

Misallocation and Inequality

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- Differences in labor earnings across individuals are key sources of income inequality (Hoffmann et al 20)
- Firms shape earnings distribution
 - not all firms pay the same wage to workers with similar characteristics (Abowd et al 99, Card et al 13, Song et al 19)
 - large-firm wage premium (Bloom et al 18)
- Firms look very different across countries. In richer countries:
 - larger firm size (Bento and Restuccia 16)
 - firms more likely to train their workers (Ma et al 20)
- How do firms affect labor earnings distribution along development?

In this paper

- We document how the distribution of wage & salary income varies with GDP p.c.
 - the median increases faster than the mean
 - the GINI coefficient declines
 - inequality at the top shrinks, inequality at the bottom expands
- We build a model of firm dynamics and labor frictions to interpret this evidence
 - heterogeneous firms and workers
 - on-the-job human capital accumulation (learning + training)
 - wage dispersion within and across firms
- Cross-country patterns can be reproduced by two sources of misallocation
 - firm-level correlated distortions
 - larger search frictions

Three main channels

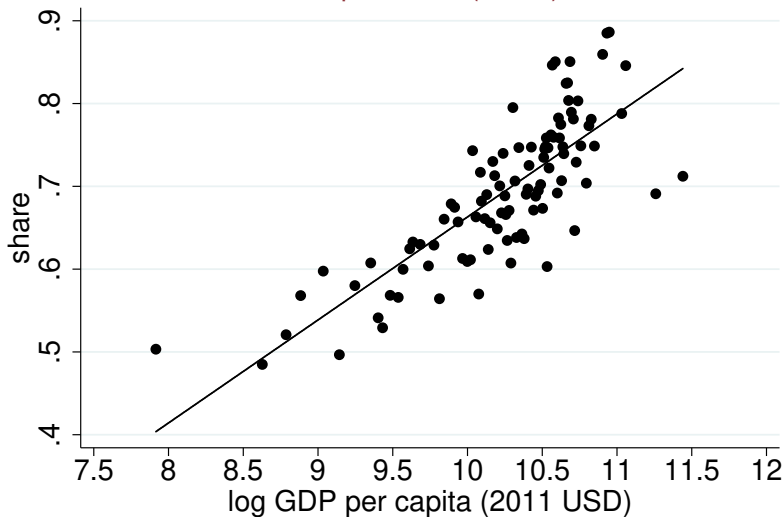
- Removing distortions affects:
 - Reallocation: increase in revenue dispersion across firms (inequality \uparrow)
- Removing frictions affects:
 - Non-employment duration: increase in participation in wage employment and human capital accumulation of low-skill workers (inequality \downarrow)
 - Sorting: increase in correlation between workers ability and firm productivity (inequality \uparrow)
- On-the-job training amplifies these patterns
 - it account up to 35% of changes in earnings inequality across countries

Earnings dataset

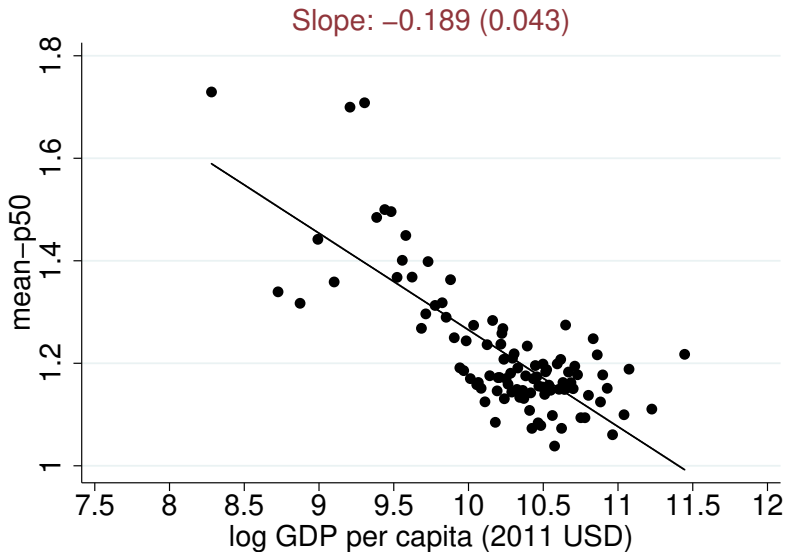
- Coverage: 57 countries, 1981-2016 ●
 - India (1993), GDP per capita: 1845 (2011, USD)
 - Luxembourg (2007), GDP per capita: 97864 (2011, USD)
- Source: IPUMS International, Survey on Income and Living Conditions (SILC), Luxembourg Income Study Database (LIS)
- Sample: all workers with non-missing wage & salary income, 18-64 y.o.
- *Earnings*: gross wages & salaries (including extra pay, tips, commissions, bonuses, piece-rate payments, occasional earnings) ●
- *Employees*: those with positive earnings
- Demographics: gender, age, education, labor market status, job characteristics

Wage and salary employees ●

Slope: 0.127 (0.020)

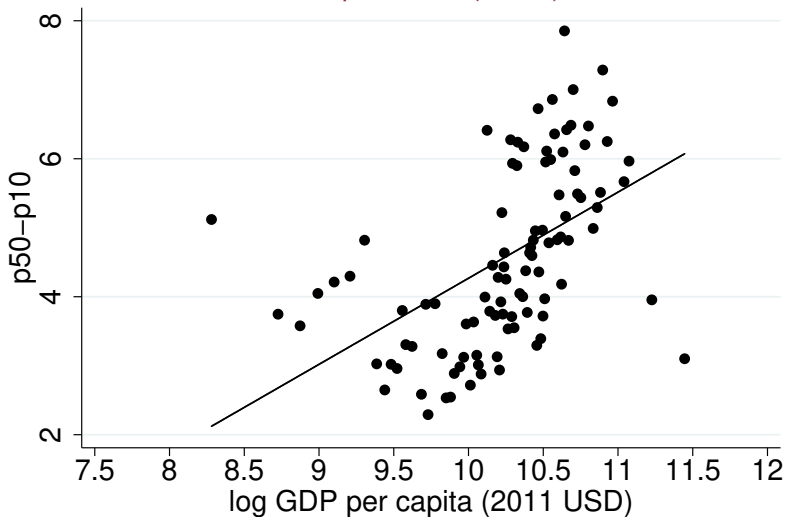


The median earnings grow faster than the mean ●



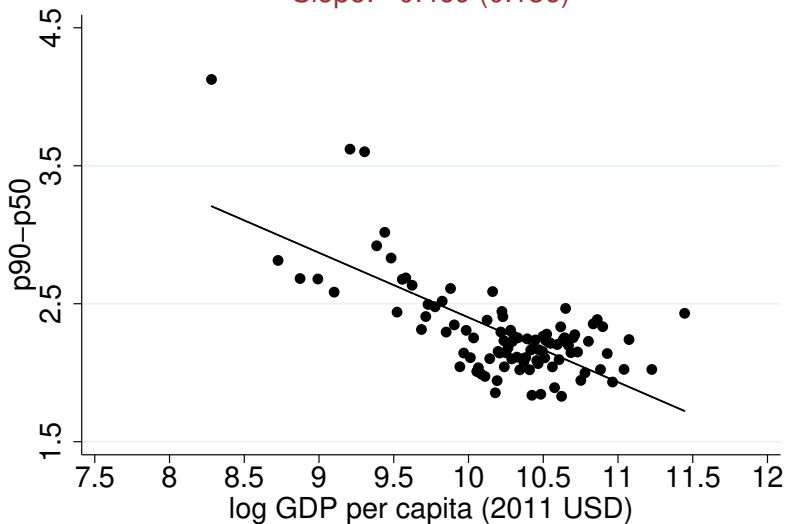
Inequality at the bottom increases... ●

Slope: 1.248 (0.440)



...while inequality at the top declines ●

Slope: -0.469 (0.136)



Evidence

- How does the earnings distribution change with development?
 - mean-median earnings ratio decline with development
 - earnings inequality at the bottom increases (p50-p10 ratio) while declining at the top (p90-p50)
- Robustness:
 - across sectors: no-agriculture, only industries ●
 - across education: non-college, college ●
 - across demographics: only males, only household heads, prime-age ●
 - other measures: p90-p60 vs p40-p10 ratios, p80-p50 vs p50-p20 ratios ●
 - conditional on controls ●
- Other evidence:
 - GINI ●
 - variance log-earnings ●

Model - Key Elements

- Search frictions as a source of misallocation (Lise et al 16, Poschke 19, Martellini and Menzio 20)
 - share of wage and salary employees increases with GDP p.c. ●
- Human capital accumulation and training (Bagger et al 14, Flinn et al 17)
 - life-cycle wage growth higher in richer countries (Lagakos et al 18)
 - on-the-job training increases with GDP p.c. ●
- Industry dynamics (Restuccia and Rogerson 08, Hsieh and Klenow 14, Fajgelbaum 20)
 - larger firms in richer countries (Bento and Restuccia 2018)
 - dispersion and skewness of firm size increase with GDP p.c. (Poschke 18)
 - larger firms pay higher wages (Bloom et al 18)
 - larger firms provide more on-the-job training ●

- Unitary measure of heterogeneous workers
 - stochastic life-cycle in the labor market
 - employed or non-employed
 - ex-ante exogenous skill, $a^0 \in \mathcal{A} = \{a_0, a_1, \dots, a_H\}$
 - life-cycle dynamics of skills:
 - on-the-job learning, with prob. p^e
 - on-the-job training, with prob. p^t
 - depreciation when non-employed, with prob. p^d
- Endogenous measure of heterogeneous firms
 - innate productivity, z , and training cost ξ
 - a firm is a collection of ℓ workers i , with distribution of skills $\psi_a^e(\cdot|z, \xi, \ell)$
 - entry-exit dynamics
 - exogenous firm exit, δ_f
 - exogenous and endogenous separation
 - workers' retirement, δ_w , exogenous destruction of a match, δ_s
 - endogenous destruction of a match if there is not enough surplus
 - firm growth bounded by convex vacancy costs

Production

- Firm-level production technology

$$y = \int_0^\ell g(z, i) \psi_a^e(i|z, \xi, \ell) di$$

where $\psi_a^e(i|z, \xi, \ell)$ is the pdf of workers i in a firm (z, ξ) with total workforce ℓ

- Firm-worker match production:

$$g(z, i) = za(i)$$

where $a(i)$ is the human capital of worker i

- Linearity of technology:

$$y = z\bar{a}\ell$$

where \bar{a} is the average human capital of workers employed in the firm

$$\bar{a} = \int_0^1 a(i) \psi_a^e(i|z, \xi, \ell) di$$

Distortions and frictions

- Firms subject to output distortions (Guner et al 16, Bento and Restuccia 18)
 - Each firm retains a fraction $1 - \tau(z)$ of its output

$$\tau(z) = 1 - z^{-\zeta}$$

where ζ is the elasticity of firm's distortion to its productivity

- Search and matching frictions (Mortensen and Pissarides 99)
 - CRS matching functions between searchers U (only non-employed) and aggregate vacancies v

$$m(U, v) = \chi \frac{Uv}{(U^\eta + v^\eta)^{\frac{1}{\eta}}}$$

where χ governs the efficiency of matching function

- flow value of non-employed, home production, b

Bargaining, training and hiring ●

- Wages are the solution to a Nash bargaining problem

$$w(z, \xi, a) = \arg \max_w \left[\underbrace{J^{e,h}(z, \xi, a; w) - J^{u,h}(a)}_{\text{worker surplus}} \right]^{\beta} \left[\underbrace{V^h(z, \xi, a; w)}_{\text{firm surplus}} \right]^{1-\beta}$$

where $\beta \in (0, 1)$ is the workers' bargaining power

- Training decision at a match level (Flinn et al 17)

$$\mathbf{1}^t(z, \xi, a) = \arg \max_{\mathbf{1}^t \in \{0,1\}} \mathbf{1}^t p^t [S^h(z, \xi, a+1) - S^h(z, \xi, a)] - \mathbf{1}^t \xi$$

p^t is the probability of skill jump and

$$S^h(z, \xi, a) = J^{e,h}(z, \xi, a) - J^u(a) + V^h(z, \xi, a)$$

- Match formation decision: $\mathbf{1}^h(z, \xi, a) = \begin{cases} 1 & \text{if } S^h(z, \xi, a) \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Firm vacancy posting and entry

