Online Appendix of
Twin Peaks: Covid-19 and the Labor Market

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O.1 Solving the model under lockdown

In the notation that follows, variables with a superscript L denote variables pertaining to the model under lockdown. Those without such superscripts refer to variables coming from the baseline model without lockdown described in Section 2 of Bradley et al. (2020).

O.1.1 Surplus functions of the lockdown model

This section proceeds by listing and describing the value functions associated with workers and vacancies of particular types. We then derive the corresponding surplus equations.

Retired workers

A retired worker that has recovered has value that comes from four sources. The first is their flow benefit of retirement. The second is the value associated with death by natural causes. The third is their value associated with the lockdown period ending, in which case they receive the baseline value of their status. The fourth is their continuation value. The formal representation follows

$$rR^L_{rt} = b_o + \chi(0 - R^L_{rt}) + \Lambda(R_r - R^L_{rt}) + \dot{R}^L_{rt}$$

An infected retired worker faces five terms associated with their value function. In addition to their flow benefit and continuation value, they face the value changes associated with the possibilities of death by natural causes, death by COVID-19, recovery and the economy leaving lockdown. The expression is given by

$$rR^L_{it} = b_o + (\chi + \gamma_o)(0 - R^L_{it}) + \rho_o(R^L_{rt} - R^L_{it}) + \Lambda(R_{it} - R^L_{it}) + \dot{R}^L_{it}$$

A retired worker that is susceptible receives a flow value, continuation value, faces a possibility of death and infection and the changes from the economy leaving lockdown. The value function can be written as

$$rR^L_{st} = b_o + \chi(0 - R^L_{st}) + \lambda^L_0 \ell_{it}(R^L_{it} - R^L_{st}) + \Lambda(R_{st} - R^L_{st}) + \dot{R}^L_{st}.$$

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Recovered young workers

The value of unemployment for a young worker is comprised of six terms. The first, the flow benefit of unemployment, is the same as in the baseline model. Their option value of finding work is now split into two terms, capturing the two possibilities of matching with a firm in the locked and unlocked sectors. The worker can retire at the rate η and the economy can come out of lockdown, in which case the worker receives the value of being recovered in the baseline model. They also receive their continuation value. The formal representation is given below

$$\begin{aligned} rU^L_{rt} &= b_u + \phi_t \beta \pi \int\int \max\{S^L_{rt}(\alpha, x; L), 0\} f(\alpha, x) d\alpha dx \\ &\quad + \phi_t \beta (1-\pi) \int\int \max\{S^L_{rt}(\alpha, x; U), 0\} f(\alpha, x) d\alpha dx \\ &\quad + \eta(R^L_{rt} - U^L_{rt}) + \Lambda(U_{rt} - U^L_{rt}) + \dot{U}^L_{rt} \end{aligned}$$

A recovered worker who is employed now has two separate values, depending on whether their job is in the locked or unlocked sector. A worker with match characteristics (α, x) has contract with wage w and stipulation $m \in \{0, 1\}$ where 1 means working away from home and 0 means working from home. If the match is in the locked sector, then the value for being employed is given by

$$\begin{aligned} rW^L_{rt}(w, \alpha, x, m; L) &= w + \delta(U^L_{rt} - W^L_{rt}(w, \alpha, x, m; L)) + \eta(R^L_{rt} - W^L_{rt}(w, \alpha, x, m; L)) \\ &\quad + \nu\left(\beta \max\{S^L_{rt}(\alpha, x, 0; L), S^L_{rt}(\alpha, x, 1; L), 0\} + U^L_{rt} - W^L_{rt}(w, \alpha, x, m; L)\right) \\ &\quad + \Lambda(W_{rt}(w, \alpha, x, m) - W^L_{rt}(w, \alpha, x, m; L)) + \dot{W}^L_{rt}(w, \alpha, x, m; L) \end{aligned}$$

where the argument L inside the parentheses signifies that the job is in the locked sector. Notice that, although the working arrangement regarding m can be negotiated in the contract, a match that is locked is unable to operate away from home for the duration of the lockdown. However, once the lockdown ends, we assume that they immediately start producing using the contracted arrangement regarding m. A match that is unlocked has the value

$$\begin{aligned} rW^L_{rt}(w, \alpha, x, m; U) &= w + \delta(U^L_{rt} - W^L_{rt}(w, \alpha, x, m; U)) + \eta(R^L_{rt} - W^L_{rt}(w, \alpha, x, m; U)) \\ &\quad + \nu\left(\beta \max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), 0\} + U^L_{rt} - W^L_{rt}(w, \alpha, x, m; U)\right) \\ &\quad + \Lambda(W_{rt}(w, \alpha, x, m) - W^L_{rt}(w, \alpha, x, m; U)) + \dot{W}^L_{rt}(w, \alpha, x, m; U), \end{aligned}$$

where this match will involve working away from home during the lockdown when the contract has $m = 1$.

Value of a filled vacancy with a recovered worker

The value of a filled job is affected directly by whether or not the job is locked. A locked job has flow value that comes from home production less the wages paid to the employee. The match will break when either the exogenous separation shock is realized, or if the worker retires. It has option value associated with re-negotiation as well as with the economy leaving lockdown; there is also an associated continuation value. The formal expression is given by

$$\begin{aligned} rJ^L_{rt}(w,\alpha,x,m;L) &= p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{rt}(w,\alpha,x,m;L)) \\ &+ \nu\left((1-\beta)\max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L),0\} + V^L_t - J^L_{rt}(w,\alpha,x,m;L)\right) \\ &+ \Lambda(J_{rt}(w,\alpha,x,m) - J^L_{rt}(w,\alpha,x,m;L)) + \dot{J}^L_{rt}(w,\alpha,x,m;L). \end{aligned}$$

where notice that V^L_t denotes the value of a vacancy under lockdown. A match that's in the unlocked sector may either be producing from home or away from home depending on the characteristics of the match; it faces no restriction. The value function for an unlocked match for contract stipulation m is

$$\begin{aligned} rJ^L_{rt}(w,\alpha,x,m;U) &= p(\alpha,x,m) - w + (\delta+\eta)(V^L_t - J^L_{rt}(w,\alpha,x,m;U)) \\ &+ \nu\left((1-\beta)\max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U),0\} + V^L_t - J^L_{rt}(w,\alpha,x,m;U)\right) \\ &+ \Lambda(J_{rt}(w,\alpha,x,m) - J^L_{rt}(w,\alpha,x,m;U)) + \dot{J}^L_{rt}(w,\alpha,x,m;U) \end{aligned}$$

where notice that the match's output varies with m.

Surplus of a match with a recovered worker

Imposing the equilibrium free entry condition, that $V^L_t = 0$, then combining expressions for retired workers, young workers and filled vacancy value functions gives the surplus equation

$$\begin{aligned} (r+\delta+\eta+\nu+\Lambda)S^L_{rt}(\alpha,x,m;L) &= p(\alpha,x,0) - b_u \\ &- \phi_t\beta\pi\int\int \max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L),0\}f(\alpha,x)d\alpha dx \\ &- \phi_t\beta(1-\pi)\int\int \max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U),0\}f(\alpha,x)d\alpha dx \\ &+ \nu\max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L),0\} \\ &+ \Lambda S_{rt}(\alpha,x,m) + \dot{S}^L_{rt}(\alpha,x,m;L) \end{aligned}$$

where again notice that the output comes from home production irrespective of the contractual choice of $m \in \{0,1\}$. Using the same logic as in appendix A.1 of the main paper, one can show that $S^L_{rt}(\alpha,x,1;L) \geq S^L_{rt}(\alpha,x,0;L)$ since $S_{rt}(\alpha,x,1) \geq S_{rt}(\alpha,x,0)$. Consequently, we write $S^L_{rt}(\alpha, x; L) = S^L_{rt}(\alpha, x, 1; L)$ henceforth. Then the surplus for an unlocked match with m is given by

$$
\begin{aligned}
(r+\delta+\eta+\nu+\Lambda)S^L_{rt}(\alpha, x, m; U) = p(\alpha, x, m) - b_u \\
&- \phi_t \beta\pi \int\int \max\{S^L_{rt}(\alpha, x, 0; L), S^L_{rt}(\alpha, x, 1; L), 0\} f(\alpha, x) d\alpha dx \\
&- \phi_t \beta(1-\pi) \int\int \max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), 0\} f(\alpha, x) d\alpha dx \\
&+ \nu \max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), 0\} \\
&+ \Lambda S_{rt}(\alpha, x, m) + \dot{S}^L_{rt}(\alpha, x, m; U).
\end{aligned}
$$

Again, we write that $S^L_{rt}(\alpha, x; U) = S^L_{rt}(\alpha, x, 1; U)$ here onward given that $p(\alpha, x, 1) \geq p(\alpha, x, 0)$ and $S_{rt}(\alpha, x, 1) \geq S_{rt}(\alpha, x, 0)$.

