text{separated workers}}$$

- Dynamics of unemployment

$$du = \delta_f(1 - u) + \delta_w - (\lambda_u + \delta_w)u$$

where in steady-state: $du = 0$

- Cumulative distribution of vacant jobs across employers

$$P(z^*) = \int_{\underline{z}}^{z^*} \frac{s(z)}{\bar{s}} d\Gamma(z)$$

where $\bar{s} = \int_{\underline{z}}^{\bar{z}} s(z) d\Gamma(z)$ and $P(\bar{z}) = 1$

- Cumulative distribution employment across employers

$$\begin{aligned}(1-u)dG(z) &= u\lambda_u P(z) + (1-u)\lambda_e P(z) G(z) \\ &\quad - (1-u)(\delta_w + \delta_f + \lambda_e(1 - P(z))G(z)\end{aligned}$$

where in steady-state: $dG(z) = 0$

A steady-state competitive equilibrium consists of a value function $V(z)$, contact rates λ_u, λ_e, unemployment rate u, measure of firms M, employment and sampling distributions, $G(z)$ and $P(z)$, s.t.:

- **Optimality**: the value of a match $V(z)$ attains its maximum
- **Free-entry**: $\Pi^e = c_e$
- **Stationarity**: employment and sampling distributions, $G(z)$ and $P(z)$, replicate themselves over time through firms' hiring decisions and workers' mobility decisions, and unemployment is equal to

$$u = \frac{\delta_f + \delta_w}{(\lambda_u + \delta_f + \delta_w)}$$

- **Labor market clearing**: λ_e and λ_u are consistent with workers flows in- and out-of-employment, job-to-job movements and firms' hiring decisions

Output loop

- **Step 1**: Make a guess for workers contact rates, λ_u^0 and λ_e^0, and the unemployment rate, u^0
- **Step 2**: Solve the **Inner loop** to obtain firm contact rate q_u^* and optimal search effort $s^*(z)$
- **Step 3**: Construct the average search effort $\bar{s}^*$ and use q_u^* to back out the equilibrium number of firms M^*, i.e.

$$M^* = \frac{\lambda_u^0 u^0}{\bar{s}^* q_u^*}$$

- **Step 4**: Obtain new guesses for workers contact rates, λ_u^1 and λ_e^1, and unemployment rate, u^1, using $\bar{S}$, M^*, the definition of matching function and the stationarity condition in the labor market
- **Step 5**: Iterate till convergence

Inner loop

- **Step 1**: Guess firms' contact rate with unemployed, q_u^0 and distribution of search effort, $P^0(z)$. Construct distribution of employment, $G^0(z)$
- **Step 2**: Solve for the value of the match $V(z)$ and value of unemployment U
- **Step 3**: Solve the problem of the firms and compute the optimal the search effort, $s^*(z)$
- **Step 4**: Construct discounted value of profits of new hires, $\pi(z)$
- **Step 5**: Jointly update the guesses till convergence:
  - **Step 5.1**: compute the value of entry, Π^e and check if free-entry condition is satisfied:
    - if no, make a new guess q_u^1 and go back to step 2
    - if yes, store $q_u^* = q_u^0$
  - **Step 5.2**: use the optimal the search effort $s(z)$ to update $P^1(z)$ and got back to step 2. Store $P^*(z)$ once converged.

- Given λ_e, changes in the contact rate from unemployment λ_u impact the employment allocation $n(z)$ only through general-equilibrium adjustment in U
- Suppose the value U is not an active margin:
  - pinned down by an outside sector
  - no bargaining power for worker, $\beta = 0$

