Incomplete Markets as Correlated Distortions

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- **What drives poor allocative efficiency?**

Introduction

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- Extensive evidence on firm-specific distortions and or constraints, such as:
    - size-dependent labor regulation (Guner et al 08, Gourio and Roys, 14)
    - financial constraints (Moll 14, Midrigan and Xu 14)
    - informational frictions (David et al 16)

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- **Alternative channel: incomplete markets**
    - undiversifiable entrepreneurial risks are paramount (Castro et al 15)
        - idiosyncratic shocks account $\approx 80\%$ of overall uncertainty faced by plants
    - missing insurance markets distorts decisions to engage in risky projects and to become entrepreneurs (Robinson 24)

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What is the misallocation cost of incomplete markets?

In this paper...

- We build a span of control model of heterogeneous risk-averse entrepreneurs subject to incomplete markets and time-to-build constraints
  - market incompleteness prevents the equalization of the marginal value of resources across firms and operates as a *correlated distortion*
  - larger idiosyncratic uncertainty exacerbates misallocation across heterogeneous entrepreneurs

- We use firm-level data for Portugal (QP) to document that industry and location with higher firm-level volatility have
  - lower correlation between firm-level capital and productivity
  - higher dispersion of the (log) marginal product of capital

- We quantify aggregate losses from imperfect insurance
  - completing markets increases output p.c. and TFP by 97% and 64%
  - output gains are robust to alternative values of capital elasticity and to allowing ex-post labor adjustment

Model

- Unit measures of risk-averse entrepreneurs' *types* $i \in \mathcal{I}$, characterized by permanent productivity, z_i
- Within each type i, there is a unit mass of ex-ante identical entrepreneurs, differing ex-post in the realization of an idiosyncratic iid shock, $s \sim \Gamma(s)$
- Production function: $y_{is} = sf(z_i, k_i), \quad \alpha \in (0, 1)$
  - production function f, increasing and concave in both arguments + supermodular
  - time-to-build: capital, k_i, chosen before uncertainty is realized
- Idiosyncratic income: $\pi_{is} = y_{is} - rk_i + T$
  - r denotes rental price of capital
  - T denotes from revenues from renting capital, rebated lump-sum
- Utility function u, increasing and concave in consumption
- Exogenous supply of capital, $\bar{K}$

The problem under complete markets

- Under complete markets, entrepreneurs choose
  - capital, k_i;
  - consumption, $\{c_i(s)\}_{s\in\mathcal{S}}$;
  - a full set of state-contingent assets, $\{\theta_i(s)\}_{s\in\mathcal{S}}$;

  to maximize their expected utility, i.e.,

$$\max_{k_i,\{\theta_i(s),c_i(s)\}_{s\in\mathcal{S}}} \int_{s\in\mathcal{S}} u(c_i(s))\Gamma(s)ds$$

$$\text{s.t.} \quad c_i(s) + \int_{s'\in\mathcal{S}} q(s')\theta_i(s')ds' \leq sf(z_i,k_i) - rk_i + T + \theta_i(s), \quad \forall s \in \mathcal{S},$$

- $q(s)$ is the price of a contingent asset that pays a unit of consumption if state s realizes, zero otherwise.

A competitive equilibrium under complete markets is...

... a list of capital choices, $\{k_i\}_{i\in\mathcal{I}}$, state-contingent assets, $\{\theta_i(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$, and consumption plans, $\{c_i(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ across entrepreneurs, a rental price of capital r, and a list of price of contingent assets $\{q(s)\}_{s\in\mathcal{S}}$ such that:

- $\{k_i\}_{i\in\mathcal{I}}$, $\{\theta_i(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ and $\{c_i(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ solve the problem of each entrepreneur i, i.e, they satify the following FOCs:

$$u_c(c(s))\Gamma(s) = q(s)\int_{s'\in\mathcal{S}} u_c(c(s'))\Gamma(s')ds', \quad \forall s\in\mathcal{S},$$

$$\int_{s\in\mathcal{S}} u_c\left(c(s)\right)[sf_k(z,k)-r]\Gamma(s)ds = 0$$

$$c(s) + \int_{s'\in\mathcal{S}} q(s')\theta(s')ds' \le sf(z,k) - rk + T + \theta(s), \quad \forall s\in\mathcal{S},$$

