Macroeconomics of “Large Firms”

Lecture 1: The basics of firm dynamics

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CEMFI, PhD short course

- Hopenhayn H. 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium". Econometrica. Vol. 60, No. 5, pp. 1127-1150
  - Workhorse model of industry dynamics
  - Partial equilibrium
  - Focus on steady-state: wages and prices are constant
  - Individual dynamics: firms enter, grow, decline, and exit
  - Competitive firms, no strategic interactions

- Time is discrete
- Wage is the model numeraire ($w = 1$)
- Output price p endogenous
- Endogenous measure of heterogeneous firms
  - DRS production function
  - Perfect competition in product and labor markets
  - No aggregate risk
  - Idiosyncratic risk: firm productivity follows a Markov process
  - Entry-exit dynamics
  - Fixed cost to enter
  - Fixed cost to operate each period
- Partial equilibrium: exogenous industry demand

- Firms differ in productivity z
- Firm-level output

$$f(z, n) = zn^{\alpha} \quad \alpha \in (0, 1)$$

- Static firm-level profits

$$\pi(z; p) = \max_{n \geq 0} \quad pf(z, n) - n - pc_o$$

where c_o denotes per-period operating costs.

- Let $n(z; p)$ denote optimal employment

$$n(z; p) = (p\alpha z)^{\frac{1}{1-\alpha}}$$

- Let $y(z; p) = f(z, n(z; p))$ denote associated output

Problem of the incumbents

- Incumbents enter the period with states z_{-1}
- Exit decision:
  - if stay, firms draw new productivity level $z \sim \Gamma(z|z_{-1})$
- Employment decision (conditional on staying)
  - choose new employment level n conditional on z
  - pay operating costs c_o and produce $y(z; p)^*$

- $V(z;p)$: value function for firm in states z and aggregate price p

$$V(z,p) = \pi(z;p) + \frac{1}{1+r}\tilde{v}(z)$$

where

$$\tilde{v}(z) = \max\left\{0, \sum_{z'} V(z';p)\Gamma(z'|z)\right\}$$

- Solution to this problem is policy function for optimal exit: $\mathbf{1}^x(z;p)$
- An exit productivity threshold z^* exists such that:

$$\sum_{z'} V(z';p)\Gamma(z'|z^*) = 0$$

- Potential entrants are ex-ante identical
- New entrants $M \geq 0$ pay c_e and enter
- Draw productivity level z from $\Gamma^e(z)$ (ergodic distribution obtained from $\Gamma(z|z_{-1})$)
- Start producing next period
- Free entry condition:

$$v^e(p) = \frac{1}{1+r} \sum_z V(z;p)\Gamma^e(z) \leq c_e$$

with equality if $M > 0$.

- Let $\mu_t(z;p)$ be the measure of firms over individual state z when the goods price is p at time t
- Evolution of distribution over time:

$$\mu_{t+1}(z';p_t) = T_t(\mu_t(z;p_t), M_t, p_t)$$

where

$$T_t(\mu_t(z;p_t), M_t, p_t) = \sum_z \psi_t(z'|z;p_t)d\mu_t(z;p_t) + M_t\Gamma^e(z)$$

and

$$\psi_t(z'|z;p_t) = \Gamma(z'|z)[\mathbf{1}_t^x(z;p_t) = 0]$$

denotes the transition function from the states z to z'

- Industry demand curve exogenous: $Y^d(p) = \bar{Y}$
- Industry supply curve endogenous:

$$Y^s(p) = \sum_z y(z;p)\mu(z;p)$$

- Inelastic labor supply function: $N^d = 1$
- Industry labor demand, endogenous,

$$L^s(p) = \sum_z n(z;p)\mu(z;p) + Mc_e$$

A recursive stationary equilibrium for this economy is characterized by a measure of entrants M^*, a distribution of incumbent firms $\mu^*(z;p)$, a price p^* and a productivity threshold z^* such that the following four conditions hold:

- **Optimality**: $\forall z < z^*$, $\mathbf{1}^x(z) = 1$
- **Free-entry** $v^e = c_e$;
- **Goods market clearing**: $Y^s(p^*) = \bar{Y}$;
- **Aggregate consistency**: $\mu^*(z;p^*) = T(\mu^*(z;p^*), M^*, p^*)$.