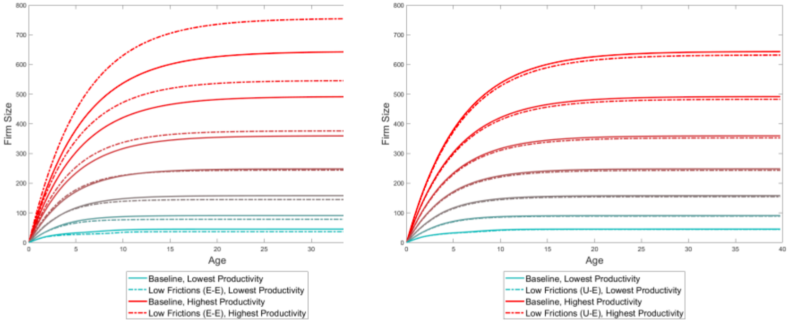
  then λ_u has no impact on firm growth: changes in λ_u don't affect q_u which is pinned down by the free-entry condition
- λ_u scales the size of the employment pool, but it does not alter its composition
  - easier to hire from employment (through a larger employment pool)
  - easier to hire from unemployment (through a higher job filling rate with the unemployed)

- Given λ_u, changes in the contact rate from employment λ_e impact the employment allocation $n(z)$ regardless U
- Because workers transit to high-value jobs, a higher rate of contact on the job, q_e, speeds up transitions
  - employment distribution becomes skewed towards more productive
- Firms' meeting rate with the employed not affected by free entry:

$$q_e/q_u = \frac{\lambda_e}{\lambda_u}\frac{(1-u)}{u} = \frac{\tilde{\lambda}_e}{\tilde{\lambda}_u}\frac{\frac{\lambda_u}{\lambda_u+\delta_f+\delta_w}}{\frac{\delta_f+\delta_w}{\lambda_u+\delta_f+\delta_w}} = \underbrace{\frac{\tilde{\lambda}_e}{\tilde{\lambda}_u}\lambda_u}_{\lambda_e}\frac{\frac{1}{\lambda_u+\delta_f+\delta_w}}{\frac{\delta_f+\delta_w}{\lambda_u+\delta_f+\delta_w}} = \frac{\lambda_e}{\delta_f+\delta_w}$$

Figure 2: Impact of 50% Reduction in Frictions on Firm Growth

(a) Firm Size by Age (Only E-E) (b) Firm Size by Age (Only U-E)



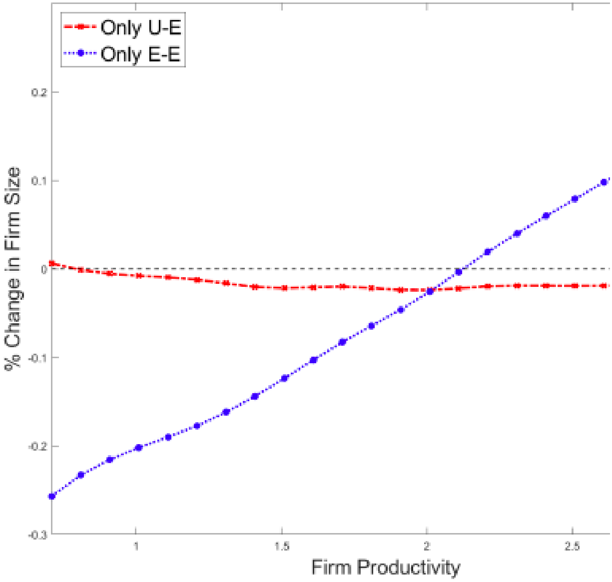
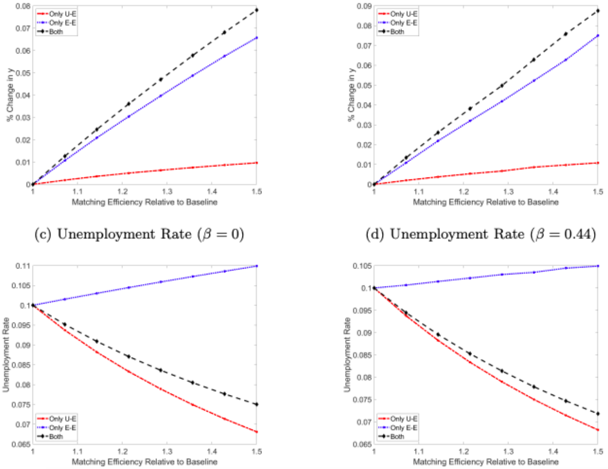