- the price of contingent assets is fair, i.e., $q(s) = \Gamma(s)$;
- the market for capital clears, i.e., $\int_i k_i di = \bar{K}$

Proposition 1

- Under complete markets, and fair price of insurance,
  - constant consumption across states of the world, i.e.

$$c(s) = u_c^{-1}\left(\int_{s' \in \mathcal{S}} u_c(c(s'))\Gamma(s')ds'\right) \quad \forall s \in \mathcal{S}$$

  - the expected marginal products of lands are equalized across farmer types, i.e.,

$$\int_{s \in \mathcal{S}} [sf_k(z,k) - r]\Gamma(s)ds = 0$$

    - the expected total income is maximized;
    - more productive entrepreneurs hold more capital.

- Under incomplete markets, entrepreneurs choose
  - capital, k_i;
  - consumption, $\{c_i(s)\}_{s\in\mathcal{S}}$;

  to maximize their expected utility, i.e.,

$$\max_{k_i,\{c_i(s)\}_{s\in\mathcal{S}}} \int_{s\in\mathcal{S}} u(c_i(s))\Gamma(s)ds$$

$$\text{s.t.} \quad c_i(s) \leq sf(z_i, k_i) - rk_i + T \quad \forall s \in \mathcal{S}$$

A competitive equilibrium without insurance is ...

... a list of capital choices, $\{k_i\}_{i\in\mathcal{I}}$ and consumption plans, $\{\{c_i(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ across entrepreneurs, and a rental price of capital r, such that:

- $\{k_i\}_{i\in\mathcal{I}}$ and $\{c_i(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ are the solution to the problem of each entrepreneur i;
- the market for capital clears, i.e.,

$$\int_i k_i di = \bar{K}$$

Characterization of equilibrium without insurance

- Assumptions:
  - CRRA utility function, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 0$
  - shock s is log-normally distributed, i.e., $s \sim \log \mathcal{N}(0, \sigma^2)$
- The problem of the entrepreneurs can be re-wrriten in terms of consumption equivalent, i.e.

$$\max_k \quad \bar{s}f(z,k) - rk + T - \frac{(\gamma-1)}{2}f(z,k)^2\sigma_s^2$$

where $\bar{s}$ and σ_s^2 are mean and variance of s

- First-order condition with respect to capital

$$f_k(z,k)[\bar{s} - (\gamma-1)\sigma_s^2 f(z,k)] - r = 0,$$

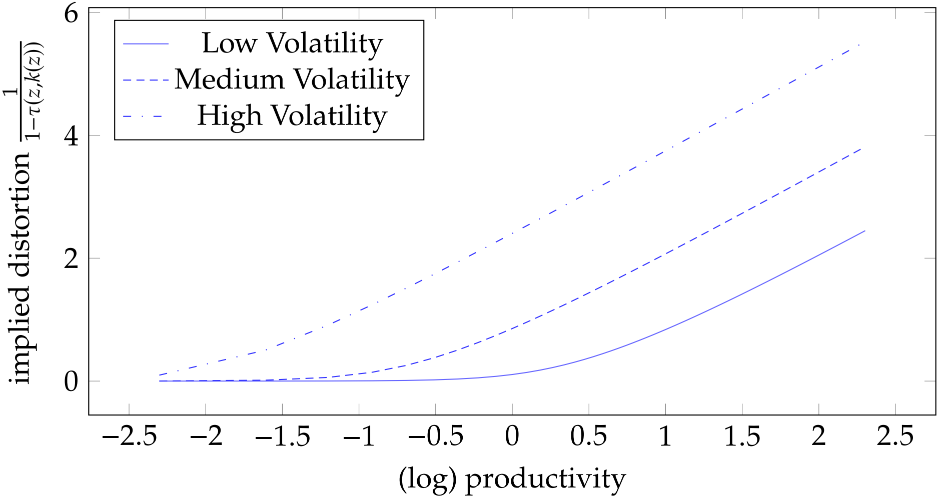
Proposition 2

- Relative to the case of full insurance, incomplete markets introduce an *endogenous wedge* between the marginal product of capital and its marginal cost

