Advanced Macroeconomics

Frictions

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Labor adjustment costs

- Hopenhayn H. and Rogerson R. 1993. "Job Turnover and Policy Evaluation: A General Equilibrium Analysis". Journal of Political Economy. Vol.101, N.5, pp. 915-938
  - General equilibrium version of Hopenhayn (1992)
  - Non-convex adjustment cost (firing costs) $\Longrightarrow$ firm-level employment additional state variable
  - No aggregate shocks
  - Optimal employment policy characterized by *inaction* region
  - Endogenous *Misallocation* across heterogeneous plants

- Large volume of job creation/destruction at firm-level
- Policies that make more costly to adjust employment level, i.e.
  - legislated severance payments
  - advance notice
  - plant closing legislation
- What are the effects of such policies on
  - employment
  - aggregate output
  - productivity
- Can labor market regulations explain heterogeneity in labor market performance across countries?

- Time is discrete
- Wage is the model numeraire ($w = 1$)
- Output price p endogenous
- Representative household
  - Consumption/labor supply decision
  - No savings
- Endogenous measure of heterogeneous firms
  - Perfect competition in product and labor markets
  - Time-varying productivity
  - Entry-exit dynamics
  - Employment adjustment costs (firing tax)
  - Free-entry

- Utility function of consumption C and labor supply N

$$U(C, N) = \log C - AN$$

where A denotes disutility from supplying labor.

- Discount factor: $\beta = 1/1 + r$
- Budget constraint:

$$pC \leq N + \Pi \quad (w = 1 \quad \text{is the numeraire})$$

where Π are aggregate profits, re-distributed to HH lump-sum

- Problem of the HH:

$$U(C,N) = \max_{C,N} \quad \log C - AN$$
$$\text{s.t.} \quad pC \leq N + \Pi$$

- First order conditions (at the interior) imply:

$$C = \frac{1}{Ap} \quad \textit{(:consumption demand)}$$
$$N^s = \frac{1}{A} - \Pi \quad \textit{(:labor supply)}$$

- Firms differ in productivity z and employment n
- Firm-level output:

$$f(z, n) = zn^{\alpha} \quad \alpha \in (0, 1)$$

with $f'_n > 0, f''_{nn} < 0$

- Static firm-level profits

$$\pi(z, n, n_{-1}) = pf(z, n) - n - pc_o - g(n, n_{-1})$$

where c_o denotes per-period operating costs.

- Adjustment costs (expressed in units of labor)

$$g(n, n_{-1}) = \tau \max\{0, n_{-1} - n\}$$

- Incumbents enter the period with states (z_{-1}, n_{-1})
- Exit decision:
  - if exit, firms pay $g(0, n_{-1})$
  - if stay, firms draw new productivity level $z \sim \Gamma(z|z_{-1})$
- Employment decision (conditional on staying)
  - choose new employment level n conditional on (z, n_{-1})
    - expanding firms $(n > n_{-1})$ subject to no cost
    - shrinking firms $(n < n_{-1})$ subject to firing costs
  - pay operating costs pc_o and produce $f(z, n)$

- $V(z, n_{-1}; p)$: value function for firm in states (z, n_{-1}) and aggregate price p

$$V(z, n_{-1}; p) = \max_{n \geq 0} \quad \pi(z, n, n_{-1}) + \frac{1}{1+r}\tilde{v}(z, n)$$

where

$$\tilde{v}(z, n) = \max \left\{ -g(0, n), \sum_{z'} V(z', n; p)\Gamma(z'|z) \right\}$$

- Solution to this problem:
  - Policy function for optimal employment policy: $n = g_n(z, n_{-1}; p)$
  - Policy policy for optimal exit: $\mathbf{1}^x(z, n_{-1}; p)$

- Potential entrants are ex-ante identical
- New entrants $M \geq 0$ pay c_e and enter
- Draw productivity level z from $\Gamma^e(z)$ (ergodic distribution obtained from $\Gamma(z|z_{-1})$)
- Hire n workers and produce
- Free entry condition:

$$v^e(p) = \frac{1}{1+r} \sum_z V(z, 0; p) \Gamma^e(z) \leq c_e$$

with equality if $M > 0$.

- Let $\mu(z, n; p)$ be the measure of firms over individual states z and n when the goods price is p
- Solution of the following linear system:

$$\mu(z', n'; p) = T(\mu(z, n; p), M, p)$$

where

$$T(\mu(z, n; p), M, p) = \sum_z \int_n \psi(z', n'|z, n; p) d\mu(z, n; p) + M\Gamma^e(z)\mathbf{1}[g_n(0, z; p) = n']$$

and

$$\psi(z', n'|z, n; p) = \mathbf{1}[g_n(n, z'; p) = n']\Gamma(z'|z)[\mathbf{1}^x(z, n) = 0]$$

denotes the transition function from the states (z, n) to z', n'

- Aggregate output:

$$Y = \sum_z \int_{n_{-1}} [f(z, g_n(z, n_{-1}) - c_f] d\mu(z, n_{-1}; p)$$
$$+ M \sum_z f(z, g_n(z, 0)\Gamma^e(z)$$

- Labor demand:

$$N^d = \sum_z \int_{n_{-1}} g_n(z, n_{-1}) d\mu(z, n_{-1}; p)$$
$$+ M \sum_z g_n(z, 0)\Gamma^e(z) + Mc^e$$

- Aggregate firing tax:

$$R = \sum_z \int_{n_{-1}} r(z, n_{-1}) d\mu(z, n_{-1}; p) +$$

where

$$\begin{aligned} r(z, n_{-1}) &= \mathbf{1}^x(z, n) g(0, n_{-1}) \\ &\quad + (1 - \mathbf{1}^x(z, n)) \sum_{z'} g(g_n(n, z'; p), n_{-1}) \Gamma(z'|z) \end{aligned}$$

- Aggregate profits: $\Pi = pY - N^d - pc_o - R$

A recursive stationary equilibrium for this economy is characterized by a measure of entrants M^*, a distribution of incumbent firms $\mu^*(z, n; p)$, and a price p^* such that the following three conditions hold:

- **Free-entry** $v^e = c_e$;
- **Labor market clearing**: $N^s = N^d$;
- **Aggregate consistency**: $\mu^*(z, n; p^*) = T(\mu^*(z, n; p^*), M^*, p^*)$.

Employment policy function without adjustment costs

- MPL is equalized across firms:

$$\frac{\partial f(z,n)}{\partial n} = \frac{1}{p}$$

- Optimal employment decision given by:

$$n' = (\alpha p z)^{\frac{1}{1-\alpha}}$$

- Labor is a fully flexible input of production

Employment policy function with adjustment costs

- MPL is the solution of two *necessary* conditions:

$$p\frac{\partial f(z,n)}{\partial n} + \frac{1}{1+r}\frac{\partial \tilde{v}(z,n)}{\partial n} = 1 \quad \text{if} \quad n > n_{-1}$$