Infected young workers

An unemployed infected worker's value function contains six terms. They receive their flow value of unemployment and continuation value. They receive value associated with the possibility of moving to recovered status, from dying from the virus, from retiring and from the economy leaving the state of lockdown. The formal expression is given by

$$
rU^L_{it} = b_u + \rho_y \left(U^L_{rt} - U^L_{it}\right) + \gamma_y(0 - U^L_{it}) + \eta(R^L_{it} - U^L_{it}) + \Lambda(U_{it} - U^L_{it}) + \dot{U}^L_{it}.
$$

An employed worker with infection status is at home on sick pay. They produce no output but continue to receive their contracted-upon wage; upon recovery, they return back to production for the firm. Consequently, whether the infected worker's match is in the locked or unlocked sector affects their value function. A worker in the locked sector receives value from their wage, continuation value, value associated with recovery, death, retirement, re-negotiation and lockdown being lifted. In the event of recovery, the worker transitions into working away from the home, given that this is the dominant mode as discussed above. The formal expression is as follows

$$
\begin{aligned}
rW^L_{it}(w, \alpha, x; L) &= w + \rho_y \left(W^L_{rt}(w, \alpha, x, 1; L) - W^L_{it}(w, \alpha, x; L)\right) \\
&+ \gamma_y(0 - W^L_{it}(w, \alpha, x; L)) \\
&+ \delta \left(U^L_{it} - W^L_{it}(w, \alpha, x; L)\right) + \eta \left(R^L_{it} - W^L_{it}(w, \alpha, x; L)\right) \\
&+ \nu \left(\beta \max\{S^L_{it}(\alpha, x; L), 0\} + U^L_{it} - W^L_{it}(w, \alpha, x; L)\right) \\
&+ \Lambda(W_{it}(w, \alpha, x) - W^L_{it}(w, \alpha, x; L)) + \dot{W}^L_{it}(w, \alpha, x; L)
\end{aligned}
$$

and similarly for an unlocked infected worker, the expression is

$$\begin{aligned}
rW_{it}^L(w,\alpha,x;U) = {} & w + \rho_y \left(W_{rt}^L(w,\alpha,x,1;U) - W_{it}^L(w,\alpha,x;U)\right) \\
& + \gamma_y(0 - W_{it}^L(w,\alpha,x;U)) \\
& + \delta \left(U_{it}^L - W_{it}^L(w,\alpha,x;U)\right) + \eta \left(R_{it}^L - W_{it}^L(w,\alpha,x;U)\right) \\
& + \nu \left(\beta \max\{S_{it}^L(\alpha,x;U),0\} + U_{it}^L - W_{it}^L(w,\alpha,x;U)\right) \\
& + \Lambda(W_{it}(w,\alpha,x) - W_{it}^L(w,\alpha,x;U)) + \dot{W}_{it}^L(w,\alpha,x;U).
\end{aligned}$$

One point to note is that the match retains its status with regard to being in the locked or unlocked sector throughout the health status changes of the worker. That is — whatever sector their match belonged to prior and during infection — the match will remain in that sector subsequent to recovery.

Value of a filled vacancy with an infected worker

A firm that has an infected worker pays their wage as sick pay for the duration of their illness in the absence of separation. The firm receives the value associated with the possibility of the worker's recovery and the match can be broken through either exogenous or endogenous separation at re-negotiation, retirement or through death of the worker from the virus. Again, note that the job moves to a working away from home arrangement in the event of the worker's recovery. The formal expression for a match with an infected worker in a locked sector is given by

$$\begin{aligned}
rJ_{it}^L(w,\alpha,x;L) = {} & -w + \rho_y \left(J_{rt}^L(w,\alpha,x,1;L) - J_{it}^L(w,\alpha,x;L)\right) \\
& + (\gamma_y + \delta + \eta)(V_t^L - J_{it}^L(w,\alpha,x;L)) \\
& + \nu \left((1-\beta)\max\{S_{it}^L(\alpha,x;L),0\} + V_t^L - J_{it}^L(w,\alpha,x;L)\right) \\
& + \Lambda(J_{it}(w,\alpha,x) - J_{it}^L(w,\alpha,x;L)) + \dot{J}_{it}^L(w,\alpha,x;L)
\end{aligned}$$

while that in the unlocked sector is

$$\begin{aligned}
rJ_{it}^L(w,\alpha,x;U) = {} & -w + \rho_y \left(J_{rt}^L(w,\alpha,x,1;U) - J_{it}^L(w,\alpha,x;U)\right) \\
& + (\gamma_y + \delta + \eta)(V_t^L - J_{it}^L(w,\alpha,x;U)) \\
& + \nu \left((1-\beta)\max\{S_{it}^L(\alpha,x;U),0\} + V_t^L - J_{it}^L(w,\alpha,x;U)\right) \\
& + \Lambda(J_{it}(w,\alpha,x) - J_{it}^L(w,\alpha,x;U)) + \dot{J}_{it}^L(w,\alpha,x;U)
\end{aligned}$$

Surplus of a match with an infected worker

The surplus can be found through using the value functions for employment, unemployment and value of a filled job for the locked and unlocked sectors to get the surplus. Using the equilibrium condition that the value to a vacancy is zero, the surplus for a locked match is

$$
\begin{aligned}
(r + \rho_y + \gamma_y + \delta + \eta + \nu + \Lambda) S_{it}^L(\alpha, x; L) &= -b_u + \rho_y S_{rt}^L(\alpha, x; L) \\
&\quad + \nu \max\{S_{it}^L(\alpha, x; L), 0\} + \Lambda S_{it}(\alpha, x) + \dot{S}_{it}^L(\alpha, x; L)
\end{aligned}
$$

while that for an unlocked match is

$$
\begin{aligned}
(r + \rho_y + \gamma_y + \delta + \eta + \nu + \Lambda) S_{it}^L(\alpha, x; U) &= -b_u + \rho_y S_{rt}^L(\alpha, x; U) \\
&\quad + \nu \max\{S_{it}^L(\alpha, x; U), 0\} + \Lambda S_{it}(\alpha, x) + \dot{S}_{it}^L(\alpha, x; U).
\end{aligned}
$$

Susceptible young workers

The value to being an unemployed susceptible worker closely resembles that of a recovered worker, with the exception of an additional value change associated with the possibility of infection. Their value function is given as

$$
\begin{aligned}
rU_{st}^L = b_u &+ \phi_t \pi \beta \int \int \max\{S_{st}^L(\alpha, x, 0; L), S_{st}^L(\alpha, x, 1; L), 0\} f(\alpha, x) d\alpha dx \\
&+ \phi_t (1 - \pi) \beta \int \int \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} f(\alpha, x) d\alpha dx \\
&+ \lambda_0^L \ell_{it} (U_{it}^L - U_{st}^L) + \eta (R_{st}^L - U_{st}^L) + \Lambda (U_{st} - U_{st}^L) + \dot{U}_{st}^L
\end{aligned}
$$

where notice that the rate of infection is given by $\lambda_0^L \ell_{it}$, using the lockdown infection parameter that exists regardless of working decisions. A worker that is employed with contract for wages and working arrangements (w, m) has value that differs based on the sector they work in. When locked, the worker's value function is given by

$$
\begin{aligned}
rW_{st}^L(w, \alpha, x, m; L) = w &+ \delta (U_{st}^L - W_{st}^L(w, \alpha, x, m; L)) + \eta (R_{st}^L - W_{st}^L(w, \alpha, x, m; L)) \\
&+ \lambda_0^L \ell_{it} \left(W_{it}^L(w, \alpha, x, L) - W_{st}^L(w, \alpha, x, m; L) \right) \\
&+ \nu \left(\beta \max\{S_{st}^L(\alpha, x, 0; L), S_{st}^L(\alpha, x, 1; L), 0\} + U_{st}^L - W_{st}^L(w, \alpha, x, m; L) \right) \\
&+ \Lambda (W_{st}(w, \alpha, x, m) - W_{st}^L(w, \alpha, x, m; L)) + \dot{W}_{st}^L(w, \alpha, x, m; L)
\end{aligned}
$$

where notice that their infection rate is the same as that of the unemployed worker given that they are unable to work away from home. In contrast, a worker in an unlocked sector has value functions that differ based on the contracted m. For $m = 0$, see that

$$
\begin{aligned}
rW_{st}^L(w, \alpha, x, 0; U) = w &+ \delta (U_{st}^L - W_{st}^L(w, \alpha, x, 0; U)) + \eta (R_{st}^L - W_{st}^L(w, \alpha, x, 0; U)) \\
&+ \lambda_0^L \ell_{it} \left(W_{it}^L(w, \alpha, x, U) - W_{st}^L(w, \alpha, x, 0; U) \right) \\
&+ \nu \left(\beta \max\{S_{st}^L(\alpha, x, 0; U), S_{st}^L(\alpha, x, 1; U), 0\} + U_{st}^L - W_{st}^L(w, \alpha, x, 0; U) \right) \\
&+ \Lambda (W_{st}(w, \alpha, x, 0) - W_{st}^L(w, \alpha, x, 0; U)) + \dot{W}_{st}^L(w, \alpha, x, 0; U)
\end{aligned}
$$

and then for $m = 1$

$$
\begin{aligned}
rW^L_{st}(w,\alpha,x,1;U) = {} & w + \delta(U^L_{st} - W^L_{st}(w,\alpha,x,1;U)) + \eta(R^L_{st} - W^L_{st}(w,\alpha,x,1;U)) \\
& + \left[\lambda^L_0 + \lambda_1\right] \ell_{it} \left(W^L_{it}(w,\alpha,x,U) - W^L_{st}(w,\alpha,x,1;U)\right) \\
& + \nu \left(\beta \max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), 0\} + U^L_{st} - W^L_{st}(w,\alpha,x,1;U)\right) \\
& + \Lambda(W_{st}(w,\alpha,x,1) - W^L_{st}(w,\alpha,x,1;U)) + \dot{W}^L_{st}(w,\alpha,x,1;U)
\end{aligned}
$$

where the distinction between the two is the higher infection rate when working away from home.