$$\tau(z,k) = (\gamma - 1)\frac{\sigma_s^2}{\bar{s}} f(z,k)$$

- When entrepreneurs are enough risk-averse (i.e., $\gamma > 1$), under incomplete markets
  - there is misallocation along the intensive margin, i.e., the expected marginal product of capital is not equalized across firms with different productivity;
  - lack of insurance operates as correlated distortions, i.e., it results in the reallocation of capital from more to less productive firms;
  - higher volatility of the idiosyncratic shock, σ_s reduces the correlation between capital holding k and productivity z.

Correlated distortions and entrepreneurial productivity



- Firm-level data from Portugal's SCIE - 2010 to 2021
  - it covers the universe of firms excluding FIRE and state-owned enterprises
  - yearly information on firm-level sales, number of employees, payroll, material expenditure, current and non-current assets, age, industry and region
  - group firms into local markets - industry × location

- Descriptive evidence:
  - distribution of firm size
  - distribution of number of establishments
  - share of firms by legal form

- **Step 1.** For every local market, estimate:

$$\log y_{it} = \beta_{j,0} + \beta_{j,1} \log k_{it} + \beta_{j,2} \log(w_{it}\ell_{it}) + \beta_{j,3} \log m_{it} + \mu_i + \mu_t + \epsilon_{it}$$

where y_{it} denotes sales of firm i at time t, k_{it} is capital stock, l_{it} is number of employees, w_{it} is the average wage, m_{it} are intermediate inputs and μ_i and μ_t are firm and time FE.

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- **Step 2.** From the estimates we get
  - Firm-level permanent productivity

$$\ln(z_i) = \hat{\mu}_i$$

  - Firm-level shock volatility

$$\hat{\sigma}_i = \sqrt{\sum_{t=1}^{T} \left(\hat{\epsilon}_{it} - \bar{\hat{\epsilon}}_{it}\right)^2}$$

- Distributions | Robustness: Persistence | Robustness: Firm Exit

- **Step 3.** Exploit variation across local markets:
  - Average volatility within markets

$$\hat{\sigma}^j = \text{mean}_{i \in \{1,\ldots,N_j\}} \left[\hat{\sigma}_i\right]$$

  - Elasticity between productivity and capital

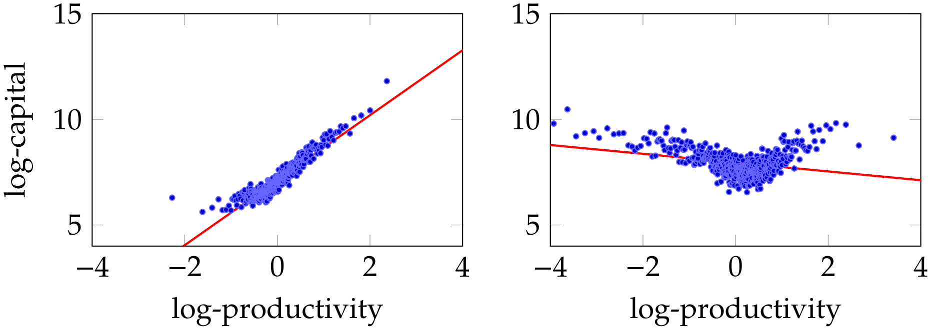
$$\overline{\ln(k_{it})} = \alpha^j + \eta^j \hat{\mu}_i + \varepsilon_i \quad \text{where } \overline{\ln(k_{it})} \equiv \frac{1}{T}\sum_{t=1}^{T} \ln(k_{it})$$

  - Dispersion of MPK

$$\delta^j = \text{std}_j \left[\overline{\ln \text{MPK}_{it}^j}\right] \quad \text{where } \overline{\ln \text{MPK}_{it}^j} \equiv \frac{1}{T}\sum_{t}^{T} \ln\left(\hat{\mu}_i + \hat{\beta}_{1j} \ln(k_{it})\right)$$