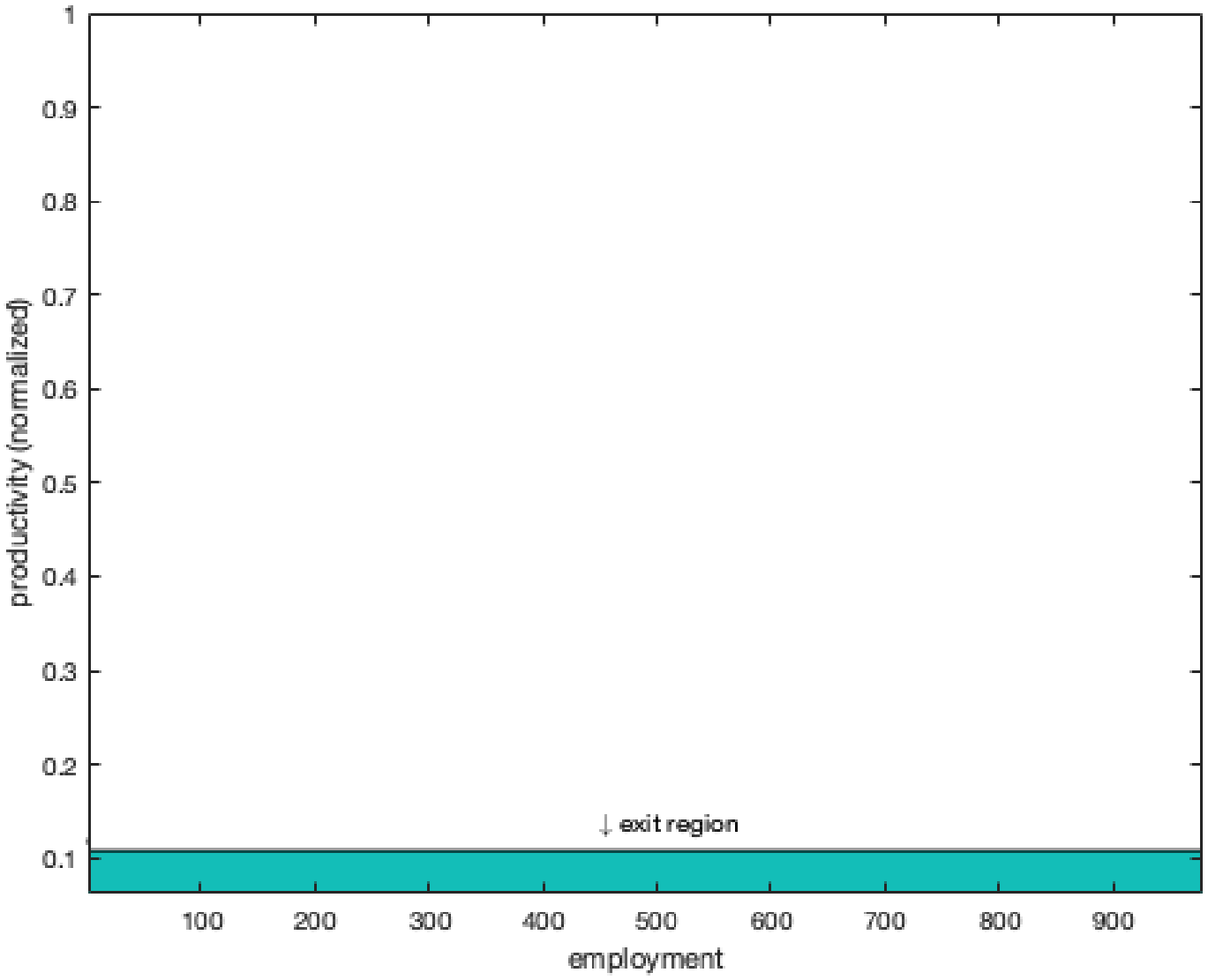
$$p\frac{\partial f(z,n)}{\partial n} + \frac{1}{1+r}\frac{\partial \tilde{v}(z,n)}{\partial n} = \left[1 + \underbrace{\frac{\partial g(n,n_{-1})}{\partial n}}_{\tau}\right] \quad \text{if} \quad n < n_{-1}$$

- Employment decision characterized by two reservation thresholds, $z_F(n)$ and $z_H(n)$, such that:

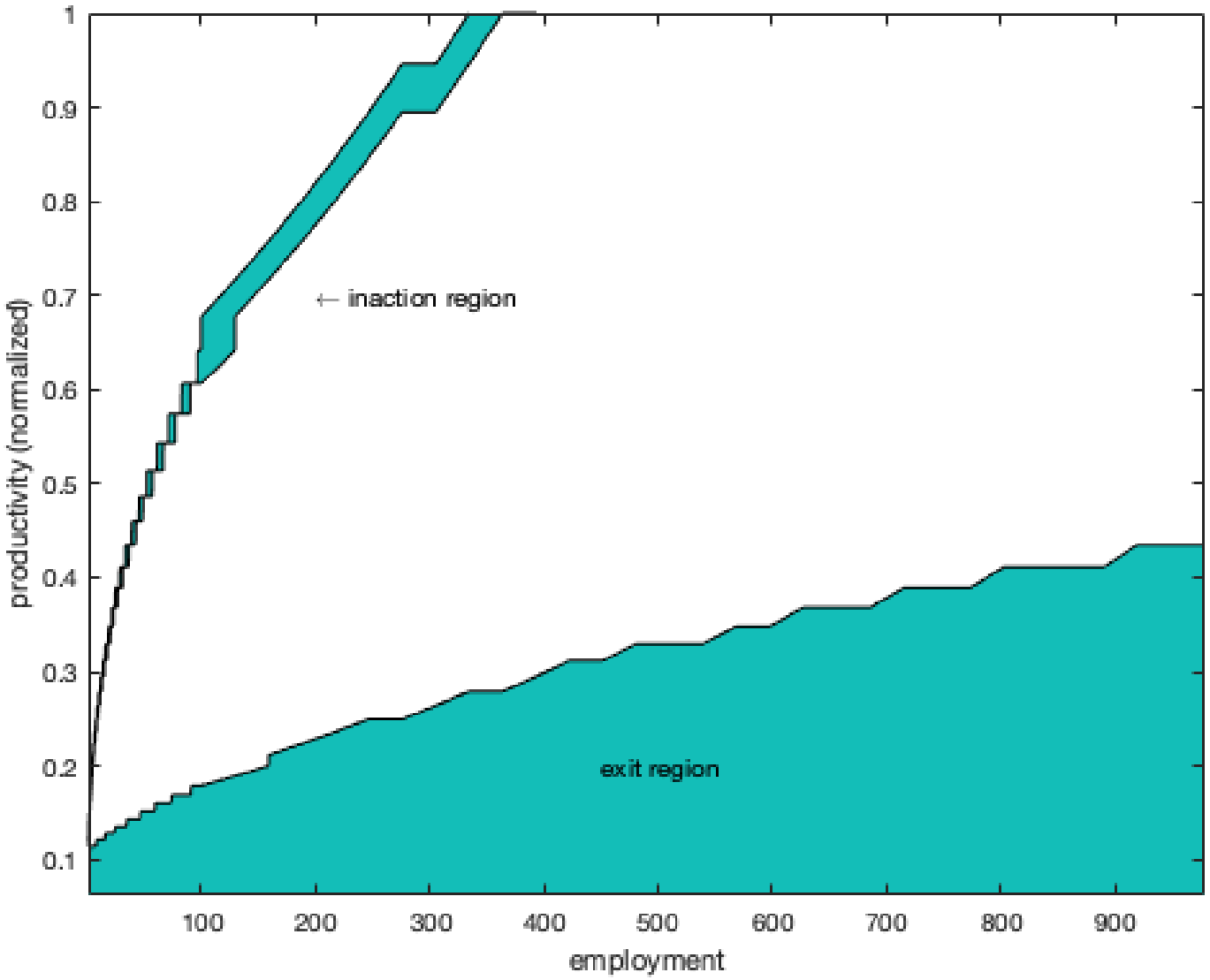
$$\begin{aligned} n' = n_F(z) \quad &\text{if} \quad z < z_F(n) \\ n' = n_{-1} \quad &\text{if} \quad z \in [z_F(n), z_H(n)] \\ n' = n_H(z) \quad &\text{if} \quad z > z_H(n) \end{aligned}$$

