Macroeconomics of “Large Firms”

Lecture 4: Firm dynamics with search and trade frictions

Alessandro Ruggieri
University of Nottingham

CEMFI, PhD short course

This lecture

- Cosar K. Guner N. and Tybout J. 2016. “Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy”, American Economic Review, Vol. 106, No. 3, pp. 625-63.
    - How do trade frictions impact unemployment, job turnover and wage inequality?
    - Do labor market institutions affect firm-level adjustment from trade?

Small-open economy in discrete time

Two sectors: industrial (tradable) and service (non-tradable)

Three agents:

- Infinitely-lived, risk-neutral homogeneous workers/consumers
    - face trade barriers to import foreign industrial varieties
    - sort into service or industrial labor market.
- Homogeneous service sector firms
    - perfect competition in the product market
    - friction-less labor market for services
- Heterogeneous industrial firms
    - monopolistic competition and trade barriers
    - time-varying idiosyncratic productivity
    - search and matching frictions in labor market
    - labor market institutions

- Utility function: Cobb-Douglas in services, s, and industrial composite good, c, i.e.

$$U = c_t^{\gamma} s^{1-\gamma} \quad \gamma \in (0,1)$$

- Industrial composite goods: CES function aggregate of N differentiated varieties available

$$c = \left(\int_0^N c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \qquad \sigma > 1$$

- Domestic demand for services and industrial consumption goods

$$s = (1-\gamma) I(i) \qquad c = \gamma \frac{I(i)}{P}$$

- Service firms' technology: $s = L^s + bL^u$ where
  - L^s, L^u: measures of workers in service sector and unemployed labor
  - $b < 1$: value home production (relative efficiency)

- Industrial firms produce differentiated varieties ω, and are defined by
  - idiosyncratic productivity, $z' = \rho_z z + \epsilon, \quad \rho_z \in (0,1)$, $\epsilon \sim N(0, \sigma_z)$
  - number of employees, $n \geq 1$.

- Industrial firms' technology: $q(z,n) = zn$

- Industrial product market subject to monopolistic competition and is internationally segmented
- $p(\omega)$ and $p^*(\omega)$: price charged in the domestic and foreign market by producer ω

- Demand function:
  - Non-exporters: $q^d(\omega) = D_h p(\omega)^{-\sigma}$
  - Exporters: $q^x(\omega) = D_f^* \tau_c^{-1} k p^*(\omega)^{-\sigma}$

  where D_h is an endogenous domestic demand shifter, D_f^* is an exogenous foreign demand shifter, τ_c is iceberg cost, k is the exchange rate (#LCU/#FCU)

- Revenue function:
  - Non-exporters: $R^d(\omega) = p(\omega)q^d(\omega) = D_h^{\frac{1}{\sigma}} q(\omega)^{\frac{\sigma-1}{\sigma}}$
  - Exporters: $R^x(\omega) = p(\omega)q^x(\omega) = D_h^{\frac{1}{\sigma}} [1 + d_f]^{\frac{1}{\sigma}} q(\omega)^{\frac{\sigma-1}{\sigma}}$
- $d_f = k^\sigma \tau_c^{1-\sigma} D_f^* / D_h$: treat it exogenously!

- Service labor market assumed to be frictionless
- Industrial labor market subject to search and matching frictions
- Job seekers, X and open vacancies, v, meet through a CRS matching function

$$h(v, X) = \frac{vX}{(v^{\theta} + X^{\theta})^{\frac{1}{\theta}}} \qquad \theta > 0$$

- Vacancy filling probability: $\phi = \frac{h(v,X)}{v}$
- Job finding probability: $\widetilde{\phi} = (1 - \phi^{\theta})^{\frac{1}{\theta}}$

- Cost of adjusting employment from n to n',

$$C(n,n') = \begin{cases} C^+(n,n') = \frac{c_h}{n^{\lambda_2}} \left(\underbrace{\frac{n'-n}{\phi}}_{v} \right)^{\lambda_1} & \text{if} \quad n' > n \\ C^-(n,n') = c^f(n-n') & \text{if} \quad n' < n \end{cases}$$

where

- c^h: scale parameter for hiring costs
- $\lambda_1 > 0$: parameter governing the convexity of C in employment
- $\lambda_2 > 0$: parameter governing the growth of small vs. large firms
- c_f: firing costs