- **Step 1**: guess a price p_0
- **Step 2**: solve for the value of the incumbent, $v(z;p^0)$. A solution to this problem implies an optimal exit rule, $\mathbf{1}^x(z;p^0)$
- **Step 3**: compute the value of entry, v^e and check if free-entry condition is satisfied:
  - if no, make a new guess p_1 and go back to step 2 till convergence
  - if yes, store $p^* = p_0$
- **Step 4**: given p^*, solve for the stationary distribution of incumbents $\mu(z;p^*)$ and measure of entrants M^e
  - guess measure of entrants M_0^e
  - calculate the stationary distribution $\mu(z;p^*, M_0^e)$
  - given $\mu(z;p^*, M_0^e)$, compute total industry supply and check the if market clearing condition is satisfied:
    - if no, make a new guess M_1^e and go back to step 4 till convergence
    - if yes, store $M^{e*} = M^e$

- Steps 4 can be speeded up exploiting the property of linear homogeneity of the function $\mu(\cdot)$ with respect to M
- Re-write in matrix notation the function μ, i.e.

$$\boldsymbol{\mu} = \boldsymbol{\Psi}\boldsymbol{\mu} + M\boldsymbol{\Gamma}$$

- Solving for $\boldsymbol{\mu}$

$$\boldsymbol{\mu} = M(\mathbf{I} - \boldsymbol{\Psi})^{-1}\boldsymbol{\Gamma}$$

- No need to use simulations to find stationary distribution $\boldsymbol{\mu}$
- No need to iterate on M. Solve for μ imposing $M = 1$
- Compute equilibrium M using market clearing condition

$$M^* = \frac{\bar{Y}}{\sum_z y(z;p^*)\mu(z;p^*)}$$

- What if corner solution, i.e. $M = 0$? No entry/exit dynamics
- The stationary distribution of firms just given by stationary distribution of Markov chain, i.e.

$$\mu(z;p) = \sum_{z'} \Gamma(z'|z) = \mu(z)$$

- Stationary distribution independent of p - no need to use free entry condition!
- Solve for p^* using the market clearing condition, i.e.

$$Y^s(p^*) = \bar{Y}$$

Productivity state-space:

- AR(1) process in logs:

$$\log z' = \mu + \rho \log z' + \epsilon' \quad \epsilon' \sim \mathcal{N}(0, \sigma^2)$$

- Markov chain approximation using Tauchen method on 201 nodes

Baseline parameters (period model = 1 year)

$$r = 0.01, \alpha = 0.78, \mu = 0, \rho = 0.995, \sigma = 0.14$$
$$c_o = 10, c_e = 80, Y^d = 500$$

What happens if we increase the entry cost c_e?

Entry costs: c_e/Y	0.36	0.41	0.46	0.50
Price	1	1.162	1.247	1.316
Output	1	1.448	1.728	1.989
Profits	1	1.453	1.732	1.993
# firms	1	0.676	0.5620	0.485
Firm Size	14.429	24.295	31.081	37.799
Exit rate	47.711	44.422	43.072	42.191
Productivity threshold	1	0.741	0.652	0.598

Increase in entry costs c_e:

- decreases exit productivity threshold — **less selection!**
- decreases number of firms and exit rate
- increases prices

Two contrasting effects on firm-size

- *price/value effects*: higher c_e increases prices, which leads to higher output and employment
- *selection effects*: higher c_e reduces productivity thresholds, keeping low-productivity firms in the industry

If the density of firms near the exit point is very small, the first effect will dominate and the average value and profits increase with c_e.

What happens if we increase the operating cost c_o?

Operating costs: c_o/Y	0.04	0.08	0.13	0.20
Price	1	1.027	1.086	1.158
Output	1	1.165	1.532	1.8160
Profits	1	1.125	1.369	1.526
# firms	1	0.881	0.696	0.547
Firm size	14.429	17.270	24.010	33.337
Exit rate	47.711	51.761	57.402	61.217
Productivity threshold	1	1.408	2.159	2.791

Increase in operating costs, c_o:

- increases exit productivity threshold — **more selection!**
- decreases number of firms
- increases entry/exit rate
- increases prices

As the c_o increases, the profit function changes, pushing profits down for every state. Productivity thresholds need to increase to maintain free-entry.