- Inaction region wider with higher τ

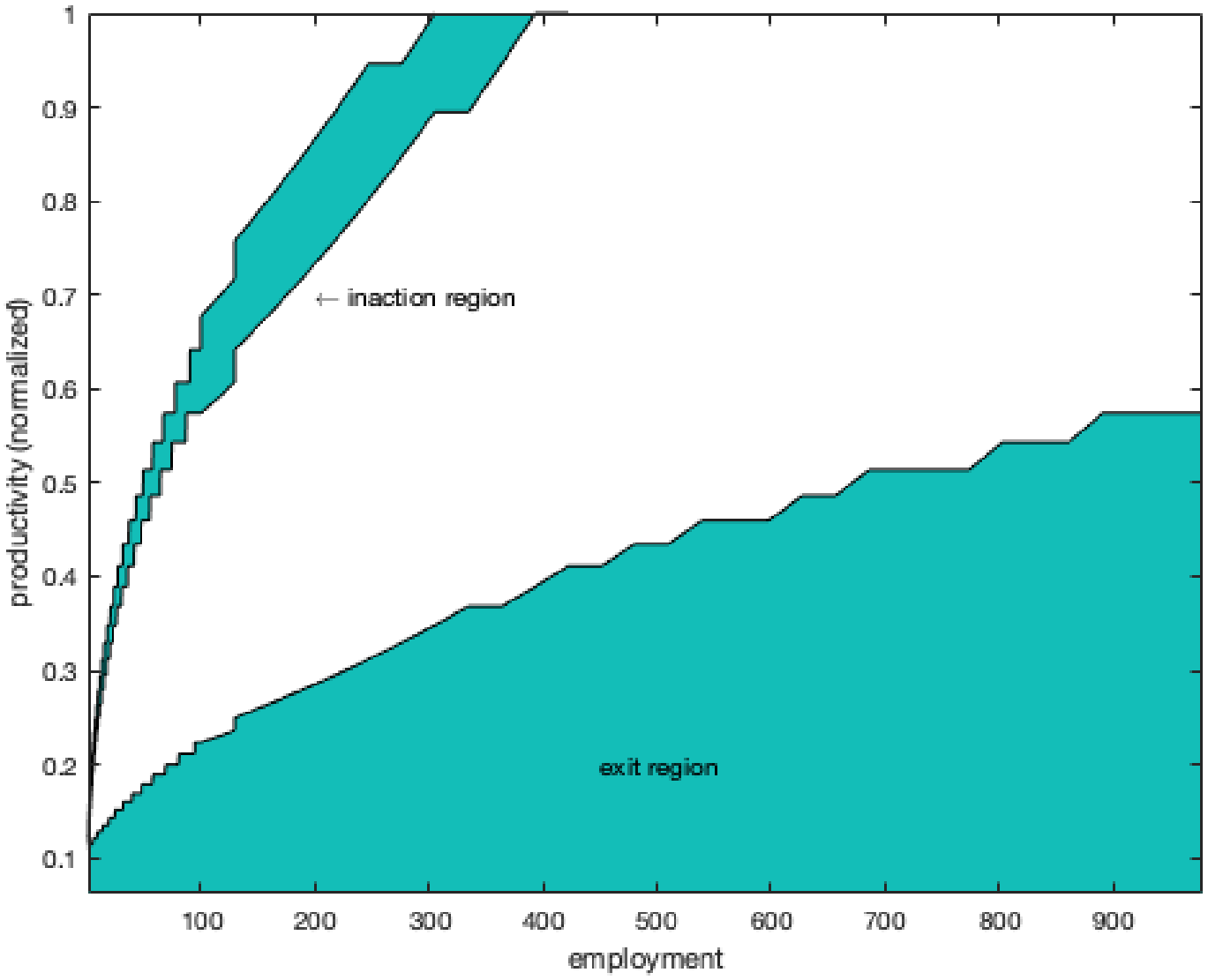
Inaction regions: $\tau = 0$



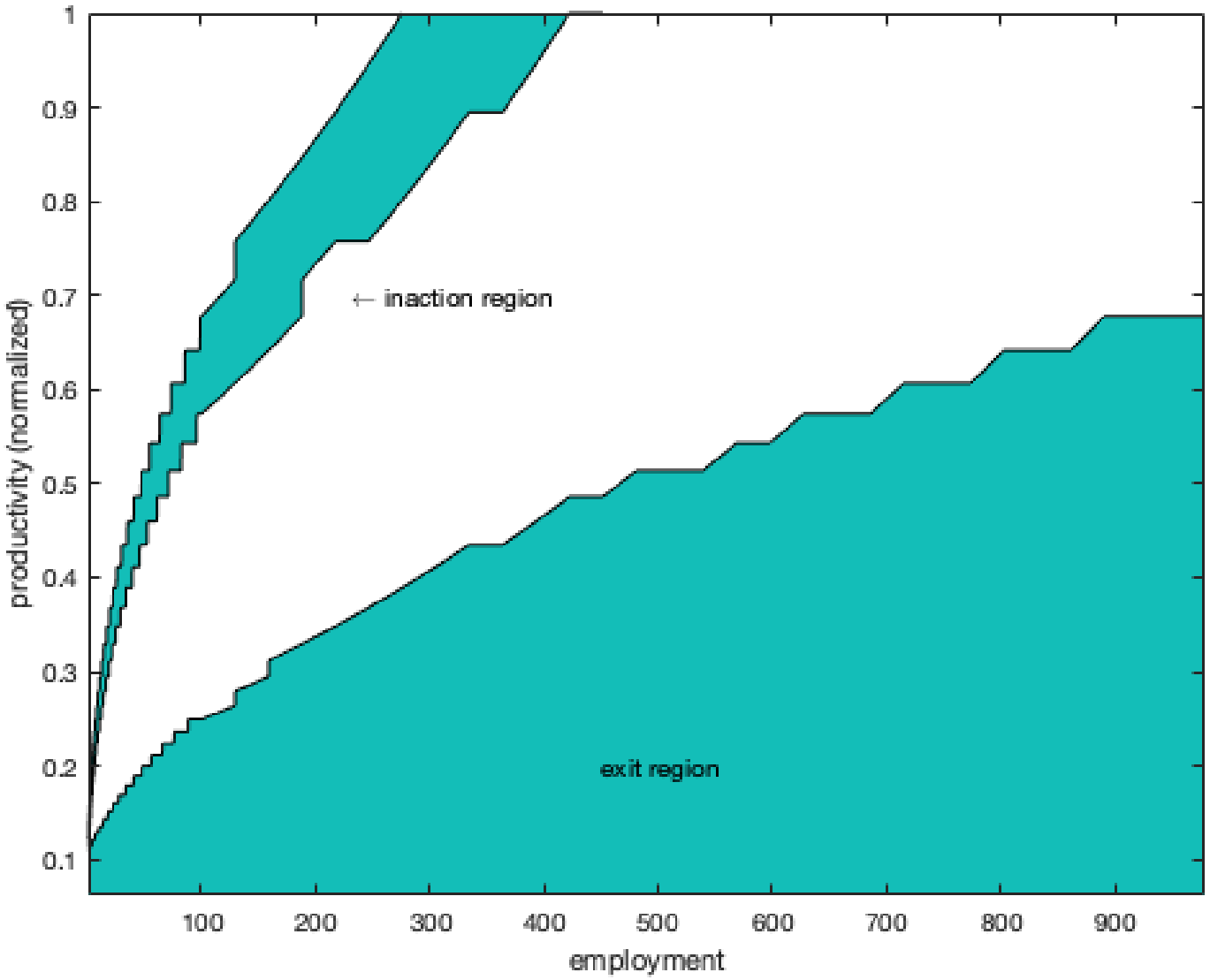
Inaction regions: $\tau = 0.1$



Inaction regions: $\tau = 0.2$



Inaction regions: $\tau = 0.3$



Productivity state-space:

- AR(1) process in logs:

$$\log z' = \mu + \rho \log z' + \epsilon' \quad \epsilon' \sim \mathcal{N}(0, \sigma^2)$$

- Markov chain approximation using Tauchen method on 51 nodes

Employment state-space:

- maximum number of employees: 3000
- 500 points (400 points between 1 and 200 employees)