- Per-period firm problem

$$\pi(z, \xi) = \max_{v \geq 0} v \phi_f \sum_{a \in \mathcal{A}} \max\{0, \underbrace{(1 - \beta) S^h(z, \xi, a)}_{V^h(z, \xi, a; w)}\} \psi_a^u(a) - c(v)$$

where

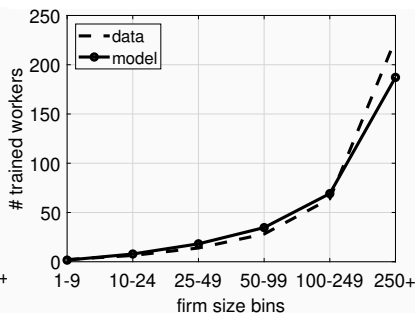
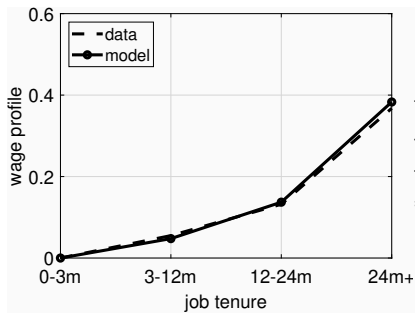
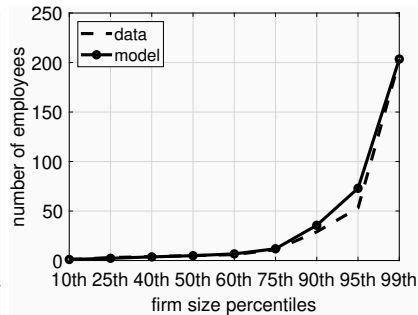
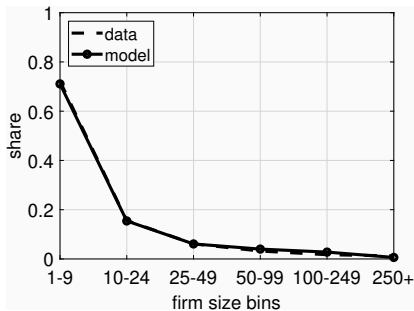
- ψ_a^u is the distribution of ability of the unemployed
 - $c(\cdot)$ are vacancy costs, with $c' > 0$, $c'' > 0$
 - ϕ_f is the vacancy contact probability
- Discounted sum of per-period aggregate profits

$$\Pi(z, \xi) = \sum_{t=0}^{\infty} \left(\frac{1 - \delta_f}{1 + r} \right)^t \pi(z, \xi) = \frac{1 + r}{r + \delta_f} \pi(z, \xi)$$

- Entry decision: $\mathbf{1}^e(z, \xi) = \begin{cases} 1 & \text{if } \Pi(z, \xi) \geq c^e \\ 0 & \text{otherwise} \end{cases}$
- No free-entry: exogenous measure of potential entrants M_e

- Baseline economy: UK, 2010-2016
 - Five-Quarter Longitudinal Labor Force Survey: workers age, employment status, job tenure, hours worked, OTJ training ●
 - The Employer Skill Survey: firm size, OTJ training ●
- Assumptions:
 - model period is a quarter
 - stationary equilibrium ●
 - no distortion ($\zeta = 0$), visibility is normalized ($\chi = 1$)
- Matching elasticity η estimated outside the model using GMM ●
- 3 parameters directly calibrated, $\theta_1 = \{r, \delta_w, \delta_f\}$
- 13 parameters estimated using MCMC (Chernozhukov and Hong 2003)

$$\theta_2 = \{\textcolor{red}{b}, c_e, \sigma_z, \underline{\xi}, \bar{\xi}, \lambda_1, M_e, \beta, \sigma_a, p^d, p^e, p^t, \delta_s\}.$$
- 40 worker- and firm-level targets ● non-targeted moments ●



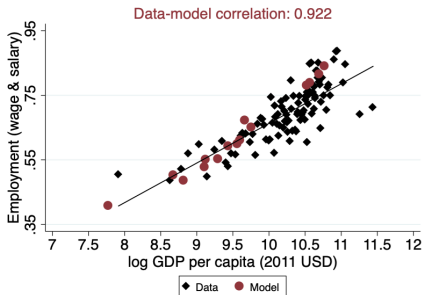
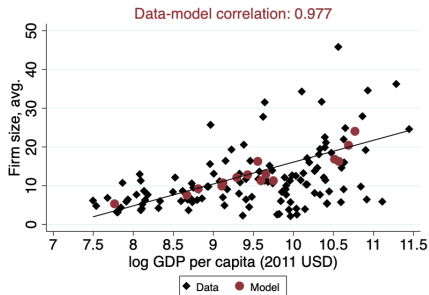
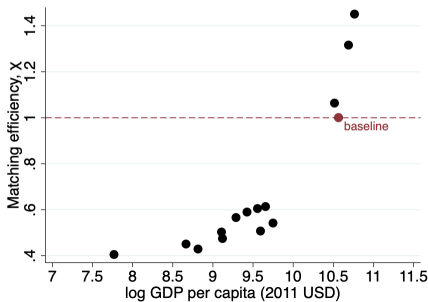
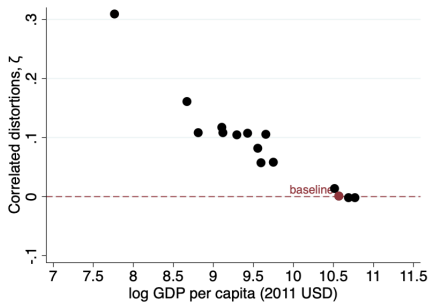
Estimates and standard errors ●

Parameters	Description	Value	Source/Targets
r	Interest rate	0.0033	annual return of 4%
δ_w	Workers retirement	0.0099	life-span of 40 years, ages 25-65
δ_f	Firm exit	0.0253	annual exit rate of 10.50% (ONS)
Parameters	Description	Estimates	Standard errors
c_e	Entry cost	39.262 (2004.21 USD)	3.6646
$\underline{\xi}$	Training cost (lower bound)	1.7346 (88.54 USD)	0.1569
$\bar{\xi}$	Training cost (upper bound)	26.668 (1361.32 USD)	2.3036
λ_1	Hiring costs, convexity	2.5246	0.1656
σ_z	Firm-productivity dispersion	1.2044	0.1060
M_e	Measure of potential entrants	0.0127	0.0444
δ_s	Match separation	0.0124	0.0012
b	Home production	20.943 (1068.92 USD)	1.8241
β	Bargaining power	0.4573	0.0416
σ_a	Initial human capital dispersion	1.1950	0.1110
p^e	Experience jump	0.2233	0.0194
p^t	Training jump	0.0282	0.0030
p^d	Depreciation jump	0.4318	0.0400

Accounting for Cross-Country Differences

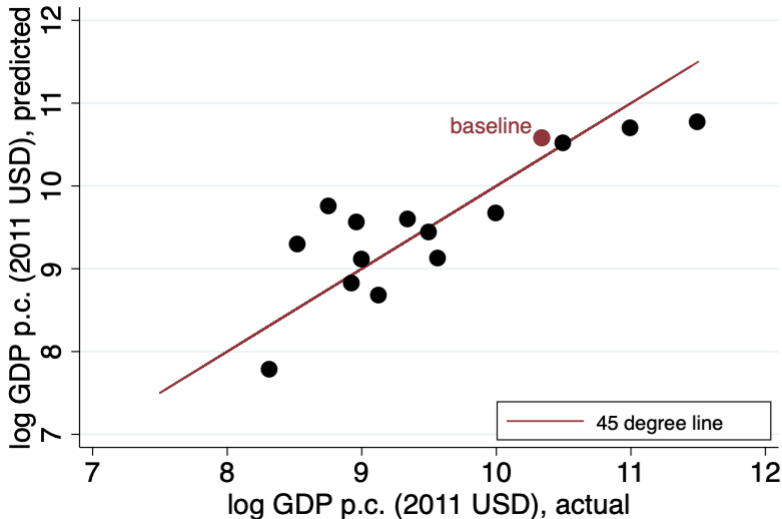
- Select ζ , extent of correlated distortions, and η , the elasticity of the matching function, to match:
 - average firm size
 - wage and salary employment
- Countries targeted: Brazil, Georgia, Indonesia, Peru, Serbia, South Africa, Poland, Mexico (+ 5 generic countries)
- We keep all other parameters, except b , at their benchmark value
 - adjust b to be the same fraction of average earnings as in the benchmark
- Identification of ζ and χ ●
- Alternative mechanisms ●
- Alternative counterfactual ●

Estimated distortions across countries

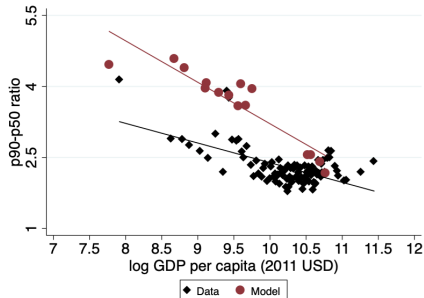
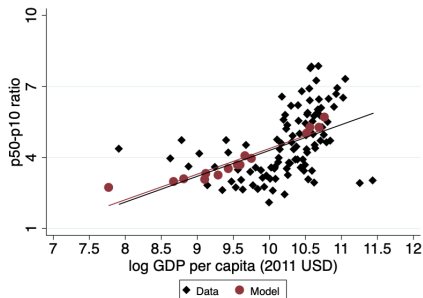
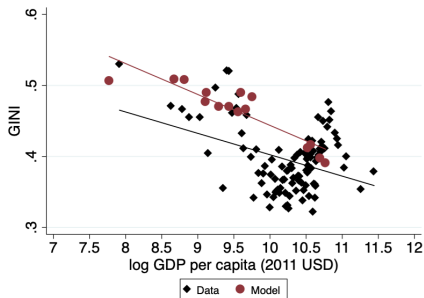
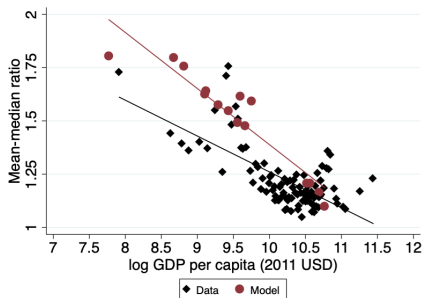


GDP p.c. across countries

Data-model correlation: 0.873



Earnings inequality across countries

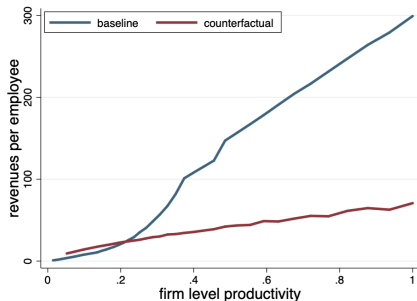


Beyond Earnings Inequality...