Extension: Adjustment Costs

- Hopenhayn H. and Rogerson R. 1993. "Job Turnover and Policy Evaluation: A General Equilibrium Analysis". Journal of Political Economy. Vol.101, N.5, pp. 915-938
  - General equilibrium version of Hopenhayn (1992)
  - Non-convex adjustment cost (firing costs)
  - No aggregate shocks
  - Optimal employment policy characterized by *inaction* region
  - *Misallocation* of resources across heterogeneous plants

- Large volume of job creation/destruction at firm-level
- Policies that make more costly to adjust employment level, i.e.
  - legislated severance payments
  - advance notice
  - plant closing legislation
- What are the effects of such policies on
  - employment
  - aggregate output
  - productivity
- Can labor market regulations explain heterogeneity in labor market performance across countries?

- Time is discrete
- Wage is the model numeraire ($w = 1$)
- Output price p endogenous
- Representative household
  - Consumption/labor supply decision
  - No savings
- Endogenous measure of heterogeneous firms
  - Perfect competition in product and labor markets
  - Time-varying productivity
  - Entry-exit dynamics
  - Employment adjustment costs (firing tax)
  - Free-entry

- Utility function of consumption C and labor supply N

$$U(C, N) = \log C - AN$$

where A denotes disutility from supplying labor.

- Discount factor: $\beta = 1/1 + r$
- Budget constraint:

$$pC \leq N + \Pi \quad (w = 1 \quad \text{is the numeraire})$$

where Π are aggregate profits, re-distributed to HH lump-sum

- Problem of the HH:

$$U(C,N) = \max_{C,N} \quad \log C - AN$$
$$\text{s.t.} \quad pC \leq N + \Pi$$

- First order conditions (at the interior) imply:

$$C = \frac{1}{Ap} \quad \textit{(:consumption demand)}$$
$$N^s = \frac{1}{A} - \Pi \quad \textit{(:labor supply)}$$

- Firms differ in productivity z and employment n
- Firm-level output

$$f(z,n) = zn^{\alpha} \quad \alpha \in (0,1)$$

- Static firm-level profits

$$\pi(z,n,n_{-1}) = pf(z,n) - n - pc_o - g(n,n_{-1})$$

where c_o denotes per-period operating costs.

- Adjustment costs (expressed in units of labor)

$$g(n,n_{-1}) = \tau \max\{0, n_{-1} - n\}$$

- Incumbents enter the period with states (z_{-1}, n_{-1})
- Exit decision:
  - if exit, firms pay $g(0, n_{-1})$
  - if stay, firms draw new productivity level $z \sim \Gamma(z|z_{-1})$
- Employment decision (conditional on staying)
  - choose new employment level n conditional on (z, n_{-1})
    - expanding firms $(n > n_{-1})$ subject to no cost
    - shrinking firms $(n < n_{-1})$ subject to firing costs
  - pay operating costs pc_o and produce $f(z, n)$

- $V(z, n_{-1}; p)$: value function for firm in states (z, n_{-1}) and aggregate price p

$$V(z, n_{-1}; p) = \max_{n \geq 0} \pi(z, n, n_{-1}) + \frac{1}{1+r}\tilde{v}(z, n)$$

where

$$\tilde{v}(z, n) = \max \left\{ -g(0, n), \sum_{z'} V(z', n; p)\Gamma(z'|z) \right\}$$

- Solution to this problem:
  - Policy function for optimal employment policy: $n = g_n(z, n_{-1}; p)$
  - Policy policy for optimal exit: $\mathbf{1}^x(z, n_{-1}; p)$

- Potential entrants are ex-ante identical
- New entrants $M \geq 0$ pay c_e and enter
- Draw productivity level z from $\Gamma^e(z)$ (ergodic distribution obtained from $\Gamma(z|z_{-1})$)
- Hire n workers and produce
- Free entry condition:

$$v^e(p) = \frac{1}{1+r} \sum_z V(z, 0; p) \Gamma^e(z) \leq c_e$$

with equality if $M > 0$.