Baseline parameters (period model = 5 years)

$$r = 0.25, \alpha = 0.64, \mu = 0.25, \rho = 0.93, \sigma = 0.17$$
$$c_o = 20, c_e = 40, A = 0.45, \tau = 0$$

Marginal product of labor: $\frac{\partial f(z,n)}{\partial n}$

	Dispersion	Percentiles			
Firing cost: τ	St.Dev.	20th	40th	60th	80th
0	0	1	1	1	1
0.1	0.0439	0.9485	0.9546	1.0171	1.0349
0.2	0.0720	0.9097	0.9349	1.0156	1.0568
0.3	0.0911	0.8735	0.9199	1.0134	1.0715

Firing costs increase the dispersion of the MPL

Counterfactual outcomes

Firing cost: τ	0	0.1	0.2	0.3
Price	1	1.0085	1.0145	1.0193
Consumption (output)	1	0.9915	0.9856	0.9810
Employment	0.6000	0.5988	0.5986	0.5988
Profits	1	1.1561	1.2758	1.4127
# firms	1	0.9935	0.9879	0.9806
Firm size	20.055	20.333	20.655	21.152
Labor productivity	1	0.9914	0.9818	0.9718
Job creation rate	0.1879	0.1403	0.1108	0.0953

Firing costs reduce aggregate productivity and aggregate output

Role of volatility σ_ϵ

- When shocks are less volatile, efficient employment does not change often
  - Adjustment costs are more important
- When shocks are more volatile, efficient employment changes much more often
  - Adjustment costs are less important
- While reducing dispersion of MPL, lower volatility also reduces selection

Firing costs: τ	0	0.1	0.1
Volatility: σ_ϵ	0.17	0.17	0.10
Price	1	1.0085	1.2309
Consumption (output)	1	0.9915	0.8124
Employment	0.6000	0.5988	0.6112
Profits	1	1.1561	0.8450
# firms	1	0.9935	1.0457
Firm size	20.055	20.333	18.501
Labor productivity	1	0.9914	0.9926
Job creation rate	0.1879	0.1403	0.0701
MPL, st.dev.	0	0.0439	0.0361

Lower volatility reduces job turnover and selection

- **Step 1**: guess a price p_0
- **Step 2**: solve for the value of the incumbent, $v(z, n; p^0)$
- **Step 3**: compute the value of entry, v^e and check if free-entry condition is satisfied:
  - if no, make a new guess p_1 and go back to step 2 till convergence
  - if yes, store $p^* = p_0$
- **Step 4**: given p^*, solve for the stationary distribution of incumbents $\mu(z, n; p^*)$ associated with $M = 1$
  - Exploit linear homogeneity of T

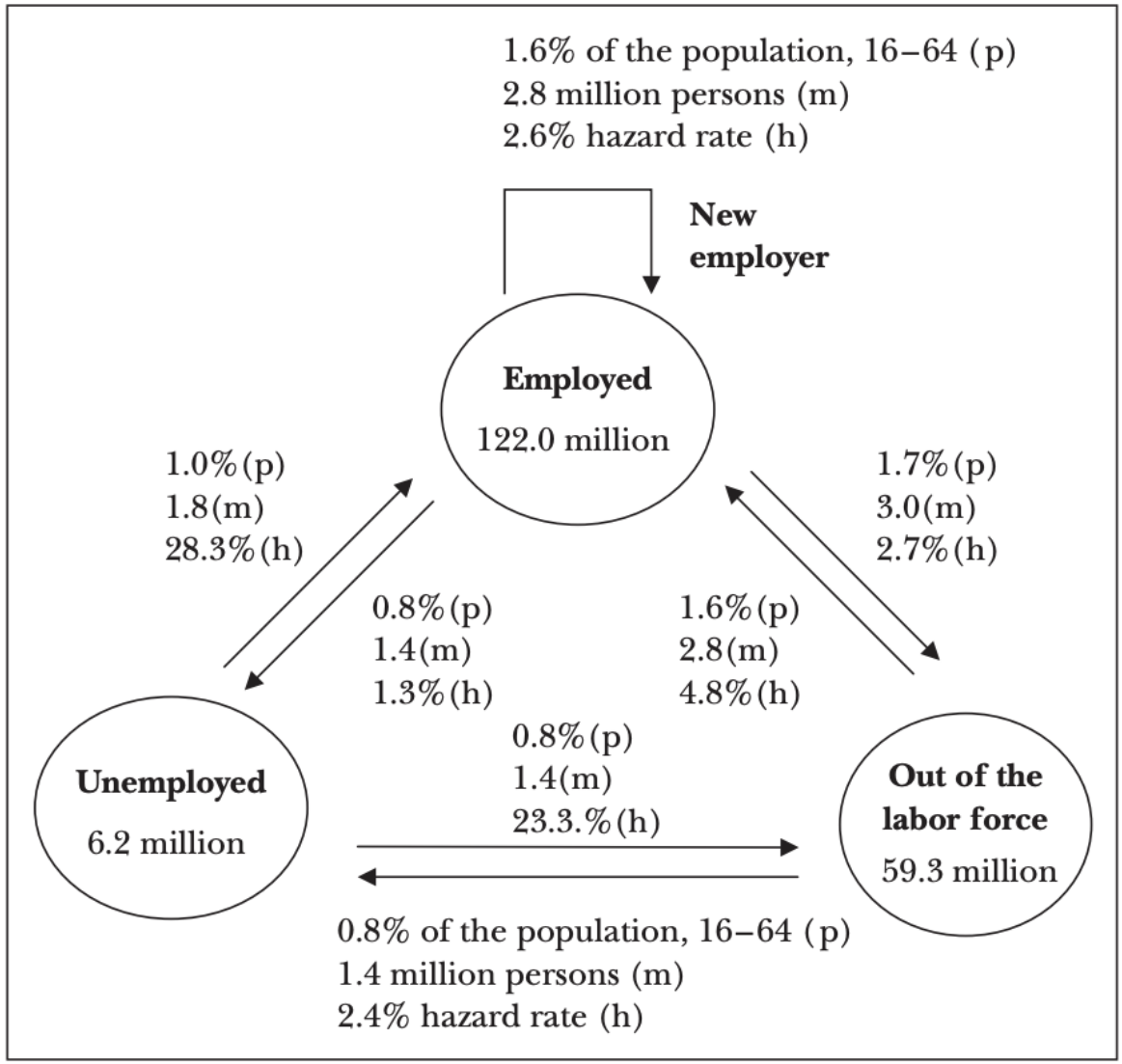
$$M\mu^*(z, n; p^*) = MT(\mu^*(z, n; p^*), M, Mp^*)$$

- **Step 5**: given $\mu^*(z, n; p^*)$, find M^* that makes the labor market clear

Search frictions

Figure 1

Average Monthly Worker Flows, Current Population Survey, 1996–2003



Source: Fallick and Fleischman (2004).

Figure 5

Monthly Unemployment Inflows and Outflows, 1976–2005

Notes: The figure depicts three-month centered moving averages of estimated gross flows of persons into and out of unemployment based on Current Population Survey (CPS) data. Shaded areas show NBER-dated recessions.