- Extension of intra-firm bargaining problem:

$$\max_{w^q(z',n')} \quad \Pi_w^{\beta}(z',n')\Pi_f^{1-\beta}(z',n')$$

$$\text{s.t.}$$

$$\textit{participation constraint:} \quad \Pi_w(z',n') \geq 0$$

$$\textit{minimum wage constraint:} \quad w^q(z',n') \geq \underline{w}$$

where

- Π_f: firm surplus flow from marginal worker
- Π_w: worker surplus flow from being employed
- $\beta \in (0,1)$: worker's bargaining power
- $\underline{w}$: minimum wage

Timing of the firms' problem

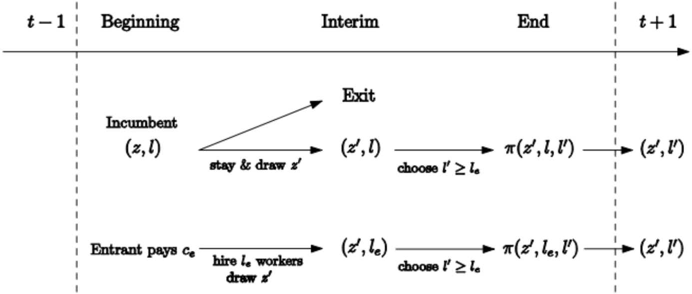

Figure 1: Within-Period Sequencing of Events for Firms

- Value of exporting firm

$$\tilde{V}^x(z', n) = \max_{n' \geq 0} \left[R^x(z', n') - c_x - w^q(z', n')n' - C(n, n') + V(z', n') \right]$$

- Value of not-exporting firm

$$\tilde{V}^d(z', n) = \max_{n' \geq 0} \left[R^d(z', n') - w^q(z', n')n' - C(n, n') + V(z', n') \right]$$

where

- c_x: fixed per-period cost of exporting
- $w^q(z', n')$: industrial wage rate
- $C(n, n')$: employment adjustment costs

Solution: optimal policy for employment, $g_n^x(z', n)$ and $g_n^d(z', n)$

- Value of firm (z', n) at the interim stage of the period:

$$\tilde{V}(z', n) = \max\{\tilde{V}^x(z', n), \tilde{V}^d(z', n)\}$$

Solution: optimal export decision, $g_x(z', n)$

- Value of the firm (z, n) at the end of the period:

$$V(z, n) = \max\left\{0, \frac{1-\delta}{1+r}\int_{z'} \tilde{V}(z', n)\Gamma(z'|z)dz\right\}$$

where

  - $r > 0$: exogenous interest rate
  - $\delta \geq 0$: exogenous firm exit probability

Solution: optimal exit decision, $g_e(z, n)$

- Value of entrant firm at the beginning of the period

$$V^e = \frac{1}{1+r} \int_z \tilde{V}(z, \bar{n}) \Gamma^e(z) dz \leq c_e$$

with equality if number of entrants N^e is strictly positive.

  - Γ^e: ergodic productivity distribution
  - $c_e > 0$: sunk entry costs, inclusive of initial hiring costs
  - $\bar{n}$: initial scale of production (assumed to be 1)

Timing of the workers' problem

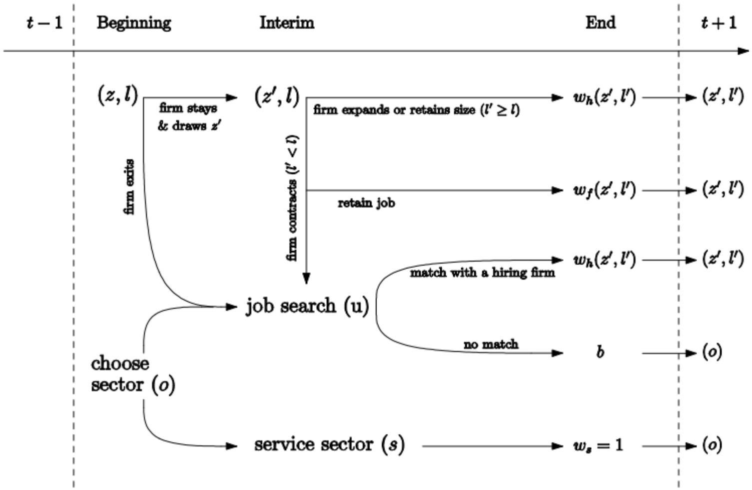

Figure 2: Within-Period Sequencing of Events for Workers

- Value of non-employed workers at beginning of time t

$$J^o = \frac{1}{1+r} \max\{J_t^s, J^u\}$$

- Value of working in the service sector

$$J^s = w^s + J^o$$

- Value of searching for an industrial job

$$J^u = (1-\widetilde{\phi})(b+J_t^u) + \widetilde{\phi} \int_{z'} \int_n [w^q(z',n') + J^q(z',n')]\nu(z',n)dz'dn$$

where

  - w^s : wage rate in service sector (= 1, numeraire)
  - ν: distribution of vacancies