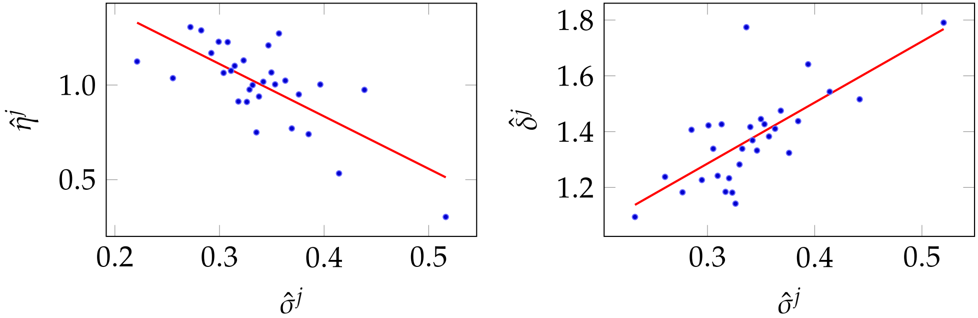
Misallocation in low- vs high-volatility markets

- High correlation, ≈ 0.49, in low-volatility economy (bottom 1% distribution)
- Low correlation, ≈ -0.09, in high-volatility economy (top 1% distribution)



(a) Low Volatility (b) High Volatility

- Markets with higher firm-level volatility are those with
  - lower correlation between firm-level capital and productivity
  - higher dispersion of the (log) marginal product of capital



(c) Capital and Productivity

(d) Dispersion of MPK

What are the aggregate costs of incomplete markets?

- Solve the model using a DRS production function: $f(z, k) = zk^{\alpha}, \quad \alpha \in (0, 1)$
- Assume permanent productivity is gamma-distributed: $z \sim \Gamma(\kappa, \theta)$
- Keep assumption log-normality of shocks: $s \sim \log N(0, \sigma)$

Parameters	Description	Value	Source/Target
	A. Parameters calibrated outside the model		
α	Output elasticity of capital	0.730	Erosa et al (2023)
σ	Volatility of shocks	0.347	Data
(κ, θ)	Distribution of permanent productivity	(1.404,1.060)	Data
	B. Parameters calibrated internally to the model		
γ	Relative risk aversion	4.024	corr. $[\log k, \log z] = 0.494$

What are the aggregate costs of incomplete markets?

- Counterfactual 1: Shut down idiosyncratic uncertainty.
- Counterfactual 2: Complete markets but keep same shock structure.

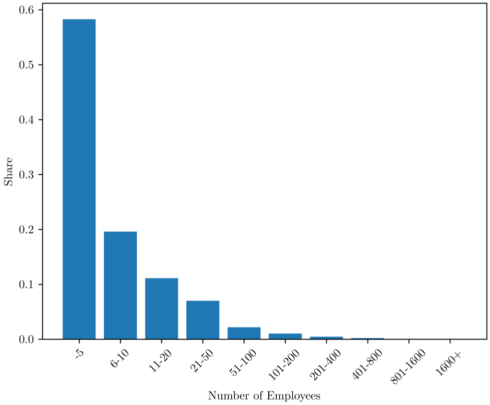
		Counterfactual	
	Baseline	No Uncertainty	Complete markets
	(1)	(2)	(3)
Volatility, σ	0.347	0	0.347
corr$[\log z, \log k]$	0.494	1.000	1.000
$\omega[\log z, \log k]$	1.114	3.700	3.700
sd$[\log k]$	1.141	3.445	3.445
Rental rate, r	0.294	1.952	2.073
TFP	1.000	1.541	1.637
Income	1.000	1.853	1.968

- Completing markets $\Rightarrow$ 64% higher TFP and 97% higher income

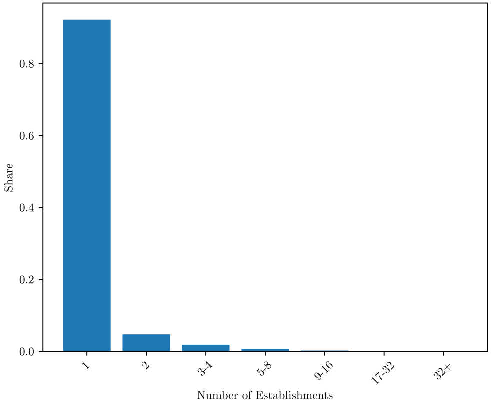
Conclusion

- We theoretically show that under entrepreneur heterogeneity, risk-aversion and time-to-build constraints market incompleteness operates as a *correlated distortion*.
- We use firm-level admin data from Portugal to document that markets with a higher degree of shock volatility have higher capital misallocation.
- We quantify aggregate gains from perfect insurance:
    - +97% in aggregate income, +64% in TFP.