- The net change in employment between two periods satisfies the following accounting identity:

$$\Delta\text{Net-Employment} = \underbrace{\text{JC}_t - \text{JD}_t}_{\text{Job flows}} = \underbrace{\text{Hires}_t - \text{Separation}_t}_{\text{Worker flows}}$$

where

$$\text{Separation}_t = \text{Quits}_t + \text{Layoffs}_t$$

- a single employer can either create *or* destroy jobs during a period, but it can *simultaneously* have positive hires and separations

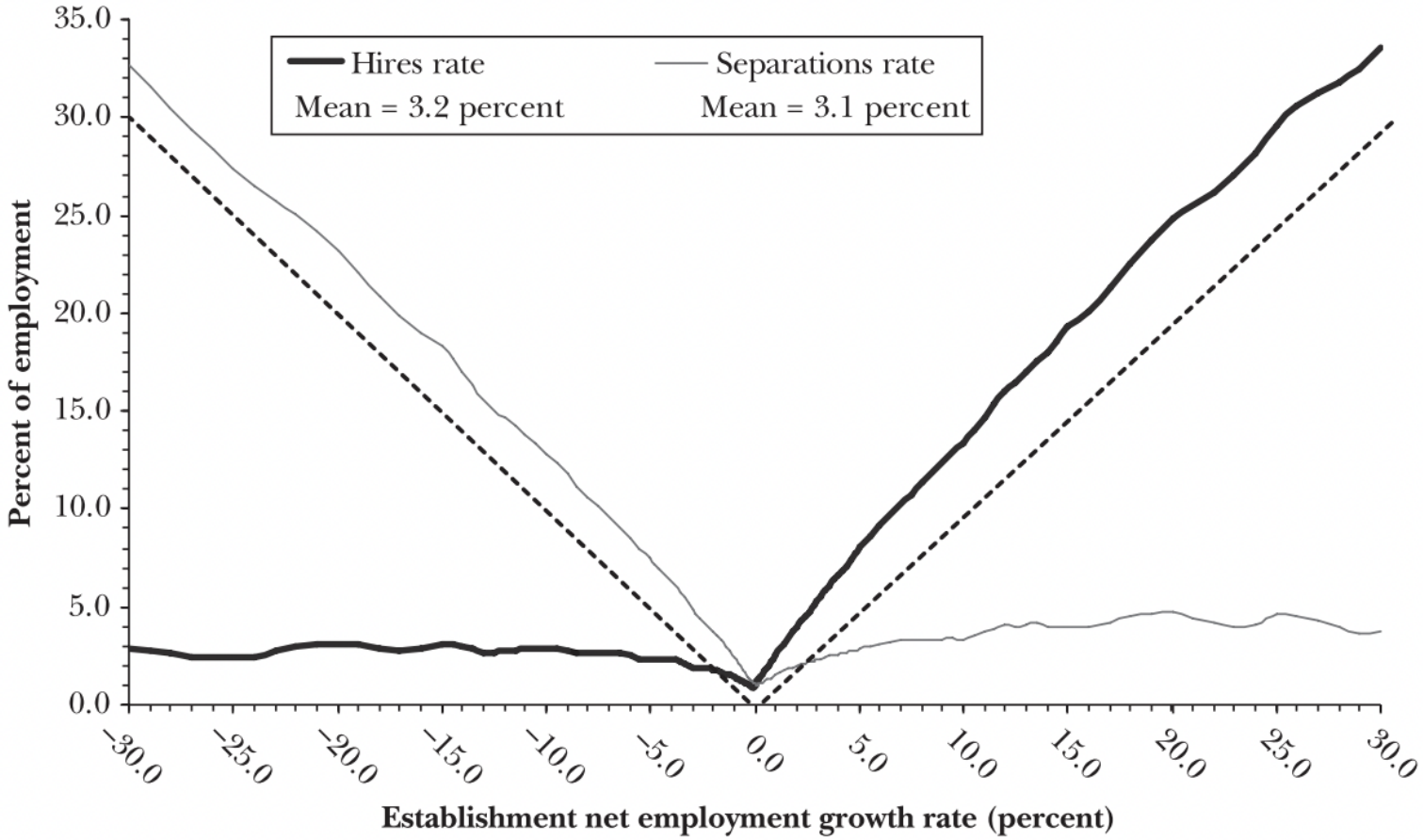
Figure 4

Monthly Worker Flow Rates, December 2000 to March 2005

Source: Published data from the BLS Job Openings and Labor Turnover Survey (JOLTS).
Note: Data are seasonally adjusted.

Figure 6

The Relationship of Hires and Separations to Establishment Growth



Notes: The curves are fitted values from nonparametric regressions of establishment-level hires and separations rates (vertical axis) on establishment-level employment growth rates (horizontal axis). The curves are fitted to monthly establishment-level JOLTS data pooled over the period from December 2000 to January 2005.

- Elsby M. and Michaels R. 2013. "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows." American Economic Journal: Macro, Vol. 5, No. 1, pp. 1-48
  - Multi-worker heterogeneous firms $\Longrightarrow$ well-defined firm-size
  - Idiosyncratic productivity shocks $\Longrightarrow$ endogenous job destruction
  - Search frictions $\Longrightarrow$ unemployment
  - Wage bargaining $\Longrightarrow$ wage dispersion

- Time is discrete
- Exogenous measure of potential workers L
  - infinitely lived
  - risk neutral
  - homogeneous
  - employed/unemployed
- Unitary measure of producers
  - heterogeneous in productivity
    - fixed productivity level, s
    - time-varying productivity level, z
  - no entry/exit dynamics
  - job creation/destruction
  - search frictions and wage bargaining
  - no aggregate shocks

- Decreasing return to scale firm-level production function

$$y = zF(n)$$

where $F'(n) > 0$ and $F''(n) < 0$

  - the marginal product of labor declines with firm employment, and generates a downward-sloped demand for labor at the firm level

- Time invariant productivity $s \sim \mathcal{H}(s)$
- Idiosyncratic productivity z assumed to follow a markov chain $z' \sim \Gamma(z'|z)$

- Labor market is subject to search and matching functions
  - Workers need to search in order to find jobs
  - Firms need to post vacancies in order to attract workers
- Only unemployed workers can search - no on-the-job search
- The matching process between vacancies V and unemployed workers U is governed by a CRS matching function $m(U, V)$:
  - increasing in both arguments

$$\frac{\partial m(U,V)}{\partial U} > 0 \quad \frac{\partial m(U,V)}{\partial V} > 0$$

  - concave in both arguments

$$\frac{\partial \partial m(U,V)}{\partial U^2} < 0 \quad \frac{\partial \partial m(U,V)}{\partial V^2} < 0$$

  - homogeneous of degree one in both arguments

- Cobb-Douglass function:

$$m(U,V) = m_0 U^{\alpha} V^{1-\alpha} \quad m_0 > 0, \alpha \in [0,1]$$

where

  - m_0 stands for matching efficiency
  - α stands for the elasticity of the matching function with respect to unemployment

- Other functional form (Den Haan et al., 2001)

$$m(U,V) = \frac{UV}{(U^{\eta} + V^{\eta})^{\frac{1}{\eta}}} \quad \eta > 0$$

where the elasticity of matching function equal to:

$$\frac{\partial m(U,V)}{\partial U} \frac{U}{m(U,V)} = \left(1 + \left(\frac{V}{U}\right)^{\eta}\right)^{-\frac{1}{\eta}}$$

- Labor market tightness: $\theta = V/U$
- Search frictions in the labor market limit the rate at which unemployed workers and hiring firms can meet:
  - Job finding probability for workers:

$$\phi_w = \frac{m(U,V)}{U} = m\left(1, \frac{V}{U}\right) = m_0\theta^{1-\alpha}$$

  - Vacancies posted by firms are filled with probability:

$$\phi_f = \frac{m(U,V)}{V} = m\left(\frac{U}{V}, 1\right) = m_0\theta^{-\alpha}$$

- Market tightness sufficient statistics for the job filling and job finding probabilities in the model

- $\Pi(s, z, n_{-1})$ denotes the value of a firm entering the period with productivity (s, z) and employment n_{-1}

- Firms choose how many vacancies v to post and current stock of employees, n, by solving the following problem

$$\Pi(s, z, n_{-1}) = \max_{n,v} \quad szF(n) - w(s, z, n)n - c_v v + \frac{1}{1+r}\tilde{\Pi}(s, z, n_{-1})$$

$$\text{and} \quad \tilde{\Pi}(s, z, n) = \sum_{z'} \Pi(s, z', n)\Gamma(z'|z)$$

where

- $w(s, z, n)$ is the wage bargained by the firm with its workers
- c_v is the cost of posting a vacancy