- Value of employed worker at the beginning of period

$$J^q(z,n) = p^o(z,n)J^u + (1 - p^o(z,n)) \int_{z'} \max\Big\{J^u, J^c(z',n)\Big\}\Gamma(z'|z)dz$$

- Value of employed worker at the interim stage of period

$$J^c(z',n) = p^f(z',n)J^u + \frac{(1 - p^f(z',n))}{1+r}[w^q(z',n') + J^q(z',\ell)]$$

where

- p^o: probability of large dismissal (firm exit)
- p^f: probability of individual dismissal (firing)

- Distribution of firms at the interim stage

$$\tilde{\psi}(z',n) = \begin{cases} (1-\delta)\int_z \Gamma(z'|z)\psi(z,n)\mathbf{1}[g_e(z,n)=1]dz, & \text{if} \quad n \neq 1 \\ \\ \frac{N^e}{N}\Gamma^e(z')+ \\ (1-\delta)\int_z \Gamma(z'|z)\psi(z,n)\mathbf{1}[g_e(z,n)=1]dz, & \text{otherwise} \end{cases}$$

where $\frac{N^e}{N}$ denotes the entry rate, equal to

$$\delta + (1-\delta)\int_z \int_n \mathbf{1}[g_e(z,n)=0]\psi(z,n)dzdn$$

- Distribution of firms at the end of the period

$$\psi(z',n') = \frac{\int_n \mathbf{1}[g_n(z',n)=n']\tilde{\psi}(z',n)dn}{\int_{z''}\int_n \mathbf{1}[g_n(z'',n)=n']\tilde{\psi}(z'',n)dz''dn}$$

- Income:

$$I = \underbrace{\int_{z',n'} w(z',n')\psi(z',n')dz'dn' + bL_u + L_s}_{\text{labor income}}$$

$$+ \underbrace{N\int_{z',n} \pi(z',n,g_n(z',n))\tilde{\psi}(z',n)dz'dn - c_e N_e}_{\text{aggregate profits}} + \underbrace{T}_{\text{tariff revenues}}$$

- Adjustment costs:

$$\bar{c} = \int_{z',n} C(n,g_n(z',n))\tilde{\psi}(z',n)dz'dn$$

- Service supply: $bL_u + L_s$
- Service demand: $(1-\gamma)I + N_e c_e + N(\bar{c} + c_o + \mu^{\text{exp}} c_x)$

- A recursive stationary equilibrium is characterized by the following conditions:
  - **market clearing**: supply matches demand for services and for each differentiated good, where supplies are determined by employment and productivity levels in each firm
  - **stationarity**: the flaw of workers into unemployment matches the flaw of workers out of unemployment
  - **free entry**: a positive mass of entrants replaces exiting firms every period, so that value of entry equals its cost aggregate expenditure
  - **no arbitrage**: workers optimally choose the sector in which they are working or seeking work s.t. there is no profitable deviation from switching sector
  - **consistency**: the distribution of firms over states reproduce themselves through the productivity shocks, the policy functions, and the productivity draws that firms receive upon entry

Loop 1: Labor market clearing

- Guess job finding rate ϕ_w^0 and compute job filling rate ϕ_f^0 using the matching function
- Given ϕ_w^0 and ϕ_f^0 solve **Loop 2** to solve for wages
- Simulate the economy for a large number of firms and compute the distribution of firms, $\psi(z', n')$ and $\psi(z', n)$
- Compute the value of searching for a job in the tradable sector

$$J^u = \phi_w^0 \int_z \int_n J^{e,h}(z,n)\psi(z,n)dzdn + \frac{(1-\phi_w^0)}{1+r}[b + J^o]$$

  using the condition $J^o = 1/r$.
- Check if no-arbitrage condition is satisfied, i.e. $J^u = J^o$
  - if no, make a new guess, ϕ_w^1, and go back to step 2 till convergence
    - $\phi_w^1 > \phi_w^0$ if $J^u < J^o$ $\quad \phi_w^1 < \phi_w^0$ otherwise
  - if yes, store $\phi_w^* = \phi_w^0$