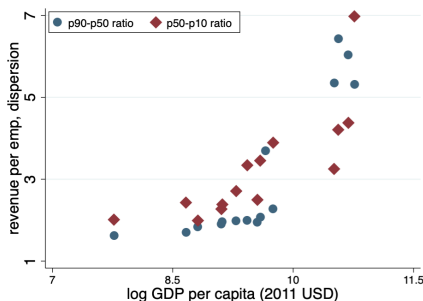
- Model is able to match several non-targeted cross-country patterns
 - changes in firm size distribution with development (Poschke 2019) ●
 - increase standard deviation of firm size
 - increase skewness of firm-size
 - changes in training patterns with development ●
 - increase share of firms providing training
 - increase share of workers receiving training
 - changes in wage growth with development ●
 - increase along life-cycle (Lagakos et al 2018)
 - decrease along job tenure (Donovan et al 2022)

- Zoom on alternative country: Indonesia
 - a low income country, about 1/10 of UK GDP p.c.
 - lower average firm size, 4.1 (versus 16.2 in UK) and lower share of wage and salary earners, 43.1% (versus 77.6% in UK)
- Correlated distortions: $\zeta = 0.308$ (vs. 0 in UK)
- Efficiency of the matching function: $\chi = 0.403$ (vs. 1 in UK)
- What happens when we move from UK to Indonesia?
- Frictions vs distortions ●

Revenue reallocation



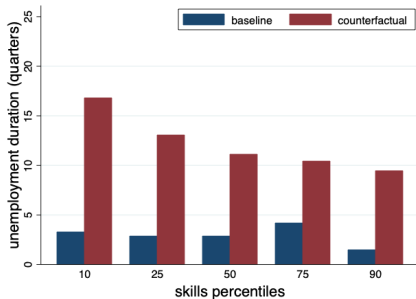
UK versus Indonesia



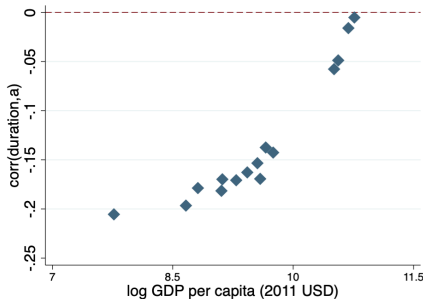
Across countries

- Higher correlated distortions imply more progressive output taxes
 - lower difference in revenues per employee between productive and unproductive firms

Non-employment duration



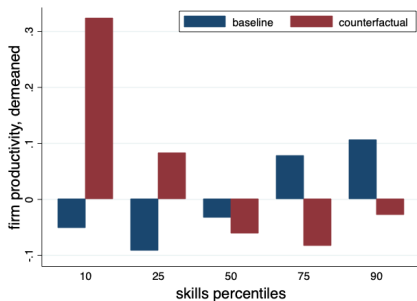
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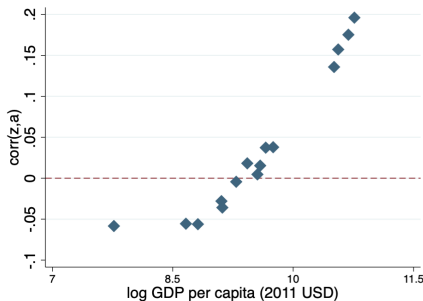
Across countries

- As a country gets richer, non-employment duration shrinks and becomes more uniform across workers with different skills
 - higher participation in wage employment allows low-skill workers to avoid skill depreciation

Worker-firm sorting



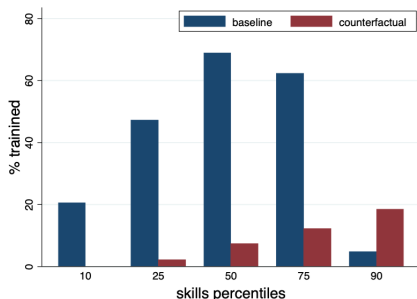
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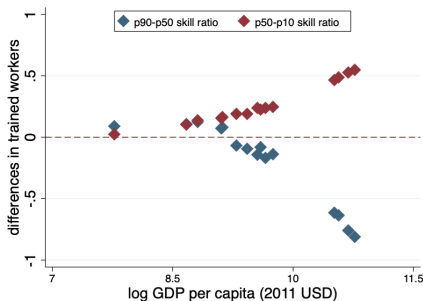
Across countries

- Negative sorting in poorer countries, correlation between workers' ability and firms' productivity increases with GDP p.c.

The role of OTJ training



UK versus Indonesia



Across countries

- Training mainly helps workers around the median of the earnings distribution. Distortions and frictions:
 - reduce the revenue gains from training, $g(z, a) = z^{1-\zeta}a$
 - lowers job finding rate ϕ_w and reduce workers outside options

The role of OTJ training

- What happens when we shut down on OTJ training?

	Explained
<i>Aggregates</i>	
Non-employment rate	37.96%
Average wage	1.838%
Income per capita	2.694%
<i>Wage profile</i>	
over experience, $E[\log(w_{25}/\bar{w}_1)]$	29.45%
over tenure ≥ 24 months	32.49%
<i>Wage inequality</i>	
Mean-median wage ratio	35.13%
GINI	20.99%

- OTJ training account up to 35% of changes in earnings inequality
- Large scale re-training program increases average wage by 16%

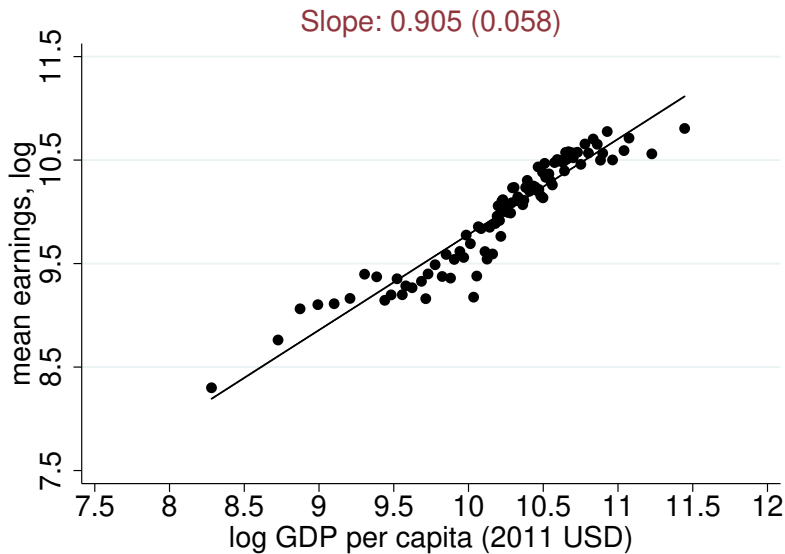
Conclusion

- We document how the distribution of labor earnings varies with development
 - inequality at the top shrinks, inequality at the bottom expands
 - the median increases faster than the mean
 - GINI declines
- We build a model of labor market to interpret this evidence
 - positive sorting between workers and firms
 - OTJ training provided by larger (and more productive) firms
- Cross-country patterns can be reproduced by two sources of misallocation
 - firm-level correlated distortions
 - lower labor market visibility
- OTJ training account up to 35% of changes in earnings inequality
- Alternative mechanisms ●

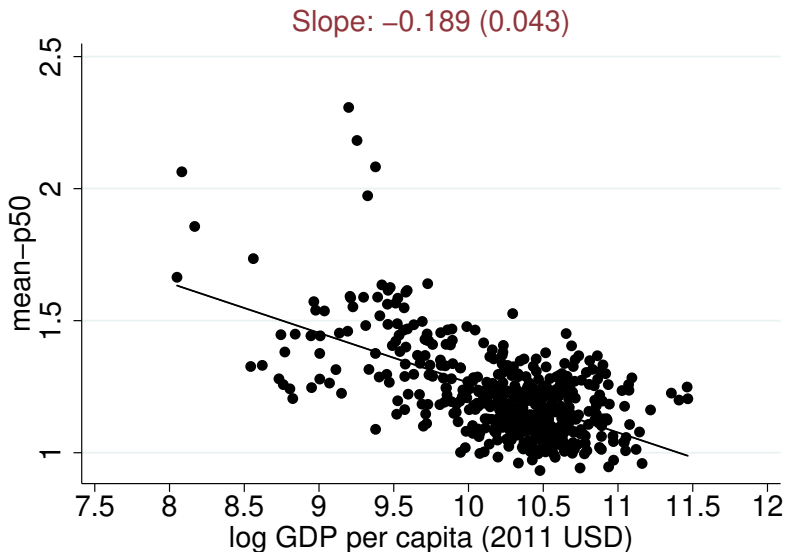
Data Source

Country	Year	Source	Country	Year	Source
Austria	2005, 2010	EU-SILC	Latvia	2006, 2010	EU-SILC
Belgium	2005, 2009	EU-SILC	Lithuania	2006, 2009	EU-SILC
Bulgaria	2007	EU-SILC	Luxembourg	2005, 2010	EU-SILC
Croatia	2010	EU-SILC	Malta	2007, 2010	EU-SILC
Cyprus	2005, 2010	EU-SILC	Netherlands	2006, 2010	EU-SILC
Czech republic	2006, 2009	EU-SILC	Norway	2005, 2010	EU-SILC
Denmark	2005, 2009	EU-SILC	Panama	1970	IPUMS
Dominican Republic	1981	IPUMS	Poland	2005, 2009	EU-SILC
Estonia	2005, 2010	EU-SILC	Portugal	2005, 2010	EU-SILC
Finland	2005, 2009	EU-SILC	Puerto Rico	1990, 2000, 2005	IPUMS
France	2005, 2010	EU-SILC	Romania	2007, 2009	EU-SILC
Germany	2005, 2009	EU-SILC	Slovakia	2006, 2009	EU-SILC
Greece	2005, 2009	EU-SILC	Slovenia	2006, 2009	EU-SILC
Hungary	2006, 2010	EU-SILC	Spain	2005, 2009	EU-SILC
Iceland	2005, 2010	EU-SILC	Sweden	2005, 2009	EU-SILC
Israel	1995	IPUMS	Switzerland	2007, 2009	EU-SILC
Italy	2005, 2009	EU-SILC	Trinidad and Tobago	2000	IPUMS
India	1993, 1999	IPUMS	USA	2000, 2005, 2010	IPUMS
Indonesia	1976, 1995	IPUMS	Uruguay	2006	IPUMS
Ireland	2005, 2009	EU-SILC	United Kingdom	2005, 2009	EU-SILC
Jamaica	1981, 1991, 2001	IPUMS			