- Firms seek a level of employment that maximizes profits subject to a dynamic constraint on the evolution of firm's employment
- Firms face frictions that limit the rate at which vacancies may be filled.
- Since vacancy posted in a given period will be filled with probability $\phi_f < 1$ prior to production, then:

$$\underbrace{n}_{\substack{\text{new stock}\\ \text{of employees}}} = \underbrace{\phi_f v}_{\text{new hires}} + \underbrace{n_{-1}}_{\substack{\text{old stock}\\ \text{of employees}}} \qquad \text{if} \quad \mathbf{1}^{+}[n > n_{-1}] = 1$$

- Notice:
  - No downward adjustment costs (e.g. firing costs) for firm-initiated separation
  - No worker-initiated separation

- Using the law of motion for firm-level employment, the problem of the firms becomes:

$$\Pi(s,z,n_{-1}) = \max_{n} \quad szF(n) - w(s,z,n)n - \frac{c_v}{\phi_f} \times \max\{0, n - n_{-1}\}$$
$$+ \frac{1}{1+r}\sum_{z'} \Pi(s,z',n)\Gamma(z'|z)$$

where $\frac{c_v}{\phi_f}$ is the (endogenous) cost of scaling employment up.

- Recall firm problem in Hopenhayn and Rogerson (1993)

$$\Pi(s,z,n_{-1}) = \max_{n} \quad szF(n) - wn - \tau \times \max\{0, n_{-1} - n\}$$
$$+ \frac{1}{1+r}\sum_{z'} \Pi(s,z',n)\Gamma(z'|z)$$

- Necessary (not sufficient) condition for a solution to the firm problem, conditional on $\Delta n \neq 0$

$$sz\frac{\partial F(n)}{\partial n} + \frac{1}{1+r}\sum_{z'}\frac{\partial \Pi(s,z',n)}{\partial n}\Gamma(z'|z) = \frac{\partial w(s,z,n)n}{\partial n} + \frac{c_v}{\phi_f}\mathbf{1}^+$$

- There is a kink in the value function around $n = n_{-1}$
  - partial irreversibility of separation decisions in the model
  - while separation is costless, it is costly to reverse such a decision because of hiring (posting vacancies) costs
  - as in Bentolila and Bertola (1990) but hiring costs endogenous

- Optimal employment policy characterized by two reservation values for firm's productivity z, $z_L(s, n_{-1})$ and $z_H(s, n_{-1})$ such that $\forall z \in (z_L(s, n_{-1}), z_H(s, n_{-1})) \implies n = n_{-1}$ (employment inaction region)

- Value of worker currently employed in a firm with productivity (s, z) and n_{-1} employees

$$J^e(s, z, n_{-1}) = w(s, z, n(s, z, n_{-1})) + \beta \tilde{J}^e(s, z, n)$$

where

$$\tilde{J}^e(s, z, n) = \sum_{z'} \underbrace{p^f(s, z', n(s, z, n_{-1}))}_{\substack{\text{endogenous} \\ \text{firing probability}}} \Gamma(z'|z) J^u +$$

$$+ \sum_{z'} (1 - p^f(s, z', n(s, z, n_{-1}))) J^e(s, z', n) \Gamma(z'|z)$$

The problem of the unemployed workers

- Value of an unemployed worker

$$J^u = b + \beta \left((1-\phi_w) J^u + \phi_w \sum_{s,z'} \int_n J^e(s,z',n) d \underbrace{\psi(s,z',n)}_{\substack{\text{endogenous} \\ \text{distribution of} \\ \text{hiring firms}}} \right)$$

or equivalently

$$J^u = \frac{b}{1-\beta} + \frac{\beta}{1-\beta} \left[\phi_w \sum_{s,z'} \int_n \underbrace{[J^e(s,z',n) - J^u]}_{\text{gain from being hired}} d\psi(s,z',n) \right]$$

- Frictions in the labor market implies makes costly for firms and workers to find alternative employment relationships.
- Quasi-rents that firm and its workers can bargain over.
- Standard search model with constant marginal product (without large firms):
  - the rents of each employment relationship are independent of the rents of all other employment relationships
  - firms can bargain with each of their workers independently
- Search model with decreasing marginal product:
  - the rents of each individual employment relationship depend on the number of workers employed!
  - the rent from "the" marginal worker lower than the rent from all infra-marginal hires due to diminishing marginal product

- Intra-firm bargaining protocol a là Stole and Zwiebel (1994)
  - generalization of the Nash solution to a setting with diminishing returns
  - Nash bargaining over the marginal surplus of firm-worker relationship
- Intuitions:
  - If the firm has only one worker, the firm and worker simply strike a Nash bargain.
  - If a second worker is added, the firm and the additional worker know that, if their negotiations break down, the firm will agree to a Nash bargain with the remaining worker. In this sense, the second employee regards herself as being on the margin.
  - By induction, then, the firm approaches negotiations with the nth worker as if that worker were marginal too
  - the wage that solves the bargaining problem is that which maximizes the marginal surplus.

- Wages are set after employment has been determined
  - Hiring costs are sunk and labor market is closed setting
- Firm marginal surplus $S^f(s, z, n; w)$

$$S^f(s,z,n;w) = \frac{\partial \Pi(s, z, n_{-1})}{\partial n}$$
$$= sz\frac{\partial F(n)}{\partial n} - w(s, z, n) - \frac{\partial w(s, z, n)}{\partial n}n + \frac{1}{1+r}\frac{\partial \tilde{\Pi}(s, z, n)}{\partial n}$$

- Worker marginal surplus $S^w(s, z, n; w)$

$$S^w(s, z, n; w) = J^e(s, z, n) - J^u$$

- Bargaining problem

$$w(s,z,n) = \arg\max_{w} \quad S^f(s,z,n;w)^{\gamma} S^w(s,z,n;w)^{1-\gamma}$$

where $\gamma \in (0,1)$ is the worker's bargaining power

- Nash splitting rule

$$S^w(s,z,n;w) = \gamma[S^f(s,z,n;w) + S^w(s,z,n;w)]$$
$$S^f(s,z,n;w) = (1-\gamma)[S^f(s,z,n;w) + S^w(s,z,n;w)]$$

- Wage solves the following ODE

$$w(s,z,n) = (1-\gamma)b + \gamma\left[sz\frac{\partial F(n)}{\partial n} - \frac{\partial w(s,z,n)}{\partial n}n + \frac{1}{1+r}c_v\phi_w\right]$$

- Wages are:
  - increasing in the worker's bargaining power
  - increasing in the marginal product of labor
  - increasing in job finding probability and marginal costs of hiring
  - increasing in home production
- Extra term: $\frac{\partial w(s,z,n)}{\partial n} n$
  - If negotiation breaks, the firm will have to pay its remaining workers a higher wage (higher marginal product)
  - Inefficient incentive to over-employ workers
- Specific solution (with $F(n) = n^{\alpha}$)

$$w(s,z,n) = (1-\gamma)b + \gamma \left[\frac{szn^{\alpha-1}}{1-\gamma(1-\alpha)} + \frac{1}{1+r} c_v \phi_w \right]$$

- Binmore et al. (1986) bargaining solution
  - alternating offers generalized to a setting when marginal returns are diminishing
- The threats are to extend bargaining rather than to terminate it in case of disagreement
  - disagreement payoffs determines the bargaining outcomes, not the outside option payoff
- Breakdown of negotiations generates a surplus to split between parties, which is equal to the marginal flow surplus

- Firm marginal flow surplus $S^f(s, z, n; w)$

$$S^f(s,z,n;w) = sz\frac{\partial F(n)}{\partial n} - w(s,z,n) - \frac{\partial w(s,z,n)}{\partial n}n$$