Value of a filled vacancy with an susceptible worker

The value to a filled job with a susceptible worker in a locked industry for arbitrary contract (w, m) is given as follows

$$
\begin{aligned}
rJ^L_{st}(w,\alpha,x,m;L) = {} & p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,m;L)) \\
& + \lambda^L_0 \ell_{it} \left(J^L_{it}(w,\alpha,x;L) - J^L_{st}(w,\alpha,x,m;L)\right) \\
& + \nu \left((1-\beta) \max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L), 0\} + V^L_t - J^L_{st}(w,\alpha,x,m;L)\right) \\
& + \Lambda(J_{st}(w,\alpha,x,m) - J^L_{st}(w,\alpha,x,m;L)) + \dot{J}_{st}(w,\alpha,x,m;L).
\end{aligned}
$$

A match that is unlocked differs based on locked status. An unlocked match with $m = 0$ delivers value of

$$
\begin{aligned}
rJ^L_{st}(w,\alpha,x,0;U) = {} & p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,0;U)) \\
& + \lambda^L_0 \ell_{it} \left(J^L_{it}(w,\alpha,x;U) - J^L_{st}(w,\alpha,x,0;U)\right) \\
& + \nu \left((1-\beta) \max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), 0\} + V^L_t - J^L_{st}(w,\alpha,x,0;U)\right) \\
& + \Lambda(J_{st}(w,\alpha,x,0) - J^L_{st}(w,\alpha,x,0;U)) + \dot{J}_{st}(w,\alpha,x,0;U).
\end{aligned}
$$

and that for an $m = 1$ contract gives

$$
\begin{aligned}
rJ^L_{st}(w,\alpha,x,1;U) = {} & p(\alpha,x,1) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,1;U)) \\
& + [\lambda^L_0 + \lambda_1] \ell_{it} \left(J^L_{it}(w,\alpha,x;U) - J^L_{st}(w,\alpha,x,1;U)\right) \\
& + \nu \left((1-\beta) \max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), 0\} + V^L_t - J^L_{st}(w,\alpha,x,1;U)\right) \\
& + \Lambda(J_{st}(w,\alpha,x,1) - J^L_{st}(w,\alpha,x,1;U)) + \dot{J}_{st}(w,\alpha,x,1;U).
\end{aligned}
$$

where the match now produces the higher away from home level of output, in addition to the rate of change to infected status being higher by λ_1.

Surplus of a match with a susceptible worker

The surplus from a match in the locked sector for $m \in \{0,1\}$ is given as

$$\begin{aligned}(r+\delta+\eta+\lambda_0^L\ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,m;L) &= p(\alpha,x,0)-b_u \\ &- \phi_t\pi\beta\int\int \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}f(\alpha,x)d\alpha dx \\ &- \phi_t(1-\pi)\beta\int\int \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}f(\alpha,x)d\alpha dx \\ &+ \lambda_0^L\ell_{it}S_{it}^L(\alpha,x;L)+\nu\max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\} \\ &+ \Lambda S_{st}(\alpha,x,m)+\dot{S}_{st}^L(\alpha,x,m;L).\end{aligned}$$

The surplus for a match in the unlocked sector with $m=0$ is

$$\begin{aligned}(r+\delta+\eta+\lambda_0^L\ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,0;U) &= p(\alpha,x,0)-b_u \\ &- \phi_t\pi\beta\int\int \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}f(\alpha,x)d\alpha dx \\ &- \phi_t(1-\pi)\beta\int\int \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}f(\alpha,x)d\alpha dx \\ &+ \lambda_0^L\ell_{it}S_{it}^L(\alpha,x;U)+\nu\max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\} \\ &+ \Lambda S_{st}(\alpha,x,0)+\dot{S}_{st}^L(\alpha,x,0;U)\end{aligned}$$

and that for the unlocked sector with $m=1$ is

$$\begin{aligned}(r+\delta+\eta+(\lambda_0^L+\lambda_1)\ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,1;U) &= p(\alpha,x,1)-b_u \\ &- \phi_t\pi\beta\int\int \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}f(\alpha,x)d\alpha dx \\ &- \phi_t(1-\pi)\beta\int\int \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}f(\alpha,x)d\alpha dx \\ &+ (\lambda_0^L+\lambda_1)\ell_{it}S_{it}^L(\alpha,x;U)+\lambda_1\ell_{it}(U_{it}^L-U_{st}^L) \\ &+ \nu\max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\} \\ &+ \Lambda S_{st}(\alpha,x,1)+\dot{S}_{st}^L(\alpha,x,1;U).\end{aligned}$$

We denote $S_{st}^L(\alpha,x;L)=\max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L)\}$ and similarly for $S_{st}^L(\alpha,x;U)$.

O.1.2 Vacant jobs

Vacant jobs get in contact with unemployed workers at a rate ϕ_t^f. Upon contracting, the firm receives fraction $(1-\beta)$ of the match's generated surplus. Notice that there are four possibilities for a given vacancy with regard to the type of match that is formed. The potential unemployed workers they can match with differ along the health dimension — they could either be susceptible or recovered. In addition, there are two possibilities from the perspective of the production being in either the locked or unlocked sectors. As such, there are five terms in the value to a vacancy: the flow cost κ as well as terms capturing these four possibilities

$$\begin{aligned}
rV_t^L = -\kappa &+ \phi_t^f(1-\beta)\frac{u_{st}^L}{u_{st}^L+u_{rt}^L}\pi\int\int \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}f(\alpha,x)d\alpha dx \\
&+ \phi_t^f(1-\beta)\frac{u_{st}^L}{u_{st}^L+u_{rt}^L}(1-\pi)\int\int \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}f(\alpha,x)d\alpha dx \\
&+ \phi_t^f(1-\beta)\frac{u_{rt}^L}{u_{st}^L+u_{rt}^L}\pi\int\int \max\{S_{rt}^L(\alpha,x;L),0\}f(\alpha,x)d\alpha dx \\
&+ \phi_t^f(1-\beta)\frac{u_{rt}^L}{u_{st}^L+u_{rt}^L}(1-\pi)\int\int \max\{S_{rt}^L(\alpha,x;U),0\}f(\alpha,x)d\alpha dx
\end{aligned}$$

where u_{st}^L and u_{rt}^L denote the measures of unemployed workers with susceptible and recovered status under the lockdown model.

O.2 Dynamics of the lockdown model

This section details the dynamics of the measures of workers in differing employment, age and health statuses in the model with lockdown. Measures with L superscripts correspond to those under lockdown, while those without are from the baseline model, (out of lockdown). We assume that the lockdown runs over $t \in [\underline{t}, \overline{t}]$ where $\overline{t} - \underline{t} = 1/\Lambda$. The measures of the three different health statuses for unemployment will be equal to their pre-lockdown measures at $\underline{t}^-$, where $\underline{t}^- = \lim_{\epsilon \to 0^-} \underline{t} + \epsilon$.

$$\begin{aligned} u^L_{s\underline{t}} &= u_{s\underline{t}^-} \\ u^L_{i\underline{t}} &= u_{i\underline{t}^-} \\ u^L_{r\underline{t}} &= u_{r\underline{t}^-} \end{aligned}$$

Similarly for the retired at the time of lockdown's commencement

$$\begin{aligned} o^L_{s\underline{t}} &= o_{s\underline{t}^-} \\ o^L_{i\underline{t}} &= o_{i\underline{t}^-} \\ o_{r\underline{t}} &= o_{r\underline{t}^-} \end{aligned}$$

which are the measures of retired people across the susceptible, infected and recovered health statuses respectively. The measures of employed workers of a given health status and match state (α, x) will be split such that fraction π will be placed into the locked sector while fraction $1 - \pi$ will be in the unlocked sector as follows

$$\begin{aligned} e^L_{0s\underline{t}}(\alpha, x; L) &= \pi e_{0s\underline{t}^-}(\alpha, x) \\ e^L_{0s\underline{t}}(\alpha, x; U) &= (1 - \pi) e_{0s\underline{t}^-}(\alpha, x) \\ e^L_{1s\underline{t}}(\alpha, x; L) &= \pi e_{1s\underline{t}^-}(\alpha, x) \\ e^L_{1s\underline{t}}(\alpha, x; U) &= (1 - \pi) e_{1s\underline{t}^-}(\alpha, x) \\ e^L_{r\underline{t}}(\alpha, x; L) &= \pi e_{r\underline{t}^-}(\alpha, x) \\ e^L_{r\underline{t}}(\alpha, x; U) &= (1 - \pi) e_{r\underline{t}^-}(\alpha, x) \\ e^L_{i\underline{t}}(\alpha, x; L) &= \pi e_{i\underline{t}^-}(\alpha, x) \\ e^L_{i\underline{t}}(\alpha, x; U) &= (1 - \pi) e_{i\underline{t}^-}(\alpha, x) \end{aligned}$$

which respectively represent the measures of susceptible workers at home in locked jobs and unlocked jobs, of susceptible workers away from home in locked and unlocked jobs, of recovered workers in locked and unlocked jobs and infected workers in locked and unlocked jobs. From the point where the lockdown commences, these measures evolve endogenously.