Wage and salary earnings

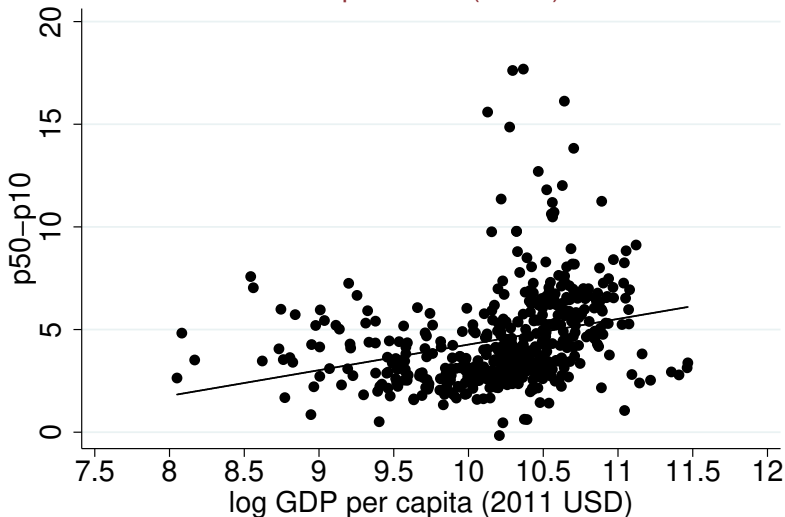


The median earnings grow faster than the mean



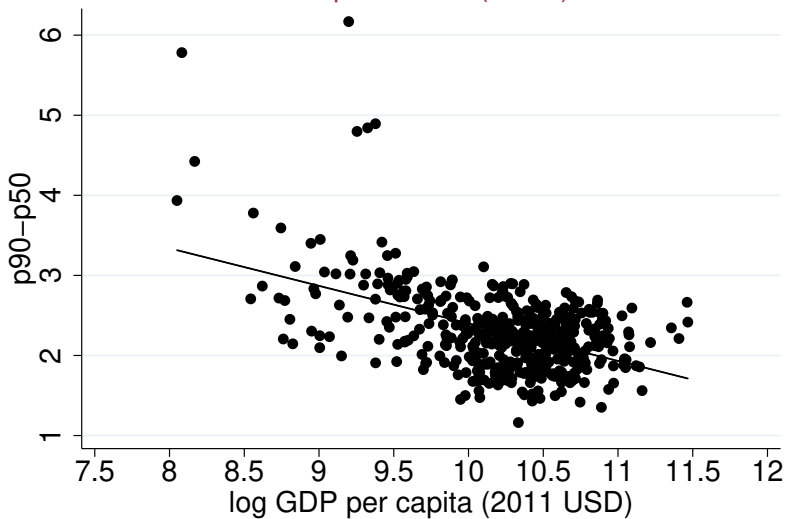
Inequality at the bottom increases...

Slope: 1.248 (0.440)

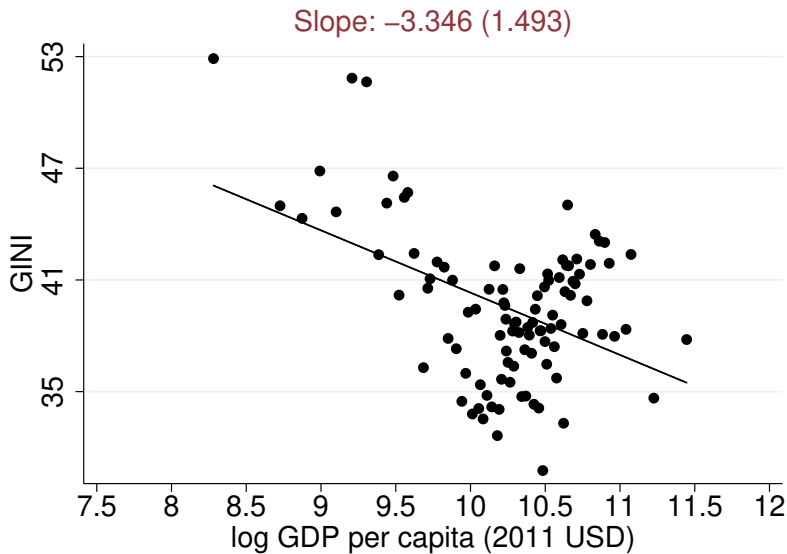


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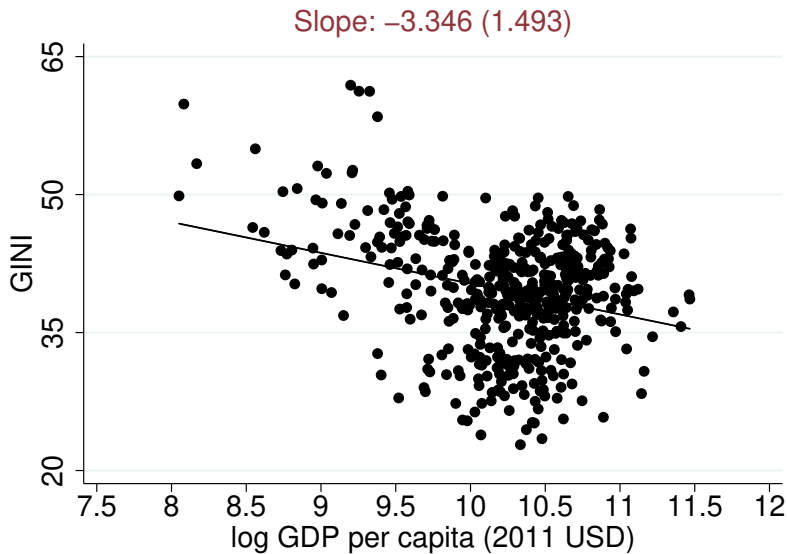
Slope: -0.469 (0.136)

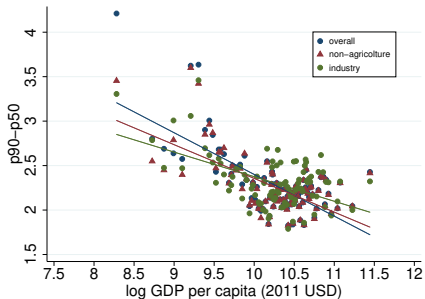
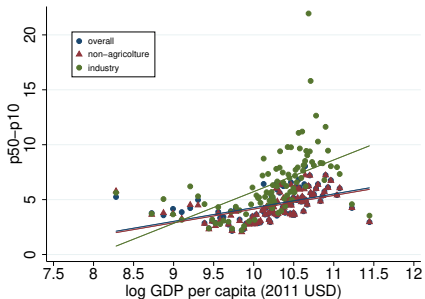
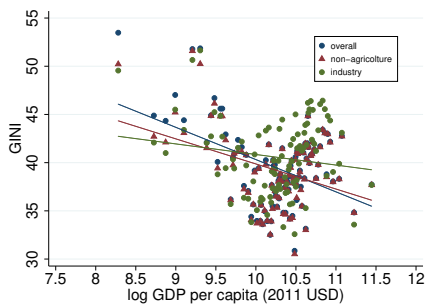
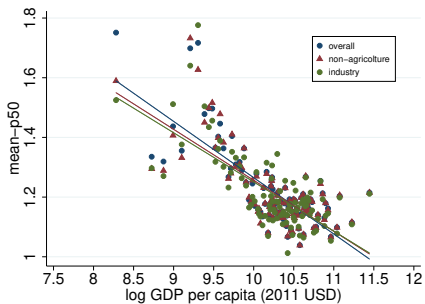


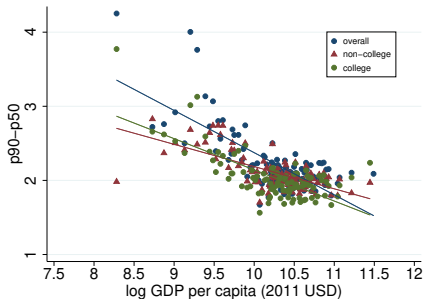
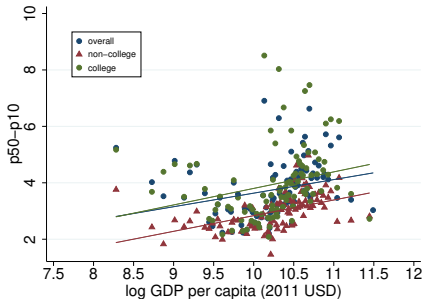
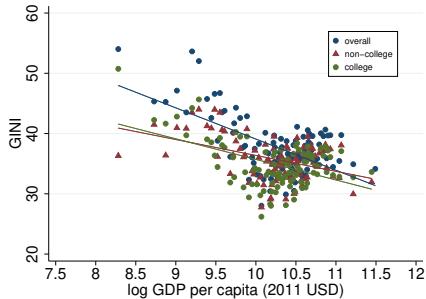
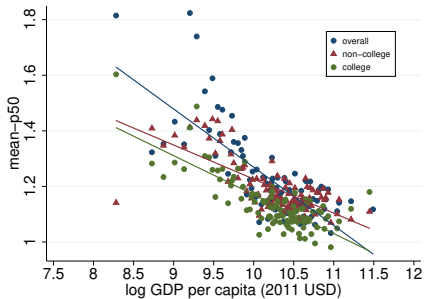
The Gini coefficient declines ●

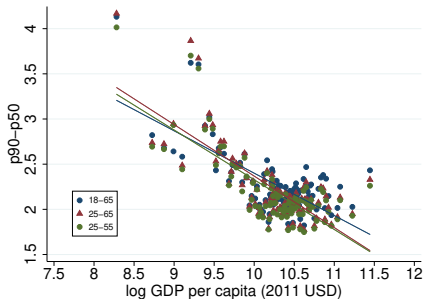
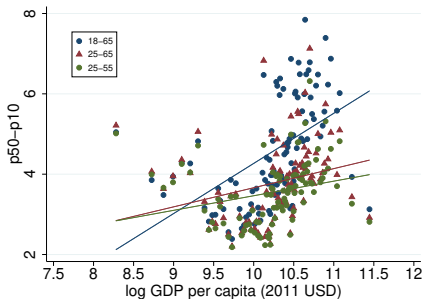
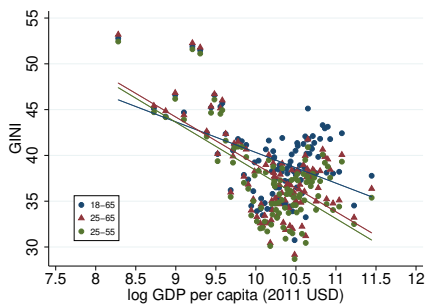
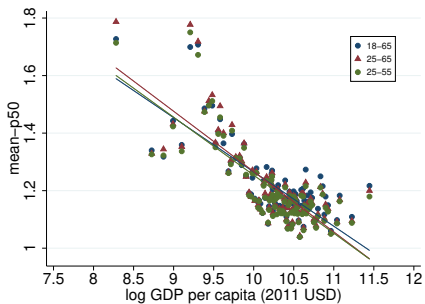