- Worker marginal flow surplus $S^w(s, z, n; w)$

$$S^w(s,z,n;w) = w(s,z,n) - b$$

- Wage solves the following ODE

$$w(s,z,n) = (1-\gamma)b + \gamma\left[sz\frac{\partial F(n)}{\partial n} - \frac{\partial w(s,z,n)}{\partial n}n\right]$$

- No influence of labor market tightness on wages (Hall and Milgrom 2008)

- Let $\mu_t(s, z, n; \theta_t)$ be the measure of firms over individual state (s, z, n) when the market tightness is θ_t at time t
- Evolution of distribution over time:

$$\mu_{t+1}(s, z, n'; \theta_{t+1}) = T_t(\mu_t(s, z, n; \theta_t), \theta_t)$$

where

$$T_t(\mu_t(s, z, n; \theta_t); \theta_t) = \sum_{s,z} \int_n \psi_t(s, z', n' | z, n; \theta_t) d\mu_t(s, z, n; \theta_t)$$

and

$$\psi_t(s, z', n' | z, n; \theta_t) = \mathbf{1}[n(s, z', n; \theta_t) = n'] \Gamma(z' | z)$$

- Aggregate employment:

$$N = \sum_{s,z,n_{-1}} n(s,z,n_{-1}) d\mu(s,z,n_{-1})$$

- Total separation:

$$S = \sum_{s,z,n_{-1}} \max\{0, n_{-1} - n(s,z,n_{-1})\} d\mu(s,z,n_{-1})$$

- Total hires: $H = \phi_w U$
- Labor market dynamics: $N' = H - S + N$
- Labor resource constrain: $N + U = L$

A steady-state competitive equilibrium is a market tightness θ, a wage schedule $w(s, z, n)$, an optimal policy function for employment, $n(s, z, n_{-1})$, and a distribution $\mu(s, z, n)$, such that:

- **Firms optimality**: $n(s, z, n_{-1})$ solves the problem of the firm
- **Bargaining**: $w(s, z, n)$ are the solution to the intra-firm Nash bargaining problem
- **Stationarity**:
  - the distribution $\mu(s, z, n)$ replicates itself through productivity shocks and hiring/firing decisions
  - workers hires and separation balance each other, i.e. $\phi_w U = S$