Only U-E
Only E-E
0.2
0.1
0
-0.1
-0.2
-0.3
% Change in Firm Size
1
1.5
2
2.5
Firm Productivity

Figure 1: Impact of Lowering Frictions



(a) Income Per Employed Worker ($\beta = 0$)

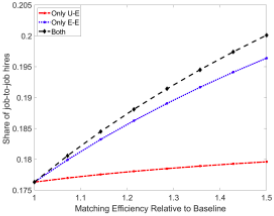
(b) Income Per Employed Worker ($\beta = 0.44$)

(c) Unemployment Rate ($\beta = 0$)

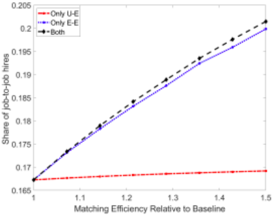
(d) Unemployment Rate ($\beta = 0.44$)

Implication: Aggregates

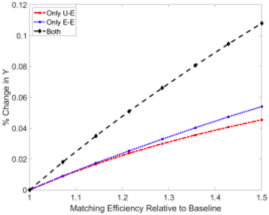
(e) Share of Job-to-job hires ($\beta = 0$)



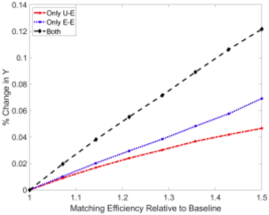
(f) Share of Job-to-job hires ($\beta = 0.44$)



(g) Income Per Capita ($\beta = 0$)



(h) Income Per Capita ($\beta = 0.44$)



Extension: Ex-ante heterogeneous workers

Production

- Workers heterogeneous in innate skills $h(i)$
- Firm-level production technology

$$y = \int_0^{\ell} g(i, z) dG(i|z)$$

where $G(i|z)$ is the CDF of worker i within firm z with size ℓ

- Firm-worker match production:

$$g(i, z) = Azh(i)$$

- Technology is a linear function of total workforce and average human capital:

$$y = Az\bar{h}\ell$$

where $h(i) = \int_0^1 h(i) dG(i|z)$

- Value of unemployment for a worker i

$$(r+\delta_w)U(i) = b + \beta\lambda_u \int_z \max\{0, V(i,z) - U(i)\} dP(z)$$

- Value of match $V(i,z) = W(w,i,z) + J(w,i,z)$ between firm with productivity z and worker i employed at wage w:

$$\begin{aligned}(r+\delta_w)V(i,z) = {} & g(i,z) + \delta_f(U(i) - V(i,z)) \\ & + \lambda_e\beta \int_{z'\in\mathcal{B}(i,z)} (V(i,z') - V(i,z)) dP(z')\end{aligned}$$

Value of the new hire

- Value of a new worker i hired from unemployment

$$S_u^f(i,z) = (1-\beta)(V(i,z) - U(i))$$

- Value of a new worker i poached from another employer z'

$$S_e^f(i,z,z') = (1-\beta)(V(i,z) - V(i,z'))$$

- Expected value of a new worker

$$S^f(z) = q_u \int_i S_u^f(i,z) dH(i) + q_e \int_{i,z' \in \mathcal{C}(z)} S_e^f(i,z,z') dG(i,z')$$

where

- $G(i,z')$ is the distribution of employed across states
- $H(i)$ is the distribution of unemployed across states

Outer loop: As before

Inner loop

- **Step 1**: Guess firms' contact rate with unemployed, q_u^0 and distribution of search effort, $P^0(z)$.
- **Step 2.1**: Solve for the value of the match $V(i, z)$ and value of unemployment $U(i)$
- **Step 2.2**: Simulate the economy for a large number of workers to construct $G(i, z)$ and $H(i)$
- **Step 3**: As before
- **Step 4**: As before
- **Step 5**: As before