- Let $\mu(z, n; p)$ be the measure of firms over individual states z and n when the goods price is p
- Solution of the following linear system:

$$\mu(z', n'; p) = T(\mu(z, n; p), M, p)$$

where

$$T(\mu(z, n; p), M, p) = \sum_z \int_n \psi(z', n' | z, n; p) d\mu(z, n; p) + M\Gamma^e(z)\mathbf{1}[g_n(0, z; p) = n']$$

and

$$\psi(z', n' | z, n; p) = \mathbf{1}[g_n(n, z'; p) = n']\Gamma(z'|z)[\mathbf{1}^x(z, n) = 0]$$

denotes the transition function from the states (z, n) to z', n'

- Aggregate output:

$$Y = \sum_{z} \int_{n_{-1}} [f(z, g_n(z, n_{-1}) - c_f] d\mu(z, n_{-1}; p)$$
$$+ M \sum_{z} f(z, g_n(z, 0)\Gamma^e(z)$$

- Labor demand:

$$N^d = \sum_{z} \int_{n_{-1}} g_n(z, n_{-1} d\mu(z, n_{-1}; p) + M \sum_{z} g_n(z, 0)\Gamma^e(z) + Mc^e$$

- Aggregate profits:

$$\Pi = pY - N^d - pc_o$$

A recursive stationary equilibrium for this economy is characterized by a measure of entrants M^*, a distribution of incumbent firms $\mu^*(z, n; p)$, and a price p^* such that the following three conditions hold:

- **Free-entry** $v^e = c_e$;
- **Labor market clearing**: $N^s = N^d$;
- **Aggregate consistency**: $\mu^*(z, n; p^*) = T(\mu^*(z, n; p^*), M^*, p^*)$.

- **Step 1**: guess a price p_0
- **Step 2**: solve for the value of the incumbent, $v(z, n; p^0)$
- **Step 3**: compute the value of entry, v^e and check if free-entry condition is satisfied:
  - if no, make a new guess p_1 and go back to step 2 till convergence
  - if yes, store $p^* = p_0$
- **Step 4**: given p^*, solve for the stationary distribution of incumbents $\mu(z, n; p^*)$ associated with $M = 1$
  - Exploit linear homogeneity of T

$$M\mu^*(z, n; p^*) = MT(\mu^*(z, n; p^*), M, Mp^*)$$

  - Fixed point/MC simulation
- **Step 5**: given $\mu^*(z, n; p^*)$, find M^* that makes the labor market clear

Productivity state-space:

- AR(1) process in logs:

$$\log z' = \mu + \rho \log z' + \epsilon' \quad \epsilon' \sim \mathcal{N}(0, \sigma^2)$$

- Markov chain approximation using Tauchen method on 51 nodes

Employment state-space:

- maximum number of employees: 3000
- 500 points (400 points between 1 and 200 employees)

Baseline parameters (period model = 5 years)

$$r = 0.25, \alpha = 0.64, \mu = 0.25, \rho = 93, \sigma = 0.17$$
$$c_o = 20, c_e = 40, A = 0.45, \tau = 0$$

- Without adjustment costs ($\tau = 0$), optimal employment decision given by:

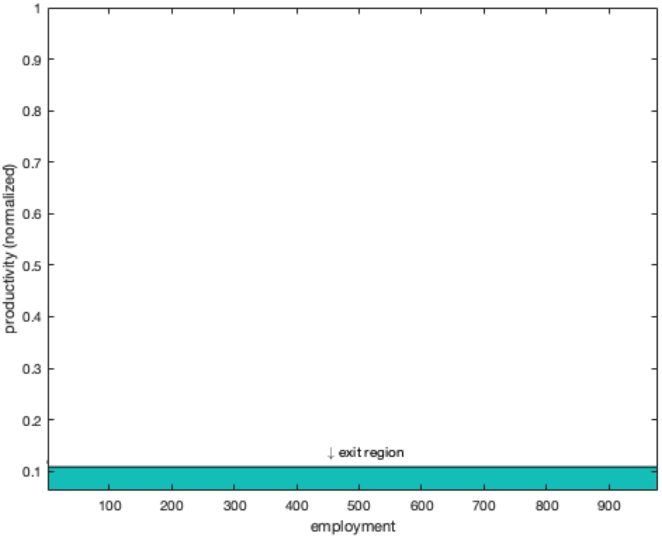
$$n' = (\alpha p z)^{\frac{1}{1-\alpha}}$$