Implication 1: Inaction region

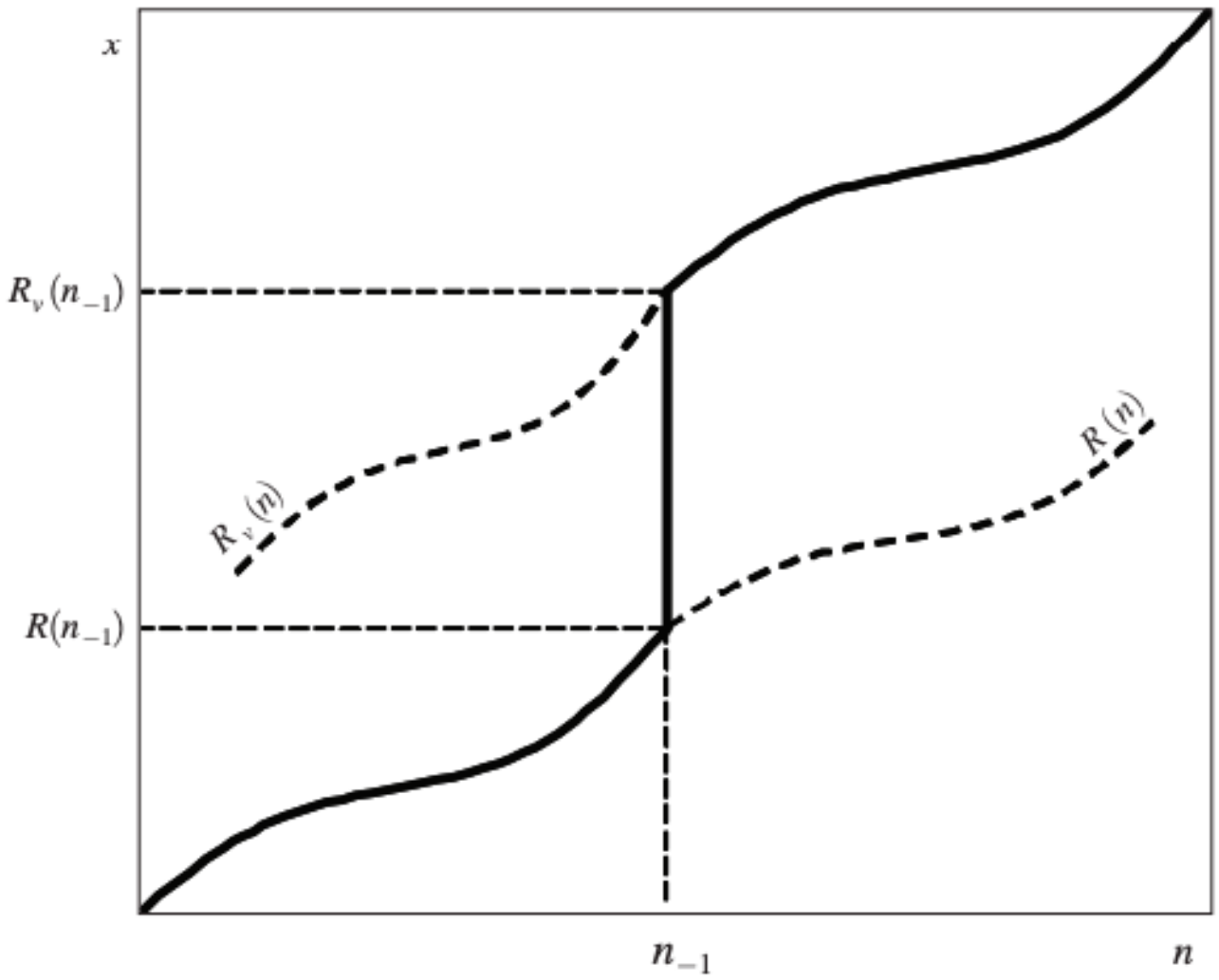


FIGURE 1. OPTIMAL EMPLOYMENT POLICY OF A FIRM

- Vacancy costs create a kink in the policy function

Implication 2: Firm size distribution

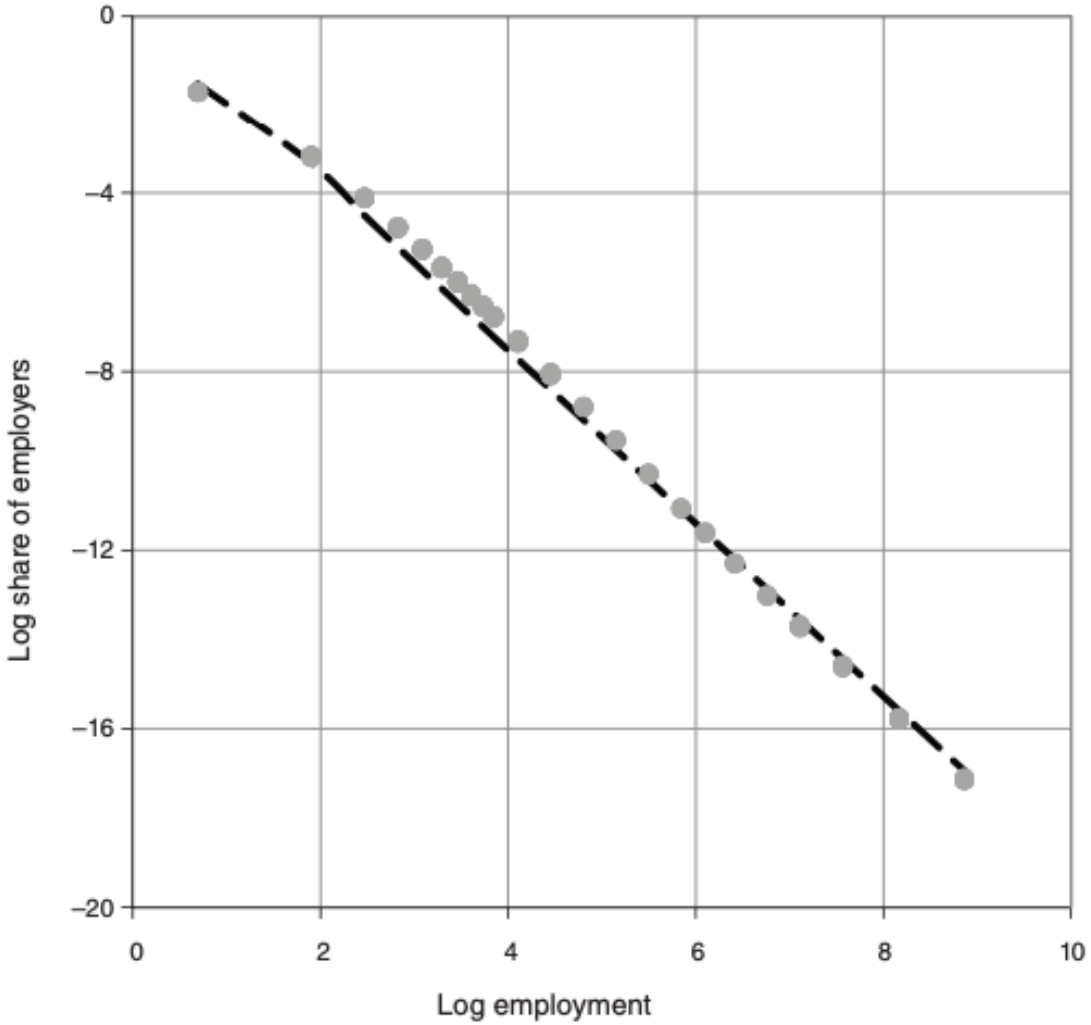


FIGURE 2. EMPLOYER SIZE DISTRIBUTION: MODEL VERSUS DATA

Notes: The dots plot data on the shares of firms in successive employment categories for the years 2002 to 2006 based on data on employment by firm-size class from the Small Business Administration. The dashed line plots the steady-state distribution of employment across firms implied by the model using the parameters reported in Table 1.

Implication 3: Firm growth distribution

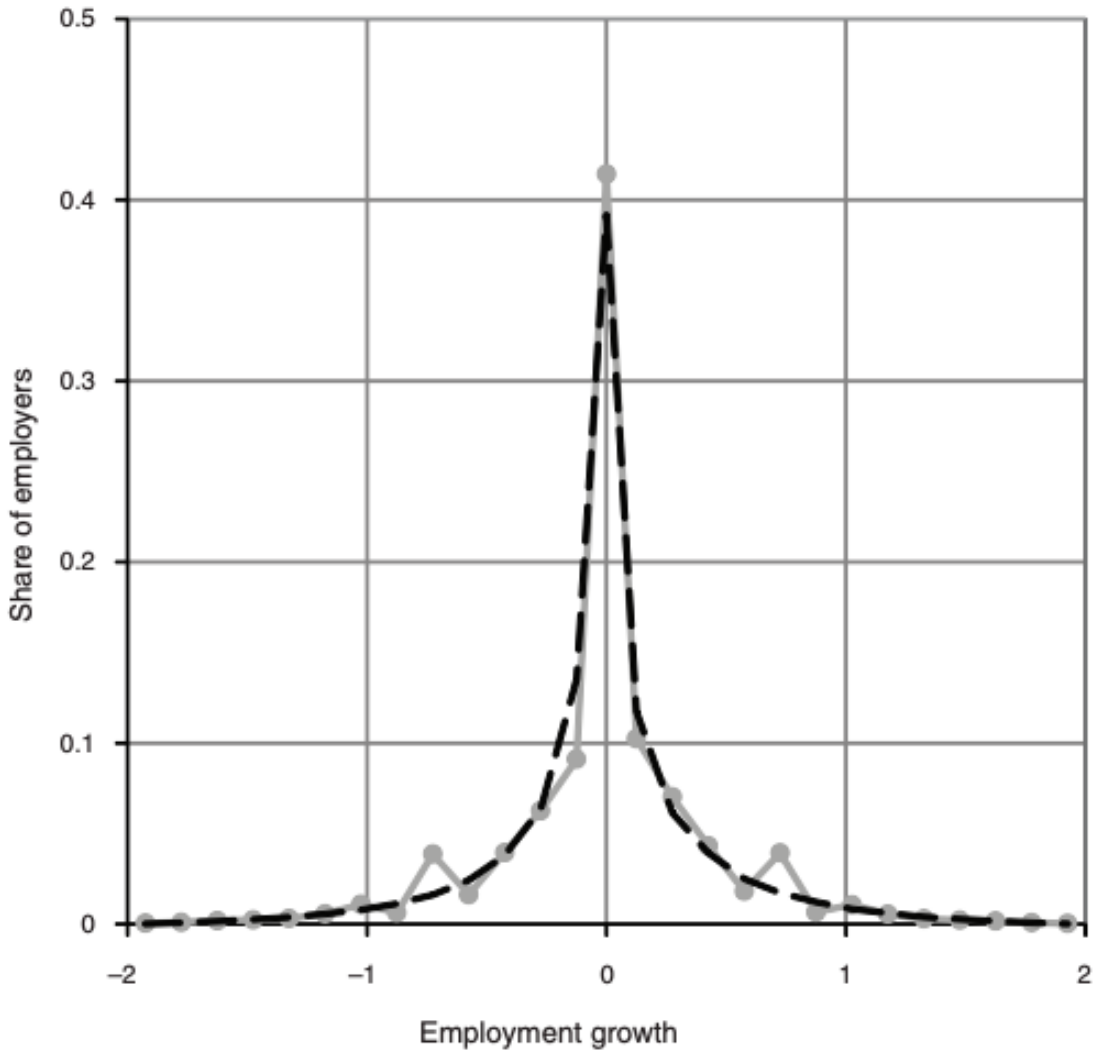


FIGURE 3. EMPLOYMENT GROWTH DISTRIBUTION: MODEL VERSUS DATA

Notes: The dotted line plots the cross-sectional distribution of employment growth based on data for continuing establishments from the Longitudinal Business Database pooled over the years 1992 to 2005. The dashed line plots the steady-state distribution of employment growth in the model using the parameters reported in Table 1.

Implication 4: Wage dispersion

- Wage dispersion between firms
  - usually accounts for 20-30% percent of overall wage dispersion
  - possible fixes:
    - workers heterogeneity
    - on-the-job search
- Counterfactual size-wage premium
  - small firms pay higher wages in the model (for some calibration)
  - two conflicting forces:
    - large firms are those more productive: wages $\uparrow$
    - large firms have lower marginal product of labor: wages $\downarrow$

- **Step 1**: guess market tightness θ^0
- **Step 2**: compute job finding and filling probabilities, ϕ_w^0, ϕ_f^0
- **Step 3**: compute wages, $w(s, z, n; \theta^0)$
- **Step 4**: solve the problem of the firm and obtain policy functions for employment $n(s, z, n_{-1})$
- **Step 5**: simulate the economy for a large number of firms and compute the stationary distribution, $\mu(s, z, n)$
- **Step 6**: use $\mu(s, z, n)$ to compute aggregate employment and total separation
- **Step 7**: obtain new value for market tightness, θ^1, using the stationarity condition and labor resource constraint
- **Step 8:** check convergence
  - if not achieved, use θ_1 and go back to step 2 till convergence
  - if achieved, store $\theta^* = \theta_0$