The laws of motion for the retired are

$$\dot{o}^L_{st} = \eta\left(u^L_{st} + \int\int e^L_{0st}(\alpha, x; L)d\alpha dx + \int\int e^L_{0st}(\alpha, x; U)d\alpha dx\right)$$
$$+ \eta\left(\int\int e^L_{1st}(\alpha, x; L)d\alpha dx + \int\int e^L_{1st}(\alpha, x; U)d\alpha dx\right) - (\lambda^L_0 \ell^L_{it} + \chi)o^L_{st}$$

$$\dot{o}^L_{it} = \eta\left(u^L_{it} + \int\int e^L_{it}(\alpha, x; L)d\alpha dx + \int\int e^L_{it}(\alpha, x; U)d\alpha dx\right)$$
$$+ \lambda^L_0 \ell^L_{it} o^L_{st} - (\gamma_o + \chi + \rho_o)o^L_{it}$$

$$\dot{o}^L_{rt} = \eta\left(u^L_{rt} + \int\int e^L_{rt}(\alpha, x; L)d\alpha dx + \int\int e^L_{rt}(\alpha, x; U)d\alpha dx\right)$$
$$+ \rho_o o^L_{it} - \chi o^L_{rt}$$

while those for the unemployed are

$$\begin{aligned}
\dot{u}_{st}^L = {} & \psi + \delta \int \int e_{0st}^L(\alpha, x; L) d\alpha dx + \delta \int \int e_{1st}^L(\alpha, x; L) d\alpha dx \\
& + \delta \int \int e_{0st}^L(\alpha, x; U) d\alpha dx + \delta \int \int e_{1st}^L(\alpha, x; U) d\alpha dx \\
& + \nu \int \int e_{0st}^L(\alpha, x; L)\{S_{st}^L(\alpha, x; L) < 0\} d\alpha dx + \nu \int \int e_{1st}^L(\alpha, x; L)\{S_{st}^L(\alpha, x; L) < 0\} d\alpha dx \\
& + \nu \int \int e_{0st}^L(\alpha, x; U)\{S_{st}^L(\alpha, x; U) < 0\} d\alpha dx + \nu \int \int e_{1st}^L(\alpha, x; U)\{S_{st}^L(\alpha, x; U) < 0\} d\alpha dx \\
& - \phi_t^L u_{st}^L \pi \int \int \{S_{st}^L(\alpha, x, L) \geq 0\} f(\alpha, x) d\alpha dx \\
& - \phi_t^L u_{st}^L (1 - \pi) \int \int \{S_{st}^L(\alpha, x, U) \geq 0\} f(\alpha, x) d\alpha dx \\
& - \lambda_0^L \ell_{it}^L u_{st}^L - \eta u_{st}^L
\end{aligned}$$

$$\begin{aligned}
\dot{u}_{it}^L = {} & \delta \int \int e_{it}^L(\alpha, x; L) d\alpha dx + \delta \int \int e_{it}^L(\alpha, x; U) d\alpha dx \\
& + \nu \int \int e_{it}^L(\alpha, x; L)\{S_{it}^L(\alpha, x; L) < 0\} d\alpha dx + \nu \int \int e_{it}^L(\alpha, x; U)\{S_{it}^L(\alpha, x; U) < 0\} d\alpha dx \\
& + \lambda_0^L \ell_{it}^L u_{st}^L - (\rho_y + \gamma(\ell_{it}^L) + \eta) u_{it}^L
\end{aligned}$$

$$\begin{aligned}
\dot{u}_{rt}^L = {} & \delta \int \int e_{rt}^L(\alpha, x; L) d\alpha dx + \delta \int \int e_{rt}^L(\alpha, x; U) d\alpha dx \\
& + \nu \int \int e_{rt}^L(\alpha, x; L)\{S_{rt}^L(\alpha, x; L) < 0\} d\alpha dx + \nu \int \int e_{rt}^L(\alpha, x; U)\{S_{rt}^L(\alpha, x; U) < 0\} d\alpha dx \\
& + \rho_y u_{it}^L - \phi_t^L u_{rt}^L \pi \int \int \{S_{rt}^L(\alpha, x, L) \geq 0\} f(\alpha, x) d\alpha dx \\
& - \phi_t^L u_{rt}^L (1 - \pi) \int \int \{S_{rt}^L(\alpha, x, U) \geq 0\} f(\alpha, x) d\alpha dx.
\end{aligned}$$

For the employed workers, we track measures across the different match characteristics and lock statuses. For measures of susceptible employed, the equations are as follows.

$$\begin{aligned}\dot{e}^L_{0st}(\alpha,x;L) &= \pi u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)<S^L_{st}(\alpha,x,0;L)\}f(\alpha,x)\\ &\quad-(\delta+\eta)e^L_{0st}(\alpha,x;L)-\nu e^L_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x,0;L)<0\}\\ &\quad-\nu e^L_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}\\ &\quad+\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)<S^L_{st}(\alpha,x,0;L)\}\\ &\quad-e^L_{0st}(\alpha,x;L)\lambda^L_0\ell^L_{it}\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{0st}(\alpha,x;U) &= (1-\pi) u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U)\}f(\alpha,x)\\ &\quad-(\delta+\eta)e^L_{0st}(\alpha,x;U)-\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x,0;U)<0\}\\ &\quad-\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}\\ &\quad+\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U)\}\\ &\quad-e^L_{0st}(\alpha,x;U)\lambda^L_0\ell^L_{it}\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{1st}(\alpha,x;L) &= \pi u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}f(\alpha,x)\\ &\quad-(\delta+\eta)e^L_{1st}(\alpha,x;L)-\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x,1;L)<0\}\\ &\quad-\nu e^L_{1st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)< S^L_{st}(\alpha,x,0;L)\}\\ &\quad+\nu e_{0st}(\alpha,x;L)\{S^L_{st}(\alpha,x;L)\geq 0\}\{S^L_{st}(\alpha,x,1;L)\geq S^L_{st}(\alpha,x,0;L)\}\\ &\quad-e^L_{1st}(\alpha,x;L)\lambda^L_0\ell^L_{it}\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{1st}(\alpha,x;U) &= (1-\pi) u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}f(\alpha,x)\\ &\quad-(\delta+\eta)e^L_{1st}(\alpha,x;U)-\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x,1;U)<0\}\\ &\quad-\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)< S^L_{st}(\alpha,x,0;U)\}\\ &\quad+\nu e^L_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}\\ &\quad-e^L_{1st}(\alpha,x;U)(\lambda^L_0+\lambda_1)\ell^L_{it}.\end{aligned}$$

For the measures of infected employed, the dynamics evolve according to

$$\begin{aligned}\dot{e}^L_{it}(\alpha,x;L) &= e^L_{0st}(\alpha,x;L)\lambda^L_0\ell^L_{it}+e^L_{1st}(\alpha,x;L)(\lambda^L_0)\ell^L_{it}-\nu e^L_{it}(\alpha,x;L)\{S^L_{it}(\alpha,x;L)<0\}\\ &\quad-(\delta+\rho_y+\gamma(\ell^L_{it})+\eta)e^L_{it}(\alpha,x;L)\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{it}(\alpha,x;U) &= e^L_{0st}(\alpha,x;U)\lambda^L_0\ell^L_{it}+e^L_{1st}(\alpha,x;U)(\lambda^L_0+\lambda_1)\ell^L_{it}-\nu e^L_{it}(\alpha,x;U)\{S^L_{it}(\alpha,x;U)<0\}\\ &\quad-(\delta+\rho_y+\gamma(\ell^L_{it})+\eta)e^L_{it}(\alpha,x;U).\end{aligned}$$

The measures of employed that are recovered evolve as follows

$$\begin{aligned}\dot{e}^L_{rt}(\alpha, x; L) &= \pi u^L_{rt}\phi_t\{S^L_{rt}(\alpha, x; L) \geq 0\}f(\alpha, x) + \rho_y e^L_{it}(\alpha, x; L) - (\delta + \eta)e^L_{rt}(\alpha, x; L)\\ &\quad - \nu e^L_{rt}(\alpha, x)\{S^L_{rt}(\alpha, x; L) < 0\}\end{aligned}$$

$$\begin{aligned}\dot{e}^L_{rt}(\alpha, x; U) &= (1 - \pi)u^L_{rt}\phi^L_t\{S^L_{rt}(\alpha, x; U) \geq 0\}f(\alpha, x) + \rho_y e^L_{it}(\alpha, x; U) - (\delta + \eta)e^L_{rt}(\alpha, x; U)\\ &\quad - \nu e^L_{rt}(\alpha, x)\{S^L_{rt}(\alpha, x; U) < 0\}.\end{aligned}$$