- With adjustment costs ($\tau > 0$), employment decision characterized by two reservation thresholds, $z_F(n)$ and $z_H(n)$, such that:

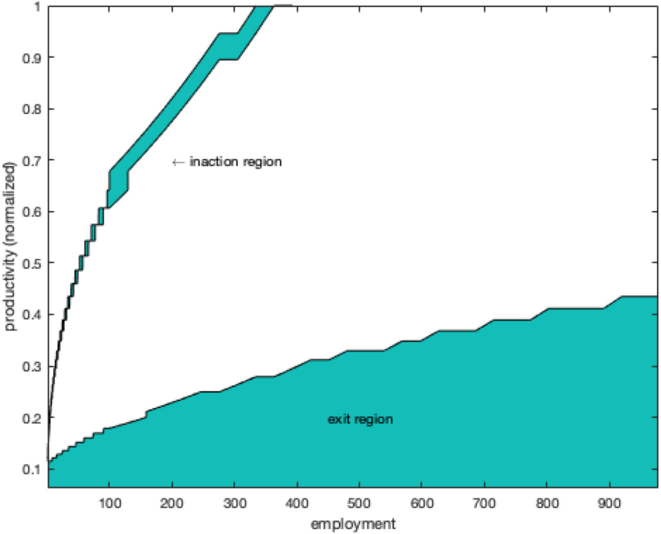
$$\begin{aligned} n' = n_F(z) \quad & \text{if} \quad z < z_F(n) \\ n' = n_{-1} \quad & \text{if} \quad z \in [z_F(n), z_H(n)] \\ n' = n_H(z) \quad & \text{if} \quad z > z_H(n) \end{aligned}$$

- Inaction region wider with higher τ

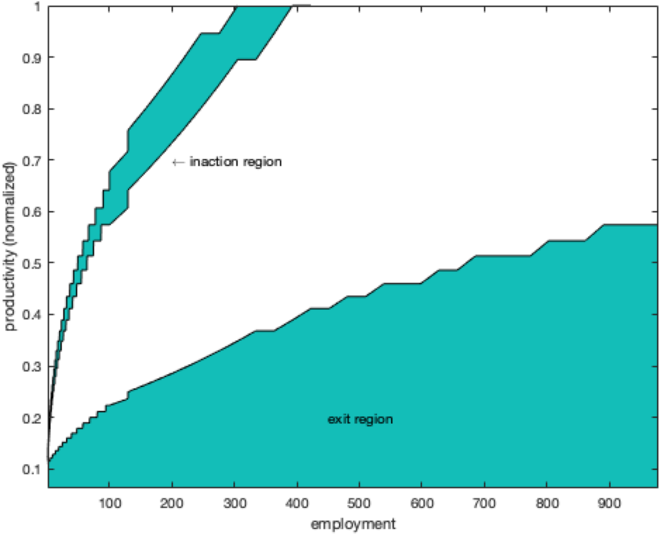
Inaction regions: $\tau = 0$



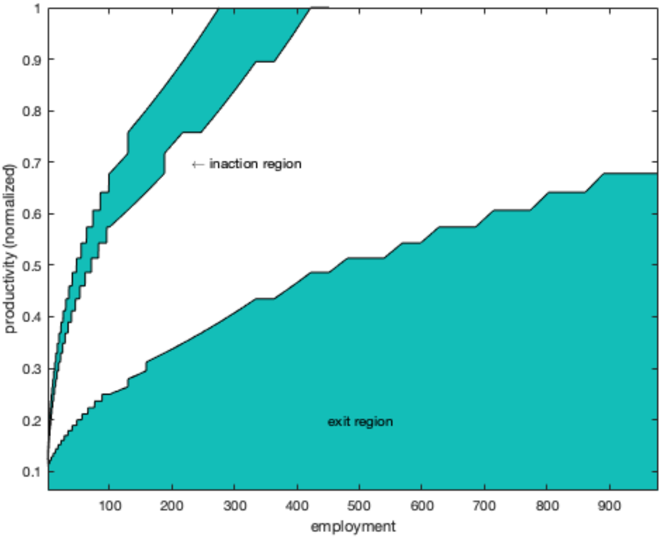
Inaction regions: $\tau = 0.1$



Inaction regions: $\tau = 0.2$



Inaction regions: $\tau = 0.3$



- Without adjustment costs ($\tau = 0$), MPL equalized across firms:

$$\frac{\partial f(z,n)}{\partial n} = \frac{1}{p}$$

- With adjustment costs ($\tau = 0$), MPL is the solution of two *necessary but not sufficient* conditions:

$$p\frac{\partial f(z,n)}{\partial n} + \frac{1}{1+r}\frac{\partial \tilde{v}(z,n)}{\partial n} = 1 \quad \text{if} \quad n' > n$$

$$p\frac{\partial f(z,n)}{\partial n} + \frac{1}{1+r}\frac{\partial \tilde{v}(z,n)}{\partial n} = \left[1 + \underbrace{\frac{\partial g(n,n_{-1})}{\partial n}}_{\tau}\right] \quad \text{if} \quad n' < n$$

Marginal product of labor: $\frac{\partial f(z,n)}{\partial n}$

	Dispersion	Percentiles			
Firing cost: τ	St.Dev.	20th	40th	60th	80th
0	0	1	1	1	1
0.1	0.0439	0.9485	0.9546	1.0171	1.0349
0.2	0.0720	0.9097	0.9349	1.0156	1.0568
0.3	0.0911	0.8735	0.9199	1.0134	1.0715

Firing costs increase the dispersion of the MPL

Firing cost: τ	0	0.1	0.2	0.3
Price	1	1.0085	1.0145	1.0193
Consumption (output)	1	0.9915	0.9856	0.9810
Employment	0.6000	0.5988	0.5986	0.5988
Profits	1	1.1561	1.2758	1.4127
# firms	1	1.1350	1.2432	1.3702
Firm size	20.055	20.333	20.655	21.152
Labor productivity	1	0.9914	0.9818	0.9718
JC rate	0.1879	0.1403	0.1108	0.0953

Firing costs reduce aggregate productivity and aggregate output

Extension: Policy Distortions

Firm dynamics with policy distortions

- Restuccia D. and Rogerson R. 2008. "Policy Distortions and Aggregate Productivity with Heterogeneous Plants." Review of Economic Dynamics. Vol 11, No. 4, pp. 707-72
    - Neoclassical model of growth
    - Firms heterogeneity in productivity
    - Exogenous idiosyncratic distortions
    - General equilibrium
    - Productivity losses of *misallocation*

- Large cross-country differences in income p.c.
- Evidence of heterogeneous distortions across countries:
  - financial markets (e.g., Parente and Prescott 1999)
  - labour regulation (e.g., Lagos 2006)
  - gov. subsidies (e.g., Guner et al. 2008)
- Assess quantitative significance of firm-level distortions on output, productivity and employment
  - misallocation across productive units
- Reduced form representation of idiosyncratic distortions to producer prices: tax/subsidy τ on output
- Results: heterogeneity in prices faced by producers can lead to decrease in TFP and output of up 50%

- Time is discrete, focus on steady-state
- Output price is the model numeraire ($p = 1$)
- Wage w and interest rate r determined endogenously
- Representative household
  - Consumption/savings decision
  - Inelastic labor supply
- Endogenous measure of heterogeneous firms
  - Perfect competition in product and labor markets
  - Innate differences in firm-level productivity
  - Entry-exit dynamics
  - Free-entry