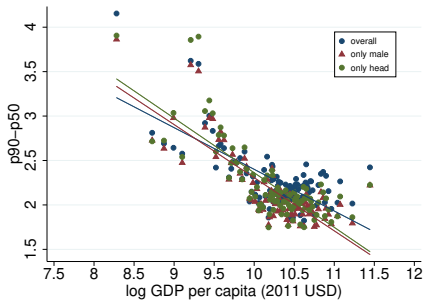
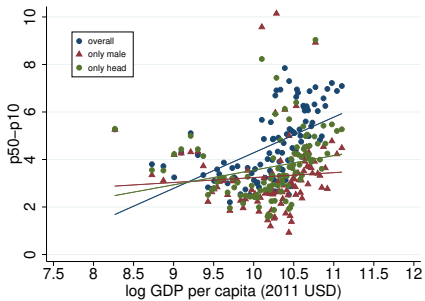
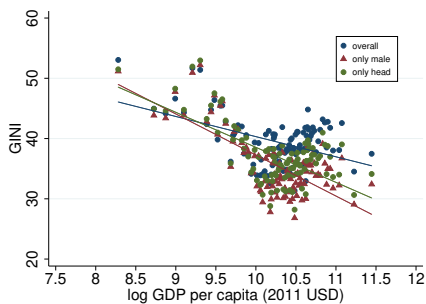
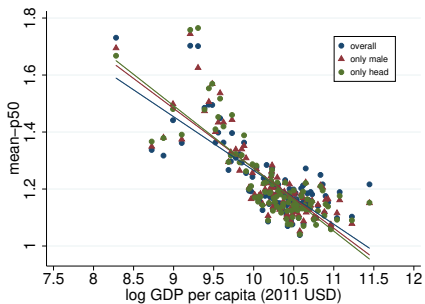
GINI coefficient

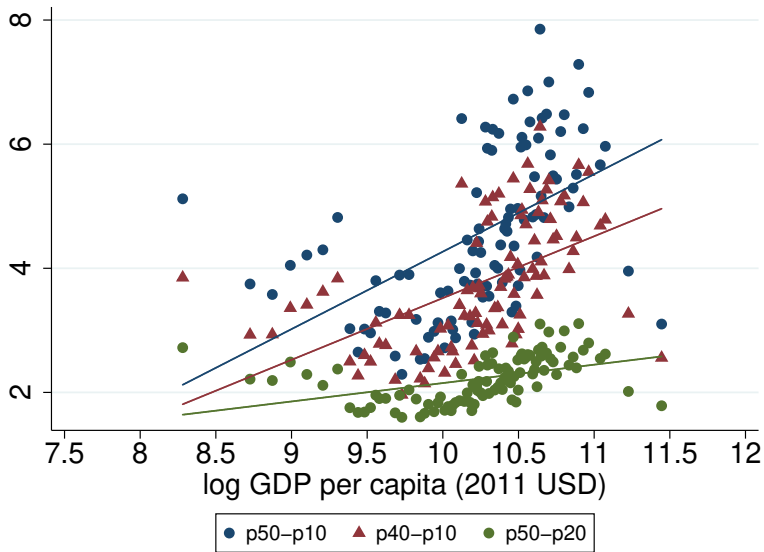


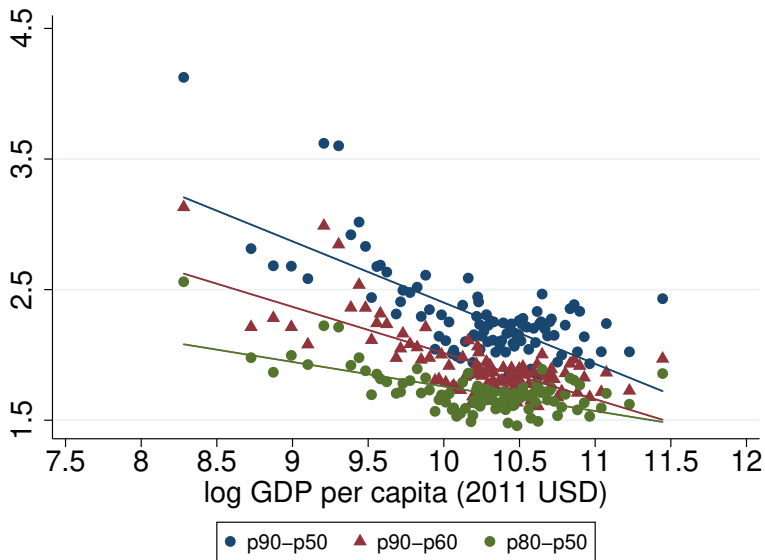












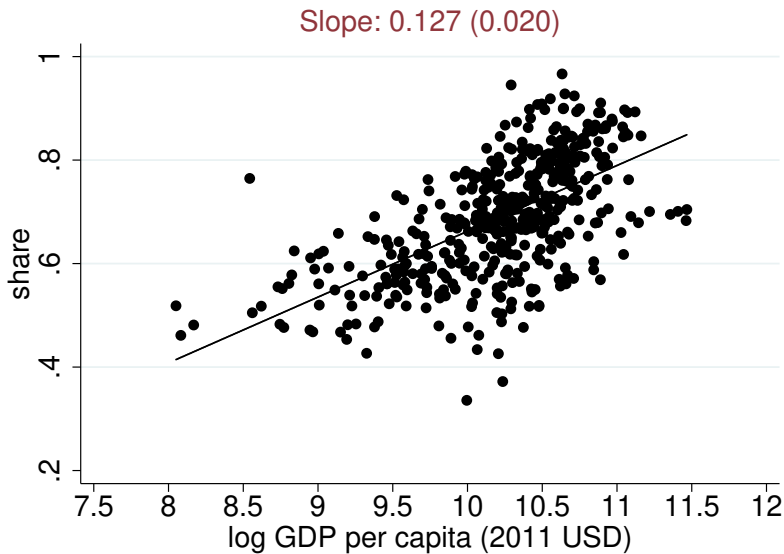
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	Mean-median ratio			GINI		
	(1)	(2)	(3)	(1)	(2)	(3)
log GDP p.c.	-0.171*** (0.0386)	-0.189*** (0.0429)	-0.229*** (0.0549)	-3.040** (1.389)	-3.346** (1.493)	-4.551* (2.603)
Observations	497	497	420	497	497	420
R-squared	0.286	0.420	0.690	0.067	0.194	0.499
Time FE		✓	✓		✓	✓
Controls			✓			✓

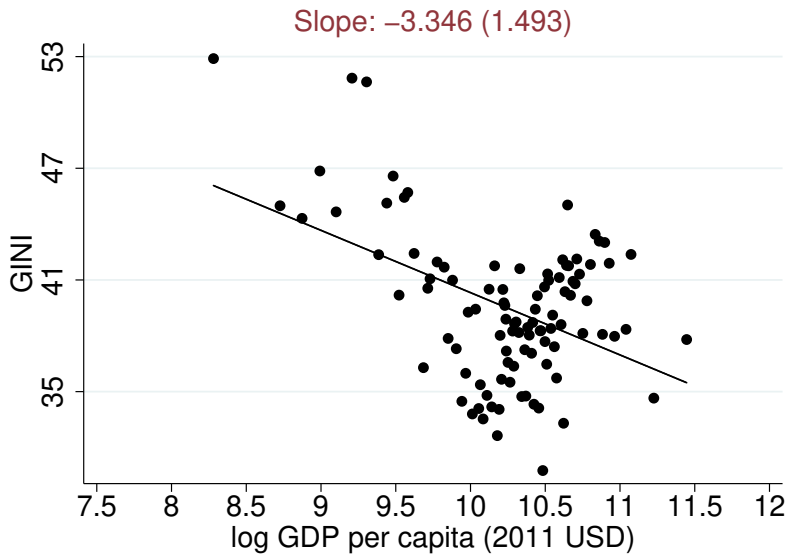
	p50-p10 ratio			p90-p50 ratio		
	(1)	(2)	(3)	(1)	(2)	(3)
log GDP p.c.	1.123*** (0.397)	1.248*** (0.440)	1.797*** (0.437)	-0.423*** (0.126)	-0.469*** (0.136)	-0.570*** (0.203)
Observations	497	497	420	497	497	420
R-squared	0.069	0.136	0.308	0.201	0.323	0.557
Time FE		✓	✓		✓	✓
Controls			✓			✓

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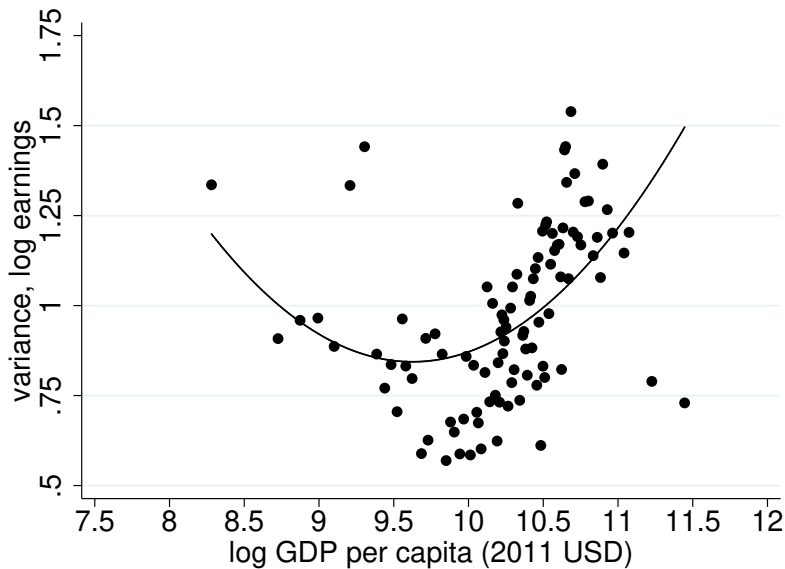
Wage and salary employees



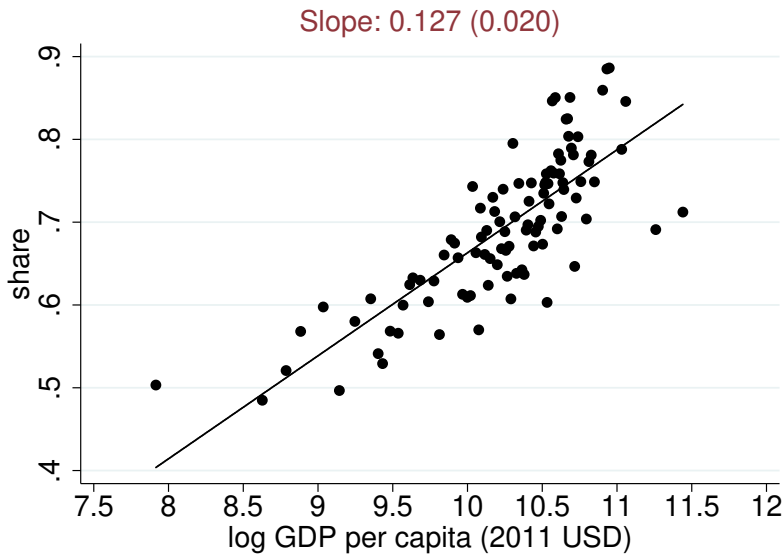
The Gini coefficient declines



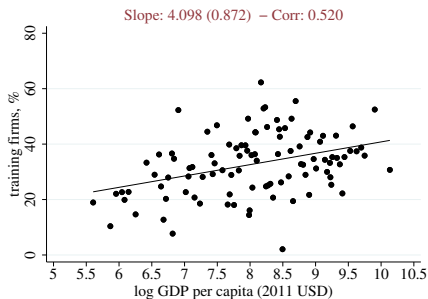
U-shape of log-variance



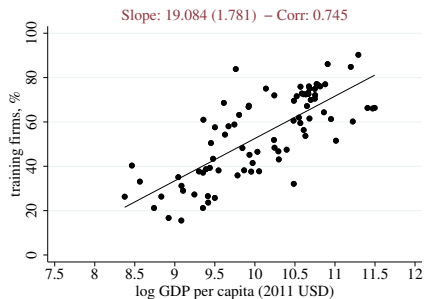
Wage and salary employees



Share of training firms



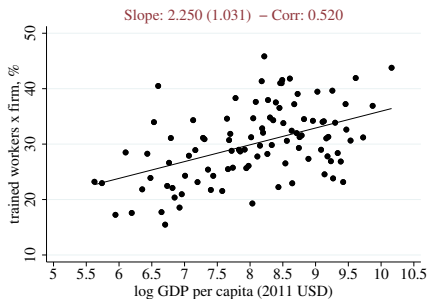
Source: WB Enterprise Survey



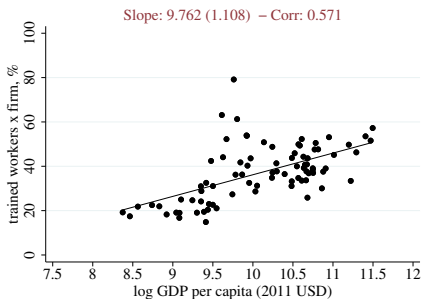
Source: EC Education and Training Dataset