Finally, recall the time the lockdown ceases is denoted by $\bar{t}$. Let $\bar{t}^+ = \lim_{\epsilon\to 0^+} \bar{t} + \epsilon$ be the time immediately after lockdown cessation. At this time, the measure of unemployed of each status for non-lockdown are equal to the values below.

$$\begin{aligned}u_{s\bar{t}^+} &= u^L_{s\bar{t}}\\ u_{i\bar{t}^+} &= u^L_{i\bar{t}}\\ u_{r\bar{t}^+} &= u^L_{r\bar{t}}\end{aligned}$$

Similarly, for the three health statuses of retired workers

$$\begin{aligned}o_{s\bar{t}^+} &= o^L_{s\bar{t}}\\ o_{i\bar{t}^+} &= o^L_{i\bar{t}}\\ o_{r\bar{t}^+} &= o^L_{r\bar{t}}.\end{aligned}$$

Lastly, for each employment health status and idiosyncratic match state, the measures of locked and unlocked matches are summed to together as follows

$$\begin{aligned}e_{0s\bar{t}^+}(\alpha, x) &= e^L_{0s\bar{t}}(\alpha, x; L) + e^L_{0s\bar{t}}(\alpha, x; U)\\ e_{1s\bar{t}^+}(\alpha, x) &= e^L_{1s\bar{t}}(\alpha, x; L) + e^L_{1s\bar{t}}(\alpha, x; U)\\ e_{r\bar{t}^+}(\alpha, x) &= e^L_{r\bar{t}}(\alpha, x; L) + e^L_{r\bar{t}}(\alpha, x; U)\\ e_{i\bar{t}^+}(\alpha, x) &= e^L_{i\bar{t}}(\alpha, x; L) + e^L_{i\bar{t}}(\alpha, x; U)\end{aligned}$$

which are for the susceptible employed working at home and away from home, the recovered and infected respectively. From time $\bar{t}^+$ onwards, the measures evolve endogenously as described in appendix A.2

O.3 Solving the model under lockdown with furlough

Here we present the surplus functions for the lockdown model with furlough. Notice that the value of a vacancy and thus the free entry condition, in this model will be the same as in section 1.2 of this online appendix.

Retired workers

The retired workers in this variant of the model have the same value functions as in section 1.1 above.

Recovered young workers

Given that new matches can only be formed in non-furloughed jobs, the unemployed value functions for all the health statuses will be the same as in section 1.1 above.

Workers now face three possible working arrangements — working from home $m = 0$, working away from home $m = 1$ and being furloughed $m = 2$. In the event of indifference, we assume for workers of all health statuses that working away from home is the preferred option, followed by working from home, followed by furlough.

A locked recovered worker derives value from their flow wages, they can exogenously separate from the match or retire. At re-negotiation, they have the additional option value associated with furlough. The values of employment in a contract $m \in \{0, 1\}$ for locked and unlocked jobs are equal to

$$\begin{aligned} rW^L_{rt}(w,\alpha,x,m;L) &= w + \delta(U^L_{rt} - W^L_{rt}(w,\alpha,x,m;L)) + \eta(R^L_{rt} - W^L_{rt}(w,\alpha,x,m;L)) \\ &\quad + \nu\left(\beta\max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L), S^L_{rt}(\alpha,x,2;L), 0\} + U^L_{rt} - W^L_{rt}(w,\alpha,x,m;L)\right) \\ &\quad + \Lambda(W_{rt}(w,\alpha,x,m) - W^L_{rt}(w,\alpha,x,m;L)) + \dot{W}^L_{rt}(w,\alpha,x,m;L) \end{aligned}$$

and

$$\begin{aligned} rW^L_{rt}(w,\alpha,x,m;U) &= w + \delta(U^L_{rt} - W^L_{rt}(w,\alpha,x,m;U)) + \eta(R^L_{rt} - W^L_{rt}(w,\alpha,x,m;U)) \\ &\quad + \nu\left(\beta\max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U), S^L_{rt}(\alpha,x,2;U), 0\} + U^L_{rt} - W^L_{rt}(w,\alpha,x,m;U)\right) \\ &\quad + \Lambda(W_{rt}(w,\alpha,x,m) - W^L_{rt}(w,\alpha,x,m;U)) + \dot{W}^L_{rt}(w,\alpha,x,m;U). \end{aligned}$$

Consider instead a recovered individual under furlough ($m = 2$). At the cessation of lockdown, since furlough is no longer an option, the match must re-decide on their working arrangement from $m \in \{0, 1\}$. The value of employment for a recovered individual in locked and unlocked jobs under furlough are equal to

$$\begin{aligned} rW^L_{rt}(w,\alpha,x,m;L) &= w + \delta(U^L_{rt} - W^L_{rt}(w,\alpha,x,m;L)) + \eta(R^L_{rt} - W^L_{rt}(w,\alpha,x,m;L)) \\ &\quad + \nu\left(\beta \max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L), S^L_{rt}(\alpha,x,2;L), 0\} + U^L_{rt} - W^L_{rt}(w,\alpha,x,m;L)\right) \\ &\quad + \Lambda(W_{rt}(w,\alpha,x,1) - W^L_{rt}(w,\alpha,x,m;L)) + \dot{W}^L_{rt}(w,\alpha,x,m;L) \end{aligned}$$

and

$$\begin{aligned} rW^L_{rt}(w,\alpha,x,m;U) &= w + \delta(U^L_{rt} - W^L_{rt}(w,\alpha,x,m;U)) + \eta(R^L_{rt} - W^L_{rt}(w,\alpha,x,m;U)) \\ &\quad + \nu\left(\beta \max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U), S^L_{rt}(\alpha,x,2;U), 0\} + U^L_{rt} - W^L_{rt}(w,\alpha,x,m;U)\right) \\ &\quad + \Lambda(W_{rt}(w,\alpha,x,1) - W^L_{rt}(w,\alpha,x,m;U)) + \dot{W}^L_{rt}(w,\alpha,x,m;U). \end{aligned}$$

Notice that the match will move to a $m = 1$ arrangement at the end of lockdown given its dominance over $m = 0$ as shown in the main appendix.

Value of a filled vacancy with a recovered worker

A locked match can generate two possible levels of output. In the event that the contractual stipulation is one of $m \in \{0,1\}$, $p(\alpha,x,0)$ is produced, while $p(\alpha,x,2)$ is produced under furlough. A filled vacancy has option value coming from the possible choice over $m \in \{0,1,2\}$ should the renegotiation shock be realized. The match turns to an unfilled vacancy when the retirement or separation shocks are realized and the working arrangement is re-optimized when lockdown ends. The formal expression for $m \in \{0,1\}$ is then given by

$$\begin{aligned} rJ^L_{rt}(w,\alpha,x,m;L) &= p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{rt}(w,\alpha,x,m;L)) \\ &\quad + \nu\left((1-\beta)\max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L), S^L_{rt}(\alpha,x,2;L), 0\} + V^L_t - J^L_{rt}(w,\alpha,x,m;L)\right) \\ &\quad + \Lambda(J_{rt}(w,\alpha,x,m) - J^L_{rt}(w,\alpha,x,m;L)) + \dot{J}^L_{rt}(w,\alpha,x,m;L). \end{aligned}$$

while that for $m = 2$ is

$$\begin{aligned} rJ^L_{rt}(w,\alpha,x,2;L) &= p(\alpha,x,2) - w + (\delta+\eta)(V^L_t - J^L_{rt}(w,\alpha,x,2;L)) \\ &\quad + \nu\left((1-\beta)\max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L), S^L_{rt}(\alpha,x,2;L), 0\} + V^L_t - J^L_{rt}(w,\alpha,x,2;L)\right) \\ &\quad + \Lambda(J_{rt}(w,\alpha,x,1) - J^L_{rt}(w,\alpha,x,2;L)) + \dot{J}^L_{rt}(w,\alpha,x,2;L) \end{aligned}$$

An unlocked match may instead produce in accordance with their work arrangement. For $m \in \{0,1\}$

$$\begin{aligned} rJ^L_{rt}(w,\alpha,x,m;U) &= p(\alpha,x,m) - w + (\delta+\eta)(V^L_t - J^L_{rt}(w,\alpha,x,m;U)) \\ &\quad + \nu\left((1-\beta)\max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U), S^L_{rt}(\alpha,x,2;U), 0\} + V^L_t - J^L_{rt}(w,\alpha,x,m;U)\right) \\ &\quad + \Lambda(J_{rt}(w,\alpha,x,m) - J^L_{rt}(w,\alpha,x,m;U)) + \dot{J}^L_{rt}(w,\alpha,x,m;U). \end{aligned}$$

while for $m = 2$

$$
\begin{aligned}
rJ^L_{rt}(w,\alpha,x,m;U) &= p(\alpha,x,m) - w + (\delta+\eta)(V^L_t - J^L_{rt}(w,\alpha,x,m;U)) \\
&\quad + \nu\left((1-\beta)\max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U), S^L_{rt}(\alpha,x,2;U), 0\} + V^L_t - J^L_{rt}(w,\alpha,x,m;U)\right) \\
&\quad + \Lambda(J_{rt}(w,\alpha,x,1) - J^L_{rt}(w,\alpha,x,m;U)) + \dot{J}^L_{rt}(w,\alpha,x,m;U).
\end{aligned}
$$

where notice again that the match optimally turns to a working away from home arrangement at the cessation of lockdown.