Appendix



(e) Employees



(f) Establishments

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Firm distribution by ownership mode

Type	Share
Sociedade por quotas	0.642
Sociedade unipessoal por quotas	0.289
Sociedade anonima	0.052
Other	0.016

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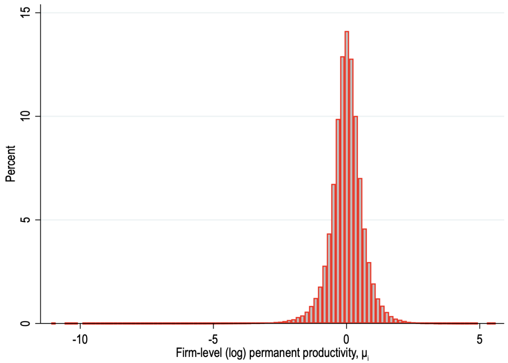
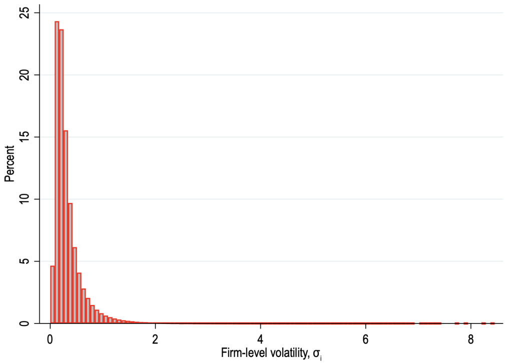
Production Function Estimates - Distributions



(g) Firm-level (log) permanent productivity



(h) Firm-level volatility

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- Allow idiosyncratic shocks to follow AR(1), follow Blundell Bond (2000) to estimate:

$$\log y_{it} = \beta_{j,0} + \beta_{j,1} \log k_{it} + \beta_{j,2} \log(w_{it} \ell_{it}) + \beta_{j,3} \log m_{it} + \mu_i + \mu_t + \nu_{it},$$

where

$$\nu_{it} = \rho \nu_{it-1} + \epsilon_{it},$$

back

- Account for non-random firm exit - jointly estimate following Wooldridge (1995):

$$\log y_{it} = \beta_{j,0} + \beta_{j,1} \log k_{it} + \beta_{j,2} \log(w_{it}\ell_{it}) + \beta_{j,3} \log m_{it} + \mu_i + \mu_t + \epsilon_{it}$$
$$d^*_{it} = \alpha_0 + \alpha_1 z_{1it} + \ldots + \alpha_N z_{Nit} + \eta_i + \epsilon_{it}, \quad d_{it} = \mathbf{1}[d^*_{it} > 0]$$

back

	$\hat{\eta}^j$		$\hat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}^j$	-3.999***	-2.530**	2.096***	1.649***
	(0.677)	(0.730)	(0.435)	(0.134)
R^2	0.433	0.809	0.534	0.902
N. Obs.	397	397	397	397
Second step	Unweighted	Weighted	Unweighted	Weighted

back

Robustness: Technology

- Introduce labor in production function
- Allow adjustment *after* uncertainty realizes

		Counterfactual	
		No	Complete
	Baseline	Uncertainty	markets
	(1)	(2)	(3)
Volatility, σ	0.347	0	0.347
corr[$\log z, \log k$]	0.494	1.000	1.000
ω[$\log z, \log k$]	0.934	3.700	3.700
sd[$\log k$]	2.603	6.816	6.816
Rental rate, r	0.743	24.11	25.61
TFP	1.000	1.343	1.348
Income	1.000	1.376	1.539

back