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Share of workers trained in the firms



Source: WB Enterprise Survey



Source: EC Education and Training Dataset

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Share of training firms, by firm size

Training firms, %							
	WB-ES					CVTS	
	LAC	ME+AFR	ASIA	others		EU15	non-EU15
Firm size (# employees)					Firm size (# employees)		
<20	34.84	18.42	19.32	26.35	<20	44.79	29.18
20-49	54.31	31.99	33.63	38.48	20-49	56.00	39.36
50-249	66.94	41.31	47.02	46.47	50-249	71.67	52.82
250-449	81.13	56.86	47.32	56.65	250-449	86.29	67.64
≥500	92.12	68.45	52.28	68.88	500-999	88.00	78.45
					≥1000	96.36	88.73

Source: World-Bank Enterprise Survey and Eurostat Education and Training Dataset.

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Share of trained workers, by firm size

Trained workers within firms, %							
	WB-ES					CVTS	
	LAC	ME+AFR	ASIA	others		EU15	non-EU15
Firm size (# employees)					Firm size (# employees)		
<20	34.36	21.01	27.95	29.63	<50	29.31	21.96
20-49	40.06	25.56	29.72	30.18	50-249	37.92	30.13
50-249	44.35	26.68	35.51	30.36	≥500	49.71	46.25
250-449	52.51	30.30	32.22	28.86			
≥500	50.73	32.37	34.34	28.98			

Source: World-Bank Enterprise Survey and Eurostat Education and Training Dataset.

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Workers value functions

- The workers' value of being not-employed

$$\begin{aligned} J^u(a) = & J^{u,h}(a) + (1 - \phi_w) p^d \underbrace{[J^{u,h}(a-1) - J^{u,h}(a)]}_{\text{loss from skill depreciation}} \\ & + \phi_w \int_{z,\xi} \mathbf{1}^h(z, \xi, a) \underbrace{[J^{e,h}(z, \xi, a; w) - J^{u,h}(a)]}_{\text{gain from match formation}} \psi_v(z, \xi) d\xi dz, \end{aligned}$$

where

$$J^{u,h}(a) = b + \frac{(1 - \delta^w)}{1 + r} J^u(a).$$

and

- $\mathbf{1}^h(z, \xi, a)$: match formation policy function
- $\psi_v(z, \xi)$: p.d.f. of open vacancies across firms' states
- job finding, $\phi_w = M(U, v)/U$

Workers value functions

- The workers' value of being employed:

$$J^e(z, \xi, a; w) = J^{e,h}(z, \xi, a; w) + (1 - \mathbf{1}^h(z, \xi, a)) \underbrace{[J^{u,h}(a) - J^{e,h}(z, \xi, a; w)]}_{\text{gain from separation}},$$

where

$$\begin{aligned} J^{e,h}(z, \xi, a; w) &= w + \frac{(1 - \delta^w)}{1 + r} J^e(z, \xi, a; w) \\ &\quad + \frac{(1 - \delta^w)}{1 + r} (\delta_f + (1 - \delta_f)\delta_s) \underbrace{[J^{u,h}(a) - J^e(z, \xi, a; w)]}_{\text{loss from separation}} \\ &\quad + \frac{(1 - \delta^w)}{1 + r} (1 - \delta_f)(1 - \delta_s) \tilde{J}^{e,h}(z, \xi, a; w) \end{aligned}$$

and

$$\begin{aligned} \tilde{J}^{e,h}(z, \xi, a; w) &= p^h(z, \xi, a) \underbrace{[J^e(z, \xi, a + 1; w') - J^e(z, \xi, a; w)]}_{\text{gain from skill accumulation}} \\ p^h(z, \xi, a) &= p^e + \underbrace{\mathbf{1}^t(z, \xi, a)}_{\text{training policy}} p^t \end{aligned}$$

Firm value functions

- The firms' value of an active match:

$$(z, \xi, a; w) = V^h(z, \xi, a; w) + (1 - \mathbf{1}^h(z, \xi, a; w)) \underbrace{[0 - V^h(z, \xi, a)]}_{\text{gain from separation}},$$

where

$$\begin{aligned} V^h(z, \xi, a; w) = & (1 - \tau(z))g(z, a) - w + \frac{(1 - \delta_w)}{1 + r} V(z, \xi, a) \\ & + \frac{(1 - \delta_w)}{1 + r} (\delta_f + (1 - \delta_f)\delta_s) \underbrace{[0 - V(z, \xi, a; w)]}_{\text{loss from separation}} \\ & + \frac{(1 - \delta_w)}{1 + r} (1 - \delta_f)(1 - \delta_s) \tilde{V}^h(z, \xi, a; w) \end{aligned}$$

and

$$\tilde{V}^h(z, \xi, a; w) = -\mathbf{1}^t(z, \xi, a)\xi + p^h(z, \xi, a) \underbrace{[V(z, \xi, a + 1; w') - V(z, \xi, a; w)]}_{\text{gain from skill accumulation}}$$

The surplus function

- The value of a match:

$$m(z, \xi, a) = J^{u,h}(a) + \max\{0, m^h(z, \xi, a) - J^{u,h}(a)\}$$

where

$$\begin{aligned} m^h(z, \xi, a) = & (1 - \tau(z))g(z, a) + \frac{(1 - \delta_w)}{1 + r}(1 - (1 - \delta_f)(1 - \delta_s))J^{u,h}(a) \\ & + \frac{(1 - \delta_w)}{1 + r}(1 - \delta_f)(1 - \delta_s)m(z, \xi, a) \\ & + \frac{(1 - \delta_w)}{1 + r}(1 - \delta_f)(1 - \delta_s)p^e[m(z, \xi, a + 1) - m(z, \xi, a)] \\ & + \frac{(1 - \delta_w)}{1 + r}(1 - \delta_f)(1 - \delta_s) \max\{0, -\xi + p^t[m(z, \xi, a + 1) - m(z, \xi, a)]\} \end{aligned}$$

and

$$S(z, \xi, a) = m(z, \xi, a) - J^{u,h}(a)$$

Equilibrium

A stationary RCE consists of workers' and firms' value functions, policy functions for job creation, training, firms' entry and vacancy posted, wage schedule, job contact probabilities for workers and firms, unemployment rate, distribution of employed and unemployed workers across states, distribution of vacancies and firms across states, s.t.:

- *optimality*: the value functions attain their maximum;
- *bargaining*: the wage schedule is the solution of the bargaining problem;
- *training*: training policies maximise surplus;
- *market clearing*: goods and labor market are cleared;
- *measure of entrants*: for all Borel sets $\mathcal{Z} \times \mathcal{E} \subset \mathcal{R}^+ \times \mathcal{R}^+$ it must be that

$$E(\mathcal{Z} \times \mathcal{E}) = M_e \int_{z \in \mathcal{Z}} \int_{\xi \in \mathcal{E}} \mathbf{1}^e(z, \xi) \psi_z(z) \psi_\xi(\xi) dz d\xi$$

where M_e is the measure of potential entrants

- *measure of incumbent*: for all Borel sets $\mathcal{Z} \times \mathcal{E} \subset \mathcal{R}^+ \times \mathcal{R}^+$ it must be that

$$\Gamma(\mathcal{Z} \times \mathcal{E}) = \frac{1}{\delta_f} E(\mathcal{Z} \times \mathcal{E})$$

- *aggregate consistency*: workers' and vacancies' distributions replicate themselves through workers' and firms' policy functions. [back](#)

Elasticity of matching function

- η is estimated to minimize the following objective function:

$$\arg \max_{\{x_0, x_1, x_2, x_3\}} \left[\left(\frac{1}{T} \sum_{t=1}^T Z'_t \epsilon_t(x) \right)' W_T \left(\frac{1}{T} \sum_{t=1}^T Z'_t \epsilon_t(x) \right) \right]$$

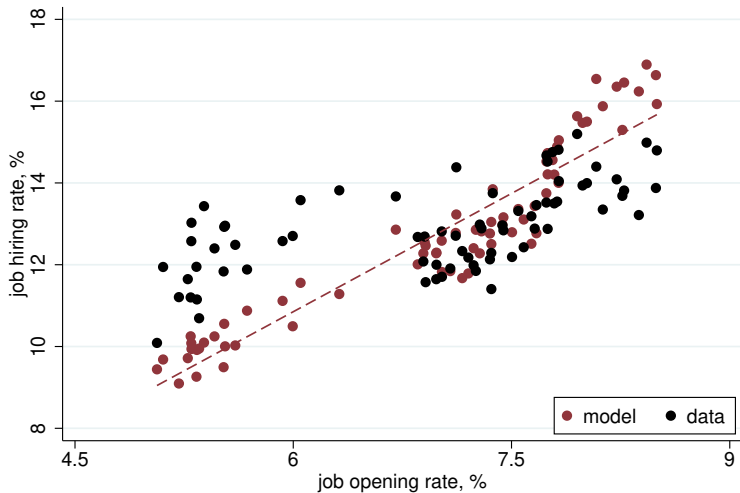
where $\epsilon_t(x)$ denotes the moment conditions, i.e.

$$\epsilon_t(x) = \left[h_t - \frac{u_t v_t}{(u_t^{x_0} + v_t^{x_0})^{\frac{1}{x_0}}} - \sum_{i=1}^4 x_i \mathbf{1}_t^{q=i} \right]$$

with h_t equal to the number of new hirings at time t , v_t the number of open vacancy and u_t the number of non-employed workers

- Seasonal effects removed by including dummies for quarters
- The vector of instruments, Z'_t includes fourth lags for non-employment and active vacancies
- Two-step GMM: estimate of $\hat{\eta} = \hat{x}_0 = 0.5417$ with a s.e. = 0.0134