Surplus of a match with a recovered worker

Combining the value functions for workers, filled matches and the free entry condition, a locked match with $m \in \{0,1\}$ generates surplus of the form

$$
\begin{aligned}
(r+\delta+\eta+\nu+\Lambda)S^L_{rt}(\alpha,x,m;L) &= p(\alpha,x,0) - b_u \\
&\quad - \phi_t\beta\pi\int\int \max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L), 0\}f(\alpha,x)d\alpha dx \\
&\quad - \phi_t\beta(1-\pi)\int\int \max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U), 0\}f(\alpha,x)d\alpha dx \\
&\quad + \nu\max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L), S^L_{rt}(\alpha,x,2;L), 0\} \\
&\quad + \Lambda S_{rt}(\alpha,x,m) + \dot{S}^L_{rt}(\alpha,x,m;L).
\end{aligned}
$$

For a recovered locked match with $m = 2$, the surplus is given by

$$
\begin{aligned}
(r+\delta+\eta+\nu+\Lambda)S^L_{rt}(\alpha,x,2;L) &= p(\alpha,x,2) - b_u \\
&\quad - \phi_t\beta\pi\int\int \max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L), 0\}f(\alpha,x)d\alpha dx \\
&\quad - \phi_t\beta(1-\pi)\int\int \max\{S^L_{rt}(\alpha,x,0;U), S^L_{rt}(\alpha,x,1;U), 0\}f(\alpha,x)d\alpha dx \\
&\quad + \nu\max\{S^L_{rt}(\alpha,x,0;L), S^L_{rt}(\alpha,x,1;L), S^L_{rt}(\alpha,x,2;L), 0\} \\
&\quad + \Lambda S_{rt}(\alpha,x) + \dot{S}^L_{rt}(\alpha,x,2;L).
\end{aligned}
$$

One can show that $S^L_{rt}(\alpha, x, 1; L) \geq S^L_{rt}(\alpha, x, 0; L)$ similarly to section 1.1 of this online appendix. For an unlocked match with $m \in \{0, 1\}$

$$\begin{aligned}(r + \delta + \eta + \nu + \Lambda)S^L_{rt}(\alpha, x, m; U) = {} & p(\alpha, x, m) - b_u \\ & - \phi_t \beta \pi \int\int \max\{S^L_{rt}(\alpha, x, 0; L), S^L_{rt}(\alpha, x, 1; L), 0\} f(\alpha, x) d\alpha dx \\ & - \phi_t \beta (1 - \pi) \int\int \max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), 0\} f(\alpha, x) d\alpha dx \\ & + \nu \max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), S^L_{rt}(\alpha, x, 2; U), 0\} \\ & + \Lambda S_{rt}(\alpha, x, m) + \dot{S}^L_{rt}(\alpha, x, m; U)\end{aligned}$$

Finally for a furloughed recovered unlocked match

$$\begin{aligned}(r + \delta + \eta + \nu + \Lambda)S^L_{rt}(\alpha, x, 2; U) = {} & p(\alpha, x, 2) - b_u \\ & - \phi_t \beta \pi \int\int \max\{S^L_{rt}(\alpha, x, 0; L), S^L_{rt}(\alpha, x, 1; L), 0\} f(\alpha, x) d\alpha dx \\ & - \phi_t \beta (1 - \pi) \int\int \max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), 0\} f(\alpha, x) d\alpha dx \\ & + \nu \max\{S^L_{rt}(\alpha, x, 0; U), S^L_{rt}(\alpha, x, 1; U), S^L_{rt}(\alpha, x, 2; U), 0\} \\ & + \Lambda S_{rt}(\alpha, x) + \dot{S}^L_{rt}(\alpha, x, 2; U).\end{aligned}$$

where again $S^L_{rt}(\alpha, x, 1; U) \geq S^L_{rt}(\alpha, x, 0; U)$ given that $p(\alpha, x, 1) \geq p(\alpha, x, 0)$. In what follows, we denote

$$\begin{aligned}S^L_{rt}(\alpha, x; L) &= \max\{S^L_{rt}(\alpha, x, m; L)\} \\ S^L_{rt}(\alpha, x; U) &= \max\{S^L_{rt}(\alpha, x, m; U)\}.\end{aligned}$$

Infected young workers

There are two possible contractual settings for infected workers — to be on sick pay (denoted by $m = 0$) or to be furloughed (denoted by $m = 2$). In the face of a renegotiation shock, the match optimizes over these two arrangements. A worker can die, have their job exogenously destroyed or retire. When lockdown ends and the furlough option is revoked by the government, the worker moves automatically to a sick leave arrangement. Finally, in the event of recovery, the match chooses m to maximize the joint surplus:

$$\begin{aligned}\widehat{m}_{rt}(L) &= \arg\max_{m \in \{0,1,2\}} S^L_{rt}(\alpha, x, m; L) \\ \widehat{m}_{rt}(U) &= \arg\max_{m \in \{0,1,2\}} S^L_{rt}(\alpha, x, m; U).\end{aligned}$$

Notice that while working away from home still dominates home working, the match could now choose to be on furlough. Then we can write the value function for a locked infected worker with $m \in \{0, 2\}$ as follows

$$
\begin{aligned}
rW_{it}^L(w, \alpha, x, m; L) = w &+ \rho_y \left(W_{rt}^L(w, \alpha, x, \widehat{m}_{rt}(L); L) - W_{it}^L(w, \alpha, x, m; L) \right) \\
&+ \gamma_y (0 - W_{it}^L(w, \alpha, x, m; L)) \\
&+ \delta \left(U_{it}^L - W_{it}^L(w, \alpha, x, m; L) \right) + \eta \left(R_{it}^L - W_{it}^L(w, \alpha, x, m; L) \right) \\
&+ \nu \left(\beta \max\{S_{it}^L(\alpha, x, 0; L), S_{it}^L(\alpha, x, 2; L), 0\} + U_{it}^L - W_{it}^L(w, \alpha, x, m; L) \right) \\
&+ \Lambda (W_{it}(w, \alpha, x) - W_{it}^L(w, \alpha, x, m; L)) \\
&+ \dot{W}_{it}^L(w, \alpha, x, m; L)
\end{aligned}
$$

and that for an unlocked match as

$$
\begin{aligned}
rW_{it}^L(w, \alpha, x, m; U) = w &+ \rho_y \left(W_{rt}^L(w, \alpha, x, \widehat{m}_{rt}(U); U) - W_{it}^L(w, \alpha, x, m; U) \right) \\
&+ \gamma_y (0 - W_{it}^L(w, \alpha, x, m; U)) \\
&+ \delta \left(U_{it}^L - W_{it}^L(w, \alpha, x, m; U) \right) + \eta \left(R_{it}^L - W_{it}^L(w, \alpha, x, m; U) \right) \\
&+ \nu \left(\beta \max\{S_{it}^L(\alpha, x, 0; U), S_{it}^L(\alpha, x, 2; U), 0\} + U_{it}^L - W_{it}^L(w, \alpha, x, m; U) \right) \\
&+ \Lambda (W_{it}(w, \alpha, x) - W_{it}^L(w, \alpha, x, m; U)) \\
&+ \dot{W}_{it}^L(w, \alpha, x, m; U).
\end{aligned}
$$