Loop 2: Wages

- Guess a wage schedule, $w(z,n)^0$
- Solve **Loop 3** to ensure free-entry
- Construct wages for hiring firms, $w(z,n)^h$
- Use the policy functions obtained in Loop 4 to compute the value of employment for workers, $J^e(z,n)$ and $J^{e,h}(z,n)$
- Construct wages for firing firms, $w(z,n)^h$
- Update guess for wages, $w(z,n)^1$
    - $w(z,n)^1 = w(z,n)^h$ if $g_n(z,n) > n$
    - $w(z,n)^1 = w(z,n)^f$ if $g_n(z,n) \leq n$
- Go back to step 2 and iterate till convergence
- Store wage schedule, $w^*(z,n)$

Loop 3: Free-entry

- Guess a domestic demand shifter, d_h^0
- Solve the problem of the firm to obtain the value function $V(z, n)$
- Compute the value of entry, V^e
- Check if free entry condition is satisfied, i.e. $V^e = c_e$
  - if no, make a new guess, d_h^1, and go back to step 2 till convergence
    - $d_h^1 > d_h^0$ if $V^e < c_e$
    - $d_h^1 < d_h^0$ otherwise
  - if yes, store $d_h^* = d_h^0$
- Store the policy functions for employment $g_n(z', n)$, export decision $g_x(z', n)$ and exit/stay decision, $g_e(z, n)$

Two main forces at play

- Sensitivity effect:
  - greater openness increases d_f for high-productivity firms
  - firms value-added functions becomes steeper with respect to employment levels
  - larger workforce adjustments in response to z shocks
- Distribution effect:
  - greater openness concentrates workers at larger firms
  - larger firms tend to be relatively stable, because of convex adjustment costs
  - lower workforce adjustments in response to z shocks

Implication 1: Firm size distribution

Table 4: **The Effects of Reforms and Globalization**

	Baseline	Labor	Tariff	Iceberg	Reforms	Reforms and Globalization
c_f (firing cost)	0.60	0.30	0.60	0.60	0.30	0.30
τ_a (ad valorem tariff rate)	1.21	1.21	1.11	1.21	1.11	1.11
τ_c (iceberg trade cost)	2.50	2.50	2.50	2.19	2.50	2.19
Size Distribution						
20th percentile	15.09	16.84	15.24	14.64	18.05	17.87
40th percentile	25.22	27.54	24.74	23.55	29.77	30.06
60th percentile	42.97	47.30	42.97	42.15	51.57	54.09
80th percentile	87.14	98.59	90.51	99.53	107.38	129.79
Average firm size	52.69	58.73	55.89	61.99	64.08	77.91
Firm Growth Rates						
(at the baseline size quantiles)						
<20th percentile	1.28	1.36	1.28	1.26	1.43	1.49
20th-40th percentile	0.25	0.23	0.26	0.28	0.26	0.29
40th-60th percentile	0.19	0.17	0.20	0.22	0.18	0.21
60th-80th percentile	0.15	0.14	0.16	0.18	0.15	0.17

Implication 2: Aggregates

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τ_c (iceberg trade cost)	2.50	2.50	2.50	2.19	2.50	2.19
Aggregates						
Rev. share of exports	1.00	1.02	1.36	2.01	1.39	2.50
Exit rate	1.00	0.96	1.02	1.13	0.96	1.03
Job turnover	1.00	0.90	1.01	1.03	0.92	0.94
Mass of firms	1.00	0.96	0.95	0.74	0.88	0.66
Share of labor, Q sector	1.00	1.06	1.01	0.88	1.07	0.98
Vacancy filling rate (ϕ)	1.00	0.93	1.04	1.11	0.99	1.09
Unemp. rate, Q sector	1.00	0.73	1.11	1.38	0.88	1.19
Std. wages (firms)	1.00	1.09	1.01	1.03	1.12	1.18
Std. wages (workers)	1.00	1.10	1.02	1.04	1.11	1.14
Std. J (firms)	1.00	1.05	1.03	1.06	1.09	1.18
Std. J (workers)	1.00	1.04	1.04	1.06	1.07	1.21
Exchange rate	1.00	1.04	0.99	0.89	1.02	0.84
Real income	1.00	0.95	1.00	1.14	0.96	1.12