- Utility function of consumption C_t

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \beta \in (0,1)$$

where β is the discount factor

- Budget constraint:

$$C_t + I_t \leq w_t + r_t K_t + \Pi_t - T_t$$

where Π_t are aggregate profits and T_t are net taxes

- Capital depreciates at rate δ

$$K_{t+1} = (1-\delta)K_t + I_t$$

- Problem of the HH:

$$\mathcal{U} = \max_{C_t, K_{t+1}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\text{s.t.} \quad C_t + K_{t+1} \leq w_t + (1 + r_t - \delta)K_t + \Pi_t - T_t$$

$$K_0 > 0 \quad \text{given}$$

- First order condition (at the interior) imply:

$$U'(C_t) = \beta U'(C_{t+1})(1 + r_t - \delta)$$

- In steady-state, $C_t = C_{t+1}$ and

$$r^* = \frac{1}{\beta} - 1 + \delta$$

- Firms differ in productivity, z and distortion, τ
- Firm-level output

$$f(z,n,k) = zn^{\alpha}k^{\gamma} \quad \alpha, \gamma \in (0,1)$$

where $\alpha + \gamma < 1$: decreasing return to scale.

- Static firm-level profits

$$\pi(z,\tau;w,r) = \max_{n\geq 0, k\geq 0} \quad (1-\tau)f(z,n,k) - wn - rk - c_o$$

where c_o denotes per-period operating costs.

- Let $n(z,\tau;w,r)$ and $k(z,\tau;w,r)$ be the optimal employment and capital demand function

- $V(z, \tau; w, r)$: value function for firm in states (z, τ) and prices (w, r)

$$V(z, \tau; w, r) = \pi(z, \tau; w, r) + \frac{(1-\lambda)}{1+r-\delta} V(z, \tau; w, r)$$

where λ is an exogenous probability of exit

- Expected discounted value of per-period profits:

$$V(z, \tau; w, r) = \frac{\pi(z, \tau; w, r)}{1 - \frac{(1-\lambda)}{1+r-\delta}}$$

- Large measure of identical entrants pay c_e and draw productivity z and distortion τ from a joint distribution

$$\Gamma(z, \tau) = P(\tau | z) H(z)$$

- Entry decision: $V^e(z, \tau; w, r) = \max\{0, V(z, \tau; w, r)\}$. Solution to this problem is a policy for optimal entry: $\mathbf{1}^e(z, \tau; w, r)$
- Measure of entry: $M \geq 0$
- Free entry condition:

$$v^e(w, r) = \sum_z \sum_\tau V^e(z, \tau; w, r) \Gamma(z, \tau) \leq c_e$$

with equality if $M > 0$.

- Let $\mu_t(z,\tau;w_t,r_t)$ be the measure of firms over individual state (z,τ) when wage and interest rate rate (w_t,r_t) at time t
- Evolution of distribution over time:

$$\mu_{t+1}(z,\tau;w_{t+1},r_{t+1}) = T_t(\mu_t(z,\tau;w_t,r_t),M_t,w_t,r_t)$$

where

$$T_t(\mu_t(z,\tau;w_t,r_t),M_t,w_t,r_t),M_t,p_t) = \\ (1-\lambda)\mu_t(z,\tau;w_t,r_t) + M\mathbf{1}^e(z,\tau;w_t,r_t)P(\tau|z)H(z)$$

- Endogenous labor and capital demand

$$L^d = \sum_{z,\tau} n(z,\tau;w,r) d\mu(z,\tau;w,r)$$
$$K^d = \sum_{z,\tau} k(z,\tau;w,r) d\mu(z,\tau;w,r)$$

- Aggregate output

$$Y^s = \sum_{z,\tau} [zn(z,\tau;w,r)^\alpha k(z,\tau;w,r)^\gamma - c_o] d\mu(z,\tau;w,r)$$

- Goods demand: $Y^d = C + \delta K + Mc_e$
- Aggregate taxes: $\sum_{z,\tau} \tau f(z, n(z,\tau), k(z,\tau)) d\mu(z,\tau;w,r)$

A steady-state competitive equilibrium is a wage rate w, a rental rate r, a lump-sum tax T, a policy function $\mathbf{1}^e(z,\tau)$, a distribution $\mu(z,\tau)$, and a mass of entry M such that:

- **Consumers optimality**: $r = 1/\beta - 1 + \delta$
- **Firms optimality**: $\mathbf{1}^e(z,\tau)$ solves the problem of the entrants
- **Free-entry**: $v^e(w,r) = c_e$
- **Markets clearing**: $L^d = 1 \quad K^d = \bar{K} \quad Y^s = Y^d$
- **Balanced budget**:

$$T = \sum_{z,\tau} \tau f(z, n(z,\tau), k(z,\tau)) d\mu(z,\tau; w, r)$$

- **Time-invariance**: $\mu(z,\tau;w,r) = M\frac{\mathbf{1}^e(z,\tau;w,r)P(\tau|z)H(z)}{\lambda}$

- **Step 0**: fix interest rate to steady-state value: r^*
- **Step 1**: guess a wage rate w_0
- **Step 2**: solve for the value of the incumbent, $v(z, \tau; w_0, r^*)$
- **Step 3**: solve the problem of the potential entrant, $\mathbf{1}^e(z, \tau; w_0, r^*)$
- **Step 4**: compute the value of entry, $v^e(w_0, r^*)$ and check if free-entry condition is satisfied:
  - if no, make a new guess w_1 and go back to step 2 till convergence
  - if yes, store $w^* = w_0$
- **Step 5**: compute the invariant distribution of plants (normalized $M = 1$)
- **Step 6**: find mass of firms such that the labor market clears

- Distortions: U.S treated as un-distorted benchmark, $\tau = 0$
- Period model: 1 Year $\implies r = 0.04, \beta = 0.96$
- Decreasing return to scale: $\alpha + \gamma = 0.85$. Two-third assigned to labor return $\implies \alpha = 0.57, \gamma = 0.28$
- Depreciation to match capital/output ratio of 2.3 $\implies \delta = 0.08$
- No operating costs, $c_o = 0$.
- Entry cost normalized to 1, $c_e = 1$ (identification issues: changes to c_e isomorphic to changes in establishment-level productivity)

Productivity state-space:

- 100 nodes
- Lowest productivity normalized to 1
- Range of values chosen to match the range of employment across establishments

$$\frac{n_i}{n_j} = \left(\frac{z_i}{z_j}\right)^{\frac{1}{1-\gamma-\alpha}}$$

In US data, biggest firms are 10000 times larger than smallest. Given α and γ, largest productivity equal to 3.98

- Distribution of productivity $H(z)$ to match observed firm-size distribution

Calibration

- Two main experiments:
  - Uncorrelated distortions, τ independent of z
    - half producers taxed, half subsidized
    - resources flow from taxed to subsidized, but no systematic effect across productivity classes
  - Correlated distortions, either positively or negatively
    - lowest half producers subsidized, top half taxed
    - systematic reallocation across productivity classes, not just within productivity class
- Size of the subsidy so that the net effect on steady-state capital accumulation is zero.

Uncorrelated distortions

		τ_t			
Variable	Description	0.1	0.2	0.3	0.4
Y	Relative Output	0.98	0.96	0.93	0.92
TFP	Relative TFP	0.98	0.96	0.93	0.92
E	Relative employment	1.00	1.00	1.00	1.00
Y_s/Y	Output of subsidized	0.72	0.85	0.93	0.97
S/Y	Subsidy share of output	0.05	0.08	0.09	0.10
τ_s	Subsidy rate	0.06	0.09	0.10	0.11

- comparatively small effect on TFP and output, no effect on E
- subsidies to undo effects on capital accumulation are smaller
- as tax increases, larger TFP-effect and larger subsidies

Variable	Description	τ_t 0.1	0.2	0.3	0.4
Y	Relative Output	0.90	0.80	0.73	0.69
TFP	Relative TFP	0.90	0.80	0.73	0.69
E	Relative employment	1.00	1.00	1.00	1.00
Y_s/Y	Output of subsidized	0.42	0.67	0.83	0.92
S/Y	Subsidy share of output	0.17	0.32	0.43	0.49
τ_s	Subsidy rate	0.40	0.48	0.52	0.53

- qualitatively similar to uncorrelated case
- larger negative effect on TFP and output
- also more costly to finance (higher subsidies)

- Non-constant capital stock
  - taxing all but some exempt producers at 40% rate and no subsidy
  - lower capital stock, wages and entry rate also fall in proportion
  - amplifies effects on TFP

- Taxes on capital and labor