Value of a filled vacancy with an infected worker

A match with an infected worker generates zero output when the worker is on sick leave, but receives amount $p(\alpha, x, 2)$ with a furlough contract. The formal expression for a sick leave locked contract is

$$
\begin{aligned}
rJ_{it}^L(w, \alpha, x, 0; L) = -w &+ \rho_y \left(J_{rt}^L(w, \alpha, x, \widehat{m}_{rt}(L); L) - J_{it}^L(w, \alpha, x, 0; L) \right) \\
&+ (\gamma_y + \delta + \eta)(V_t^L - J_{it}^L(w, \alpha, x, 0; L)) \\
&+ \nu \left((1 - \beta) \max\{S_{it}^L(\alpha, x, 0; L), S_{it}^L(\alpha, x, 2; L), 0\} + V_t^L - J_{it}^L(w, \alpha, x, 0; L) \right) \\
&+ \Lambda (J_{it}(w, \alpha, x) - J_{it}^L(w, \alpha, x, 0; L)) + \dot{J}_{it}^L(w, \alpha, x, 0; L)
\end{aligned}
$$

while that for a locked furlough arrangement is

$$
\begin{aligned}
rJ_{it}^L(w, \alpha, x, 2; L) = p(\alpha, x, 2) - w &+ \rho_y \left(J_{rt}^L(w, \alpha, x, \widehat{m}_{rt}(L); L) - J_{it}^L(w, \alpha, x, 2; L) \right) \\
&+ (\gamma_y + \delta + \eta)(V_t^L - J_{it}^L(w, \alpha, x, 2; L)) \\
&+ \nu \left((1 - \beta) \max\{S_{it}^L(\alpha, x, 0; L), S_{it}^L(\alpha, x, 2; L), 0\} + V_t^L - J_{it}^L(w, \alpha, x, 2; L) \right) \\
&+ \Lambda (J_{it}(w, \alpha, x) - J_{it}^L(w, \alpha, x, 2; L)) + \dot{J}_{it}^L(w, \alpha, x, 2; L).
\end{aligned}
$$

Similarly for unlocked matches, the value function with sick pay is

$$
\begin{aligned}
rJ_{it}^L(w,\alpha,x,0;U) = {} & -w + \rho_y \left(J_{rt}^L(w,\alpha,x,\widehat{m}_{rt}(U);U) - J_{it}^L(w,\alpha,x,0;U)\right) \\
& + (\gamma_y + \delta + \eta)(V_t^L - J_{it}^L(w,\alpha,x,0;U)) \\
& + \nu \left((1-\beta)\max\{S_{it}^L(\alpha,x,0;U), S_{it}^L(\alpha,x,2;U), 0\} + V_t^L - J_{it}^L(w,\alpha,x,0;U)\right) \\
& + \Lambda(J_{it}(w,\alpha,x) - J_{it}^L(w,\alpha,x,0;U)) + \dot{J}_{it}^L(w,\alpha,x,0;U)
\end{aligned}
$$

while that for an unlocked furlough arrangement is

$$
\begin{aligned}
rJ_{it}^L(w,\alpha,x,2;U) = {} & p(\alpha,x,2) - w + \rho_y \left(J_{rt}^L(w,\alpha,x,\widehat{m}_{rt}(U);U) - J_{it}^L(w,\alpha,x,2;U)\right) \\
& + (\gamma_y + \delta + \eta)(V_t^L - J_{it}^L(w,\alpha,x,2;U)) \\
& + \nu \left((1-\beta)\max\{S_{it}^L(\alpha,x,0;U), S_{it}^L(\alpha,x,2;U), 0\} + V_t^L - J_{it}^L(w,\alpha,x,2;U)\right) \\
& + \Lambda(J_{it}(w,\alpha,x) - J_{it}^L(w,\alpha,x,2;U)) + \dot{J}_{it}^L(w,\alpha,x,2;U).
\end{aligned}
$$

Surplus of a match with an infected worker

Using the expressions for the value functions and the free entry condition, the surplus function for a locked match with a infected worker on sick leave is

$$
\begin{aligned}
(r + \rho_y + \gamma_y + \delta + \eta + \nu + \Lambda)S_{it}^L(\alpha,x,0;L) = {} & -b_u \\
& + \rho_y \max\{S_{rt}^L(\alpha,x,1;L), S_{rt}^L(\alpha,x,2;L)\} \\
& + \nu \max\{S_{it}^L(\alpha,x,0;L), S_{it}^L(\alpha,x,2;L), 0\} \\
& + \Lambda S_{it}(\alpha,x) + \dot{S}_{it}^L(\alpha,x,0;L)
\end{aligned}
$$

while that for a locked furloughed worker is

$$
\begin{aligned}
(r + \rho_y + \gamma_y + \delta + \eta + \nu + \Lambda)S_{it}^L(\alpha,x,2;L) = {} & p(\alpha,x,2) - b_u \\
& + \rho_y \max\{S_{rt}^L(\alpha,x,1;L), S_{rt}^L(\alpha,x,2;L)\} \\
& + \nu \max\{S_{it}^L(\alpha,x,0;L), S_{it}^L(\alpha,x,2;L), 0\} \\
& + \Lambda S_{it}(\alpha,x) + \dot{S}_{it}^L(\alpha,x,2;L).
\end{aligned}
$$

Given that $p(\alpha,x,2) \geq 0$, a match optimizing over the working arrangement of an infected worker during lockdown will always choose furlough over sick pay, i.e. $S_{it}^L(\alpha,x,2;L) \geq S_{it}^L(\alpha,x,0;L)$. Denote $S_{it}^L(\alpha,x;L) = S_{it}^L(\alpha,x,2;L)$ henceforth. The surplus for an unlocked worker on sick leave is

$$
\begin{aligned}
(r + \rho_y + \gamma_y + \delta + \eta + \nu + \Lambda)S_{it}^L(\alpha,x,0;U) = {} & -b_u \\
& + \rho_y \max\{S_{rt}^L(\alpha,x,1;U), S_{rt}^L(\alpha,x,2;U)\} \\
& + \nu \max\{S_{it}^L(\alpha,x,0;U), S_{it}^L(\alpha,x,2;U), 0\} \\
& + \Lambda S_{it}(\alpha,x) + \dot{S}_{it}^L(\alpha,x,0;U)
\end{aligned}
$$

and that for an unlocked match on furlough is

$$
\begin{aligned}
(r+\rho_y+\gamma_y+\delta+\eta+\nu+\Lambda)S^L_{it}(\alpha,x,2;U) &= p(\alpha,x,2)-b_u \\
&+\rho_y\max\{S^L_{rt}(\alpha,x,1;U),S^L_{rt}(\alpha,x,2;U)\} \\
&+\nu\max\{S^L_{it}(\alpha,x,0;U),S^L_{it}(\alpha,x,2;U),0\} \\
&+\Lambda S_{it}(\alpha,x)+\dot{S}^L_{it}(\alpha,x,2;U)
\end{aligned}
$$

where we can also infer that $S^L_{it}(\alpha,x,2;U)\geq S^L_{it}(\alpha,x,0;U)$ for the same reasons as above, thus we denote $S^L_{it}(\alpha,x;U)=S^L_{it}(\alpha,x,2;U)$ in what follows.

Susceptible young workers

A susceptible worker receives their flow of wages, can separate exogenously from their match and retire. In the event of renegotiation, they optimize over their working arrangement $m\in\{0,1,2\}$. If lockdown ends, they work as per their chosen arrangement. In the event of infection, the match has the option of whether to place the worker on sick leave or furlough; they always choose the latter given that it dominates in terms of surplus. The value function for a locked worker with $m\in\{0,1\}$ is

$$
\begin{aligned}
rW^L_{st}(w,\alpha,x,m;L) &= w+\delta(U^L_{st}-W^L_{st}(w,\alpha,x,m;L))+\eta(R^L_{st}-W^L_{st}(w,\alpha,x,m;L)) \\
&+\lambda^L_0\ell_{it}\left(W^L_{it}(w,\alpha,x,2,L)-W^L_{st}(w,\alpha,x,m;L)\right) \\
&+\nu\left(\beta\max\{S^L_{st}(\alpha,x,0;L),S^L_{st}(\alpha,x,1;L),S^L_{st}(\alpha,x,2;L),0\}+U^L_{st}-W^L_{st}(w,\alpha,x,m;L)\right) \\
&+\Lambda(W_{st}(w,\alpha,x,m)-W^L_{st}(w,\alpha,x,m;L))+\dot{W}^L_{st}(w,\alpha,x,m;L)
\end{aligned}
$$

A locked worker on furlough has a similar value function, with the exception that the match chooses whether to work at home or away at the cessation of lockdown. Denote $\widehat{m}_{st}\in\{0,1\}$ as the work arrangement that maximizes the joint surplus at the end of lockdown as

$$
\widehat{m}_{st}=\arg\max_{m\in\{0,1\}}S_{st}(\alpha,x,m).
$$

We can then write the value function for a locked susceptible furloughed worker as

$$
\begin{aligned}
rW^L_{st}(w,\alpha,x,2;L) &= w+\delta(U^L_{st}-W^L_{st}(w,\alpha,x,2;L))+\eta(R^L_{st}-W^L_{st}(w,\alpha,x,2;L)) \\
&+\lambda^L_0\ell_{it}\left(W^L_{it}(w,\alpha,x,2;L)-W^L_{st}(w,\alpha,x,2;L)\right) \\
&+\nu\left(\beta\max\{S^L_{st}(\alpha,x,0;L),S^L_{st}(\alpha,x,1;L),S^L_{st}(\alpha,x,2;L),0\}+U^L_{st}-W^L_{st}(w,\alpha,x,2;L)\right) \\
&+\Lambda(W_{st}(w,\alpha,x,\widehat{m}_{st})-W^L_{st}(w,\alpha,x,2;L))+\dot{W}^L_{st}(w,\alpha,x,2;L).
\end{aligned}
$$