Data-model correlation: 0.728



Selected targeted moments

	Data	Model		Data	Model
<i>Firm-level employment</i>			<i>Worker wage distribution</i>		
Average firm size, $E(\ell_t)$	16.42	16.19	Wage at entry, $E[\log(w_1/\bar{w})]$	-0.518	-0.505
Average log-firm size, $E(\log \ell_t)$	1.739	1.700	Wage after 20 y.o., $E[\log(w_{20}/\bar{w})]$	0.107	0.109
Dispersion log-firm size, $\text{std}(\log \ell_t)$	1.220	1.392	Wage at re-emp, $E[\log(w_R/\bar{w})]$	-0.301	-0.170
<i>Firm training provision</i>			Dispersion at entry, $\text{sd}[\log w_1]$	0.582	0.675
$E\left(\frac{\# \text{training firms}}{\# \text{firms}}\right)$			Dispersion after 20 y.o., $\text{sd}[\log w_{20}]$	0.796	0.795
All firms	0.646	0.650	Dispersion at re-emp, $\text{sd}[\log w_R]$	0.834	0.833
Firms with 1-49 employees	0.611	0.644	<i>Worker-level training return</i>		
Firms with 20-249 employees	0.776	0.714	$\log w_{it} = \beta_1 \mathbf{1}_{it}^t + \epsilon_{it}$	0.199	0.208
Firms with 250+ employees	0.855	0.888	<i>Aggregate moments</i>		
$E\left(\frac{\# \text{trained employees}}{\# \text{employees}}\right)$			Job duration	6.700	6.185
All firms	0.436	0.482	Employment rate	0.776	0.788

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abor Force Survey - summary statistics

	Mean	SD	Min	Max	N
<i>Employed workers</i>					
Age	41.63	11.64	22	62	85,524
Female	0.505	0.500	0	1	85,524
Full-time	0.755	0.430	0	1	85,524
Hours worked	37.04	12.10	1	97	85,524
Log Hourly pay	2.385	0.599	0.025	7.248	85,524
Log Quarterly Earnings	8.457	0.824	3.956	13.39	85,524
Training	0.244	0.430	0	1	85,524
Tenure<3 months	0.038	0.191	0	1	85,524
Tenure \in [3,12) months	0.039	0.192	0	1	85,524
Tenure \in [12,24) months	0.109	0.311	0	1	85,524
Tenure \geq 24 months	0.815	0.388	0	1	85,524

Source: Five-Quarter Longitudinal LFS, 2010-2016

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Employer Skill Survey - summary statistics

	Mean	SD	Min	Max	N
Size (# of employees)	16.42	73.64	1	10000	182,558
Training firms, share	0.668	0.471	0	1	182,558
Trained workers, #	9.147	58.77	0	9000	171,574
Trained workers, share	0.435	0.407	0	1	171,574
Training days x worker	8.196	16.15	1	260	111,254

Source: The Employer Skill Survey, 2010-2016

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Chernozhukov and Hong's MCMC

- Simulate a chain of parameters that has the quasi-posterior density

$$p(\theta) = \frac{e^{\mathcal{L}_n(\theta)} \pi(\theta)}{\int_{\theta} e^{\mathcal{L}_n(\theta)} \pi(\theta) d\theta}$$

$\mathcal{L}_n(\theta) = |m_n(\theta) - \bar{m}|$ is the distance between simulated and observed moments

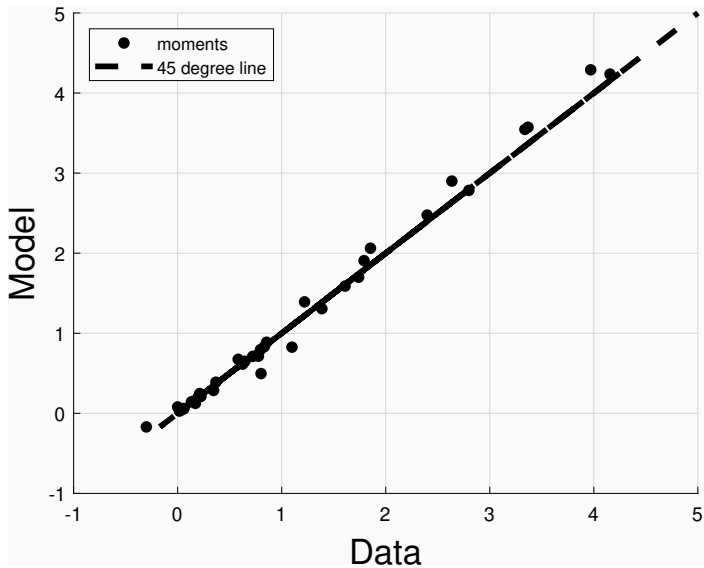
- Point estimates and st.errors are obtained as the average and st.dev. of n_s elements of the converged MCMC chain:

$$\hat{\theta} = \frac{1}{n_s} \sum_{j=1}^{n_s} \theta^j \quad \text{st.error}(\theta) = \sqrt{\frac{1}{n_s - 1} \sum_{j=1}^{n_s} (\theta^j - \hat{\theta})^2}$$

- Metropolis-Hasting algorithm to simulate a chain of θ^j with quasi-posterior $p(\theta)$
 - given last iteration θ^j , draw new guess θ' from proposal density $q(\theta'|\theta^j)$
 - if prior is uniform and proposal density is random walk, the acceptance rule is:

$$d(\theta^j, \theta') = \min\{1, e^{\mathcal{L}_n(\theta') - \mathcal{L}_n(\theta^j)}\}$$

Estimation fit



Non-targeted moments

	Data	Model
<i>Wage-size regression</i>		
<10 employees	0	0
$\in [10, 25)$ employees	0.151	0.183
$\in [25, 50)$ employees	0.244	0.342
$\in [50, 250)$ employees	0.407	0.680
≥ 250 employees	0.586	1.039
<i>Wage inequality</i>		
Log-wage dispersion, $\text{sd}[\log w_{it}]$	0.779	0.852
Mean-median wage ratio, $E[w_{it}]/p^{50}[w_{it}]$	1.276	1.207

- positive and large wage-size premium (Elsby and Michaels 2013)
- large dispersion in earnings (Hornstein et al 2011)

UK vs. Indonesia

	UK Baseline	Indonesia Counterfactual
<i>Firm-level moments</i>		
Average firm size, $E(\ell_t)$	16.19	5.179
Firm size dispersion, $\text{std}(\ell_t)$	37.16	4.576
Firm size skewness, $\text{skew}(\ell_t)$	5.178	1.652
<i>Firm training provision</i>		
$E\left(\frac{\#\text{training firms}}{\#\text{firms}}\right), \%$	65.02	6.210
<i>Wage profile over experience/tenure</i>		
Wage growth, $E[\log(w_{25}/w_1)]$	0.801	0.230
Wage at tenure ≥ 24 months	0.389	0.583
<i>Worker-level firm-size wage premium</i>		
$\log w_{it} = \beta_1 \log \ell_{it} + \epsilon_{it}$	0.066	0.139
<i>Training firm wage premium</i>		
$\log w_{jt} = \beta_1 \mathbf{1}_{jt}^t + \epsilon_{jt}$	0.039	0.083
<i>Aggregates</i>		
Non-employment rate	0.212	0.593
Average wage	1	0.124
Income per capita	1	0.061

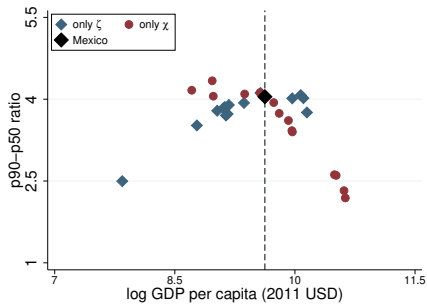
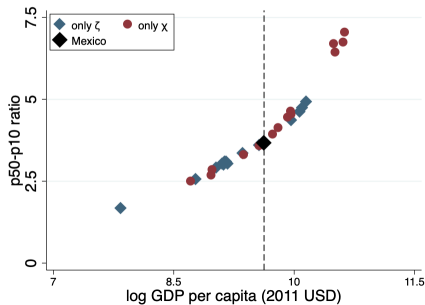
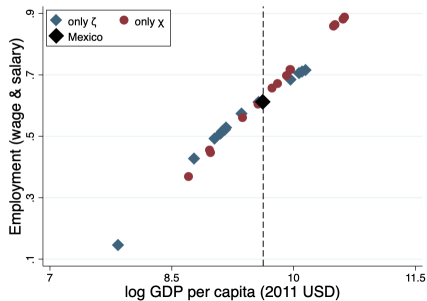
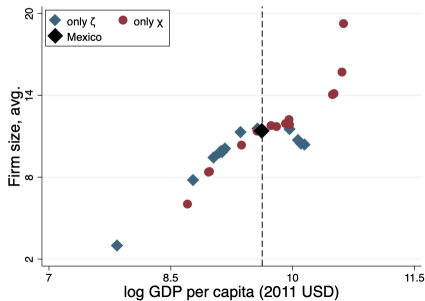
Implications for wage inequality - UK vs. Indonesia

	UK	Indonesia	
	Baseline	Counterfactual	Data
Efficiency of matching function: χ	1	0.403	-
Distortion correlation: ζ	0	0.308	-
Home production: b	20.94	3.505	-
Mean-median wage ratio, $E[w_{it}]/p^{50}[w_{it}]$	1.207	1.805	1.687
GINI	0.416	0.506	0.502
90-50 pct. wage ratio, $p^{90}[w_{it}]/p^{50}[w_{it}]$	2.551	4.462	3.182
50-10 pct. wage ratio, $p^{50}[w_{it}]/p^{10}[w_{it}]$	5.262	2.729	1.934

Understanding cross-country changes in inequality

- Recall that ζ , extent of correlated distortions, and χ , the elasticity of the matching function, are chosen to match:
 - average firm size
 - wage and salary employment
- Suppose we change only ζ or χ , what happens?
 - Take a country at the middle of the GDP per capita distribution
 - Then change only ζ (keeping χ and b fixed), and change χ (keeping ζ and b fixed)
- Focus on p50-p10 and p90-p50

Frictions vs. Distortions



Frictions vs. Distortions

- Impact on firm size, wage employment and GDP p.c.:
 - search frictions more important in richer countries
 - correlated distortions more important in poorer countries
- Impact on earnings inequality:
 - search frictions alone generate increase in bottom inequality and decline in top inequality
 - correlated distortions increase inequality at both ends of distribution

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The role of OTJ training

	Baseline with OTJ training	Counterfactual	Baseline w/o OTJ training	Counterfactual	Explained
Efficiency of matching function: χ	1	0.403	1	0.403	-
Distortion correlation: ζ	0	0.308	0	0.308	-
Home production: b	20.94	3.505	20.94	3.505	-
<i>Aggregates</i>					
Non-employment rate	0.2116	0.5925	0.2028	0.4391	37.96%
Average wage	1	0.1241	1	0.1402	1.838%
Income per capita	1	0.0611	1	0.0864	2.694%
<i>Wage profile over experience/tenure</i>					
Wage growth, $E[\log(w_{25}/\bar{w}_1)]$	0.8013	0.2797	0.7308	0.3628	29.45%
Wage at tenure ≥ 24 months	0.3893	0.4241	0.3697	0.4768	32.49%
<i>Wage inequality</i>					
Mean-median wage ratio	1.2067	1.8047	1.2795	1.6674	35.13%
GINI	0.4160	0.5061	0.4162	0.4874	20.99%

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The role of OTJ training

	Baseline	Counterfactual		Explained
Efficiency of matching function: χ	1	1	0.403	-
Distortion correlation: ζ	0	0	0.308	-
Training policy: $\mathbf{1}^t(z, \xi, h)$	baseline	counterfactual	counterfactual	-
<i>Firm-level moments</i>				
$E(\ell_t)$	16.1854	21.5297	11.187	
<i>Training provision</i>				
$E\left(\frac{\#\text{training firms}}{\#\text{firms}}\right), \%$	64.0196	34.9257	33.0791	94.03%
<i>Wage profile over experience</i>				
Wage growth, $E[\log(w_{20}/\bar{w}_1)]$	0.6141	0.5935	0.3264	7.16%
Wage growth, $E[\log(w_{25}/\bar{w}_1)]$	0.8013	0.7500	0.4244	13.61%
<i>Aggregates</i>				
Employment rate	0.7584	0.7344	0.6427	20.74%
Income per capita	1	0.9106	0.4137	15.25%
<i>Earnings inequality</i>				
Mean-p50 ratio	1.2067	1.2254	1.3793	10.83%