Extension: Dynamic Investment Decisions

- Production linear function of total workforce

$$y = Az\ell$$

- Two technologies available: $A_1 > A_0$
  - per-period sunk cost c_x of accessing more productive technology
- Firms values changes along its life-cycle. It depends on:
  - productivity z
  - time a firm plans to wait before adopting a technology
  - whether firm has adopted a new technology or not
- Match-specific state variables:
  - productivity, z
  - time of investment, h
  - age, a

- Value of match between a worker and a firm with productivity z, age a and investing after $h(z)$ periods from entry:

$$(r+\delta_w)V(z,h,a) = A_0 z + \delta_f(U - V(z,h,a)) + \lambda_e\beta\int_{z',h',a'\in\mathcal{B}(z,h,a)}(V(z',h',a') - V(z,h,a))dP(z',h',a') + \frac{\partial V(z,h,a)}{\partial a}$$

- Value of match between a worker and a firm with productivity z, age a that already invested at $h(z)$:

$$(r+\delta_w)V(z,h) = A_1 z + \delta_f(U - V(z,h)) + \lambda_e\beta\int_{z',h',a'\in\mathcal{B}(z,h)}(V(z',h',a') - V(z,h))dP(z',h',a')$$

- Discounted sum of per-period aggregate profits

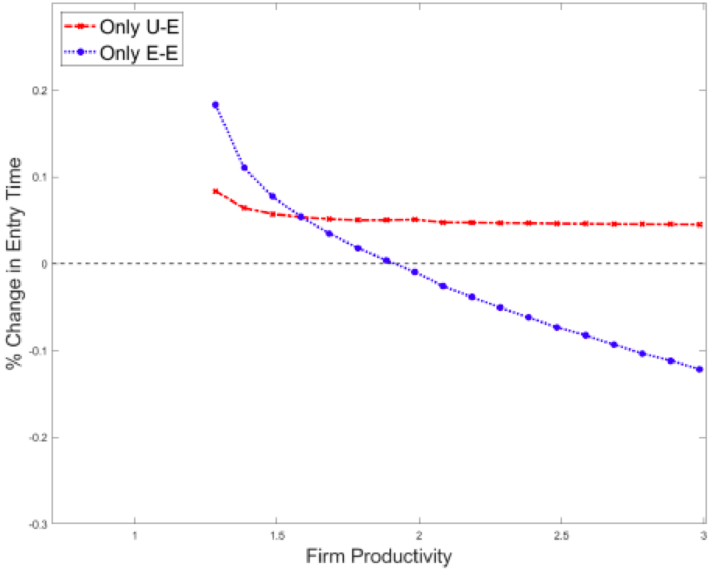
$$\Pi(z) = \max_{h \geq 0} \quad \int_0^h \pi(z,h,a) e^{-(r+\delta_f)a} da + e^{-(r+\delta_f)h} \left[\frac{\pi(z,h) - c_x}{r + \delta_f} \right]$$

- Optimality condition:

$$\underbrace{\int_0^{h^*} e^{-(r+\delta_f)x} \frac{\partial \pi(z,h^*,x)}{\partial V(z,h^*,x)} \frac{\partial V(z,h^*,x)}{\partial x} dx}_{\text{opportunity cost of delaying investment}} \leq \underbrace{c_x}_{\substack{\text{savings from} \\ \text{delaying investment}}}$$

with equality if h^* is finite

- Notice that $h^* > 0$ since $c_x > 0$.


Only U-E
Only E-E
0.2
0.1
0
-0.1
-0.2
-0.3
% Change in Entry Time
1
1.5
2
2.5
3
Firm Productivity

- Elsby M. and Gottfries A. 2021. “Firm dynamics, on-the-job search and labor market fluctuations.” Review of Economic Studies, forthcoming.
- Bilal A.G., Engbom N., Mongey, S. and Violante, G.L. 2019. “Firm and worker dynamics in a frictional labor market.” NBER working paper No. 26547.