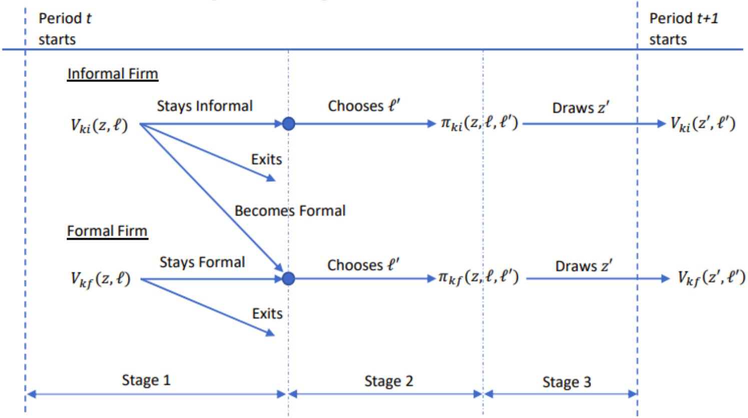
Extension: Trade and informality

- Dix-Carneiro R., Goldberg P., Meghir C. and Ulyssea G. 2021. "Trade and Informality in the Presence of Labor Market Frictions and Regulations.", NBER working paper No. 28391
    - Informal sector
        - extensive margin of informality (only)
        - formalization choice at entry and along life-cycle
        - taxes on payroll and value added versus cost of being audited (convex in firm size)
    - Unique labor market
        - probability of finding jobs in formal/informal firms
    - Collective wage bargaining at firm-level
        - split aggregate (not marginal) surplus

- Formal firms are subject to:
  - tax on value added, τ_y
  - tax on payroll, τ_w
  - firing costs, c_f
  - minimum wage, $\underline{w}$
- Informal firms are subject to:
  - cost of being audited, increasing and convex with number of employees, $\kappa(n) = a + \exp^{b(n-1)}$
- Both formal and informal firms are subject to
  - convex hiring costs, $C(n, n') = c_h n^{-\lambda_2} \left(\frac{v}{\phi}\right)^{\lambda_1}$

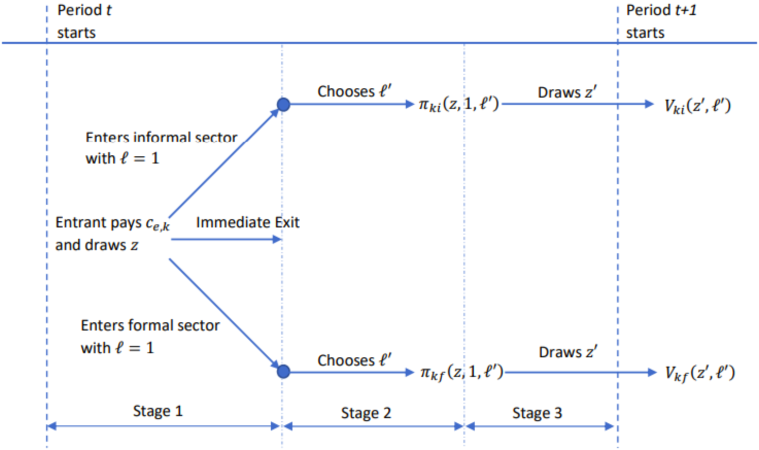
Timing of the incumbent firms' problem

Figure 1: Diagram of Firms' Behavior



Timing of the entry problem

Figure 2: Diagram of Entry Behavior



- Nash splitting rule

$$S_j^w(z,n;w) = \beta[S_j^f(z,n;w) - S_j^w(z,n;w)] \quad \forall j = f,i$$

- Worker aggregate surplus

$$S_j^w(z,n;w) = w_j(z,n) + \tilde{J}_j^e(z,n;w) - b - \tilde{J}^u \quad \forall j = f,i$$

- Firm aggregate surplus

$$\begin{aligned} S_j^f(z,n;w) &= (1-\tau_y)R_j(z,n) - (1+\tau_w)w_j(z,n)n \\ &+ \frac{1}{1+r}\int_{z'} \tilde{V}_j^f(z',n;w)d\Gamma(z'|z) \quad \forall j = f,i \end{aligned}$$

- The wage solution, $\forall j = f, i$, reads as follows:

$$(1+\beta\tau_w)w(z,n) = (1-\beta)[b + \tilde{J}^u - \tilde{J}_j^e(z,n;w)] + \beta\left[(1-\tau_y)\frac{R_j(z,n)}{n} + \frac{1}{1+r}\int_{z'}\frac{\tilde{V}_j^f(z',n;w)}{n}d\Gamma(z'|z)\right]$$

- The wage is an increasing function of:
  - average value added (not marginal)
  - average continuation value (not marginal)
- The wage is a decreasing function of:
  - value added taxes
  - payroll taxes