Re-training program for non-employed (Alfonsi et al 21)

- Assumptions: non-employed workers have the option of either searching for job or participating to a re-training program while postponing job search
- Value of being not-employed for a worker with ability h is now equal to

$$J^u(a) = \max\{J^r(a), J^s(a)\}$$

where

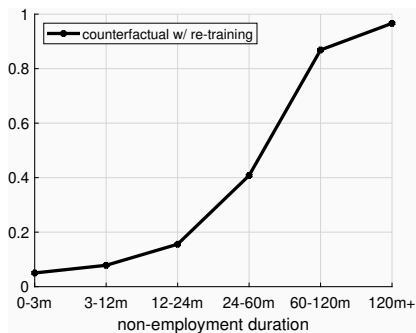
- value of re-training equal to

$$J^r(a) = p^t J^{u,h}(a+1) + (1-p^t) J^{u,h}(a)$$

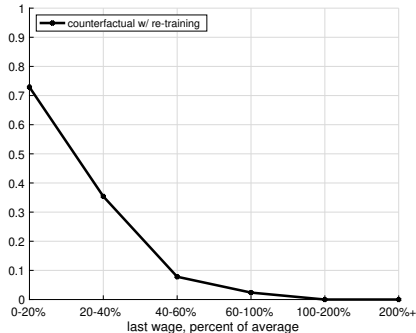
- value of searching for a job

$$\begin{aligned} J^s(a) &= J^{u,h}(a) + (1-\phi_w) p^d [J^{u,h}(a-1) - J^{u,h}(a)] \\ &\quad + \phi_w \int_{z,\xi} \mathbf{1}^h(z, \xi, a) [J^{e,h}(z, \xi, a; w) - J^{u,h}(a)] \psi_v(z, \xi) d\xi dz, \end{aligned}$$

Re-training attainment



by unemployment duration

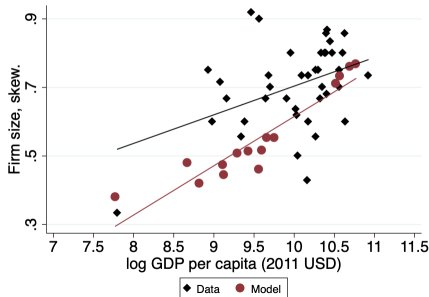
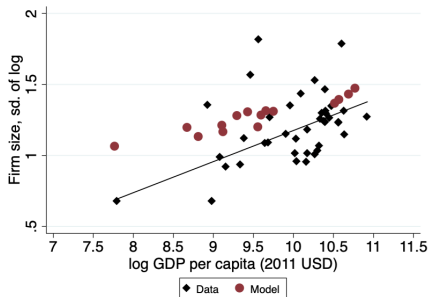


by wage of last job

- Long-term non-employed more likely to re-train
- Low-wage workers more likely to re-train

	UK Baseline (1)	Indonesia Counterfactual (2) (3)	
Efficiency of matching function: χ	1	0.403	0.403
Distortion correlation: ζ	0	0.308	0.308
Home production: b	20.94	3.505	3.505
Re-training under non-employment	no	no	yes
Cost per re-trained individual:	-	-	1024 USD
<i>Re-trained workers</i>			
$E\left(\frac{\# \text{re-trained workers}}{\# \text{non-employed workers}}\right), \%$	0	0	43.07
<i>Aggregates</i>			
Non-employment rate	0.212	0.593	0.471
Average wage	1	0.124	0.140
Income per capita	1	0.061	0.095
Income per capita (net of re-training costs)	1	0.061	0.070
<i>Wage profile over experience</i>			
Wage growth, $E[\log(w_{25}/\bar{w}_1)]$	0.801	0.280	0.329
<i>Wage inequality</i>			
Mean-median wage ratio	1.207	1.805	1.787
GINI	0.416	0.506	0.500

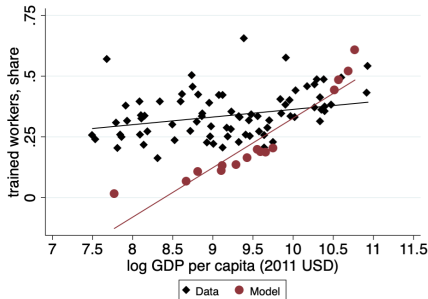
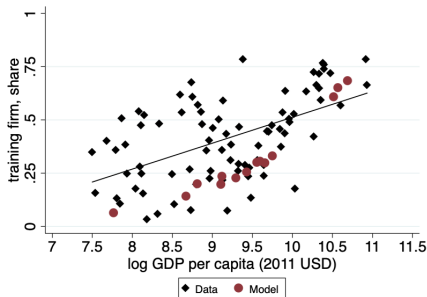
Firm size distribution



- changes in firm size distribution with development (Poschke 2019)
 - increase standard deviation of firm size
 - increase skewness of firm-size

back

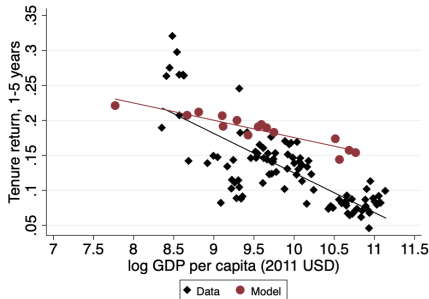
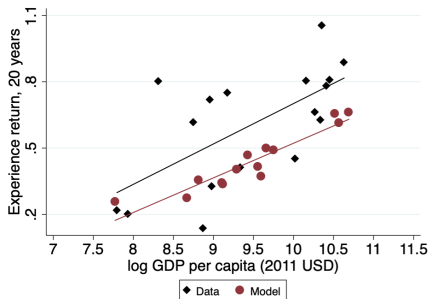
Training provision



- changes in training patterns with development
 - increase share of firms providing training
 - increase share of workers receiving training

back

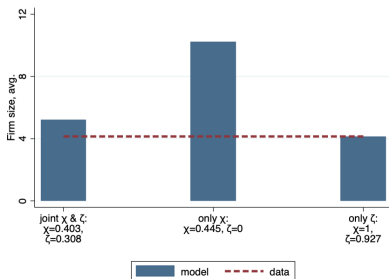
Earnings profile



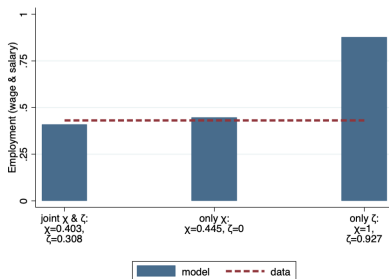
- changes in wage growth with development
 - increase along life-cycle (Lagakos et al 2018)
 - decrease along job tenure (Donovan et al 2022)

back

Identification of counterfactual parameters



Average firm-size



Wage and salary employment

- Differential effects of χ and ζ on average firm-size and employment
 - χ alone generates a much smaller drop in average firm size
 - ζ alone increases wage and salary employment

Alternative mechanisms

	UK Baseline		Indonesia Counterfactual		Indonesia Data
		Joint (χ, ζ)	Joint (δ_s, ζ)	Joint (χ, δ_f)	
	(1)	(2)	(3)	(4)	(5)
Matching frictions: χ	1	0.403	1	0.501	-
Distortion correlation: ζ	0	0.308	0.659	0	-
Separation rate: δ_s , %	1.235	1.235	5.179	1.235	-
Firm exit rate: δ_f , %	2.526	2.526	2.526	3.253	-
Home production: b	20.94	3.505	1.400	11.84	-
Average firm size, $E[\ell_t]$	16.19	5.177	4.421	10.11	4.141
Employment rate	0.788	0.408	0.666	0.452	0.431
Income per capita	1	0.061	0.051	0.232	0.100
Training provision, overall %	65.02	6.210	0	27.59	6.291
Earnings growth, $E[\log(w_{25}/w_1)]$	0.801	0.280	0.614	0.327	0.216
Mean-median ratio, $E[w_{it}]/p^{50}[w_{it}]$	1.207	1.805	1.327	1.835	1.687
GINI	0.416	0.506	0.427	0.513	0.502

- (1): reduction in worker separation over development (Donovan et al. 2020)
- (2): larger firm turnover in less developed countries (Bartelsman et al. 2009)
- (3): reduction in separation (Donovan et al. 2020) + correlated distortions

Alternative counterfactual

	UK Baseline	Indonesia Counterfactual Joint (χ, ζ)	Full	Indonesia Data
	(1)	(2)	(3)	(4)
Matching frictions: χ	1	0.403	0.382	-
Distortion correlation: ζ	0	0.308	0.252	-
Aggregate Productivity shifter: κ	1	1	0.938	-
Experience jump: p^e	0.223	0.223	0.205	-
Training jump: p^e	0.028	0.028	0.003	-
Home production: b	20.94	3.505	4.020	-
Training costs (lower bound): $\underline{\xi}$	1.735	1.735	0.232	-
Training costs (upper bound): $\bar{\xi}$	26.69	26.69	2.212	-
Entry cost: c_e	39.26	39.26	3.161	-
Average firm size, $E[\ell_t]$	16.19	5.177*	3.681*	4.141
Employment rate	0.788	0.408*	0.461*	0.431
Income per capita	1	0.061	0.087*	0.100
Training provision, overall %	65.02	6.210	7.006*	6.291
Earnings growth, $E[\log(w_{25}/w_1)]$	0.801	0.280	0.222*	0.216
Mean-median ratio, $E[w_{it}]/p^{50}[w_{it}]$	1.207	1.805	1.772	1.687
GINI	0.416	0.506	0.503	0.502

Notes: * = targeted moment.

Alternative mechanisms

	UK Baseline		Indonesia Counterfactual		Indonesia Data
		Joint (χ, ζ)	Joint (δ_s, ζ)	Joint (χ, δ_f)	
	(1)	(2)	(3)	(4)	(5)
Matching frictions: χ	1	0.403	1	0.501	-
Distortion correlation: ζ	0	0.308	0.659	0	-
Separation rate: δ_s , %	1.235	1.235	5.179	1.235	-
Firm exit rate: δ_f , %	2.526	2.526	2.526	3.253	-
Home production: b	20.94	3.505	1.400	11.84	-
Average firm size, $E[\ell_t]$	16.19	5.177	4.421	10.11	4.141
Employment rate	0.788	0.408	0.666	0.452	0.431
Income per capita	1	0.061	0.051	0.232	0.100
Training provision, overall %	65.02	6.210	0	27.59	6.291
Earnings growth, $E[\log(w_{25}/w_1)]$	0.801	0.280	0.614	0.327	0.216
Mean-median ratio, $E[w_{it}]/p^{50}[w_{it}]$	1.207	1.805	1.327	1.835	1.687
GINI	0.416	0.506	0.427	0.513	0.502

- (1): reduction in worker separation over development (Donovan et al. 2020)
- (2): larger firm turnover in less developed countries (Bartelsman et al. 2009)
- (3): reduction in separation (Donovan et al. 2020) + correlated distortions