An unlocked worker with $m = 0$ faces value function

$$\begin{aligned}
rW^L_{st}(w,\alpha,x,0;U) &= w + \delta(U^L_{st} - W^L_{st}(w,\alpha,x,0;U)) + \eta(R^L_{st} - W^L_{st}(w,\alpha,x,0;U)) \\
&+ \lambda^L_0 \ell_{it} \left(W^L_{it}(w,\alpha,x,2;U) - W^L_{st}(w,\alpha,x,0;U)\right) \\
&+ \nu \left(\beta \max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), S^L_{st}(\alpha,x,2;U), 0\} + U^L_{st} - W^L_{st}(w,\alpha,x,0;U)\right) \\
&+ \Lambda(W_{st}(w,\alpha,x,0) - W^L_{st}(w,\alpha,x,0;U)) + \dot{W}^L_{st}(w,\alpha,x,0;U)
\end{aligned}$$

while that with $m = 1$ incurs the higher infection rate associated with working outside the home

$$\begin{aligned}
rW^L_{st}(w,\alpha,x,1;U) &= w + \delta(U^L_{st} - W^L_{st}(w,\alpha,x,1;U)) + \eta(R^L_{st} - W^L_{st}(w,\alpha,x,1;U)) \\
&+ [\lambda^L_0 + \lambda_1] \ell_{it} \left(W^L_{it}(w,\alpha,x,2;U) - W^L_{st}(w,\alpha,x,1;U)\right) \\
&+ \nu \left(\beta \max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), S^L_{st}(\alpha,x,2;U), 0\} + U^L_{st} - W^L_{st}(w,\alpha,x,1;U)\right) \\
&+ \Lambda(W_{st}(w,\alpha,x,1) - W^L_{st}(w,\alpha,x,1;U)) + \dot{W}^L_{st}(w,\alpha,x,1;U).
\end{aligned}$$

Finally an unlocked furloughed worker receives

$$\begin{aligned}
rW^L_{st}(w,\alpha,x,2;U) &= w + \delta(U^L_{st} - W^L_{st}(w,\alpha,x,2;U)) + \eta(R^L_{st} - W^L_{st}(w,\alpha,x,2;U)) \\
&+ \lambda^L_0 \ell_{it} \left(W^L_{it}(w,\alpha,x,2,U) - W^L_{st}(w,\alpha,x,2;U)\right) \\
&+ \nu \left(\beta \max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), S^L_{st}(\alpha,x,2;U), 0\} + U^L_{st} - W^L_{st}(w,\alpha,x,2;U)\right) \\
&+ \Lambda(W_{st}(w,\alpha,x,\widehat{m}_{st}) - W^L_{st}(w,\alpha,x,2;U)) + \dot{W}^L_{st}(w,\alpha,x,2;U).
\end{aligned}$$

Value of a filled vacancy with a susceptible worker

A match with a locked susceptible worker with $m \in \{0,1\}$ faces value function

$$\begin{aligned}
rJ^L_{st}(w,\alpha,x,m;L) &= p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,m;L)) \\
&+ \lambda^L_0 \ell_{it} \left(J^L_{it}(w,\alpha,x,2;L) - J^L_{st}(w,\alpha,x,m;L)\right) \\
&+ \nu \left((1-\beta) \max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L), S^L_{st}(\alpha,x,2;L), 0\} + V^L_t - J^L_{st}(w,\alpha,x,m;L)\right) \\
&+ \Lambda(J_{st}(w,\alpha,x,m) - J^L_{st}(w,\alpha,x,m;L)) + \dot{J}_{st}(w,\alpha,x,m;L).
\end{aligned}$$

where they only generate $p(\alpha,x,0)$ given the inability to work away from home. A furloughed filled vacancy gives value

$$\begin{aligned}
rJ^L_{st}(w,\alpha,x,2;L) &= p(\alpha,x,2) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,2;L)) \\
&+ \lambda^L_0 \ell_{it} \left(J^L_{it}(w,\alpha,x,2;L) - J^L_{st}(w,\alpha,x,2;L)\right) \\
&+ \nu \left((1-\beta) \max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L), S^L_{st}(\alpha,x,2;L), 0\} + V^L_t - J^L_{st}(w,\alpha,x,m;L)\right) \\
&+ \Lambda(J_{st}(w,\alpha,x,\widehat{m}_{st}) - J^L_{st}(w,\alpha,x,2;L)) + \dot{J}_{st}(w,\alpha,x,m;L).
\end{aligned}$$

An unlocked filled vacancy for $m = 0$ faces value function

$$\begin{aligned}
rJ^L_{st}(w,\alpha,x,0;U) &= p(\alpha,x,0) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,0;U)) \\
&+ \lambda^L_0 \ell_{it}\left(J^L_{it}(w,\alpha,x,2;U) - J^L_{st}(w,\alpha,x,0;U)\right) \\
&+ \nu\left((1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), S^L_{st}(\alpha,x,2;U), 0\} + V^L_t - J^L_{st}(w,\alpha,x,0;U)\right) \\
&+ \Lambda(J_{st}(w,\alpha,x,0) - J^L_{st}(w,\alpha,x,0;U)) + \dot{J}_{st}(w,\alpha,x,0;U)
\end{aligned}$$

while that for a match working away from home is

$$\begin{aligned}
rJ^L_{st}(w,\alpha,x,1;U) &= p(\alpha,x,1) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,1;U)) \\
&+ [\lambda^L_0 + \lambda_1]\ell_{it}\left(J^L_{it}(w,\alpha,x,2;U) - J^L_{st}(w,\alpha,x,1;U)\right) \\
&+ \nu\left((1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), S^L_{st}(\alpha,x,2;U), 0\} + V^L_t - J^L_{st}(w,\alpha,x,1;U)\right) \\
&+ \Lambda(J_{st}(w,\alpha,x,0) - J^L_{st}(w,\alpha,x,1;U)) + \dot{J}_{st}(w,\alpha,x,1;U).
\end{aligned}$$

Finally, a match with an unlocked furloughed worker gives firm value

$$\begin{aligned}
rJ^L_{st}(w,\alpha,x,2;U) &= p(\alpha,x,2) - w + (\delta+\eta)(V^L_t - J^L_{st}(w,\alpha,x,2;U)) \\
&+ \lambda^L_0 \ell_{it}\left(J^L_{it}(w,\alpha,x,2;U) - J^L_{st}(w,\alpha,x,2;U)\right) \\
&+ \nu\left((1-\beta)\max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), S^L_{st}(\alpha,x,2;U), 0\} + V^L_t - J^L_{st}(w,\alpha,x,2;U)\right) \\
&+ \Lambda(J_{st}(w,\alpha,x,\widehat{m}_{st}) - J^L_{st}(w,\alpha,x,2;U)) + \dot{J}_{st}(w,\alpha,x,2;U).
\end{aligned}$$

Surplus of a match with a susceptible worker

Using the worker and firm value functions as well as the free entry condition, the surplus of a locked worker with $m \in \{0,1\}$ is

$$\begin{aligned}
(r+\delta+\eta+\lambda^L_0\ell_{it}+\nu+\Lambda)S^L_{st}(\alpha,x,m;L) &= p(\alpha,x,0) - b_u \\
&- \phi_t\pi\beta\int\int \max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L), 0\} f(\alpha,x)d\alpha dx \\
&- \phi_t(1-\pi)\beta\int\int \max\{S^L_{st}(\alpha,x,0;U), S^L_{st}(\alpha,x,1;U), 0\} f(\alpha,x)d\alpha dx \\
&+ \lambda^L_0\ell_{it}S^L_{it}(\alpha,x;L) + \nu\max\{S^L_{st}(\alpha,x,0;L), S^L_{st}(\alpha,x,1;L), S^L_{st}(\alpha,x,2;L), 0\} \\
&+ \Lambda S_{st}(\alpha,x,m) + \dot{S}^L_{st}(\alpha,x,m;L).
\end{aligned}$$

while that for a locked furloughed match is

$$
\begin{aligned}
(r+\delta+\eta+\lambda_0^L \ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,2;L) = {} & p(\alpha,x,2)-b_u \\
& -\phi_t \pi\beta \int\int \max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L),0\} f(\alpha,x)d\alpha dx \\
& -\phi_t(1-\pi)\beta \int\int \max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U),0\} f(\alpha,x)d\alpha dx \\
& +\lambda_0^L \ell_{it} S_{it}^L(\alpha,x;L)+\nu\max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L), S_{st}^L(\alpha,x,2;L),0\} \\
& +\Lambda\max\{S_{st}(\alpha,x,0), S_{st}(\alpha,x,1)\}+\dot{S}_{st}^L(\alpha,x,2;L).
\end{aligned}
$$

An unlocked match working at home generates surplus

$$
\begin{aligned}
(r+\delta+\eta+\lambda_0^L \ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,0;U) = {} & p(\alpha,x,0)-b_u \\
& -\phi_t \pi\beta \int\int \max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L),0\} f(\alpha,x)d\alpha dx \\
& -\phi_t(1-\pi)\beta \int\int \max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U),0\} f(\alpha,x)d\alpha dx \\
& +\lambda_0^L \ell_{it} S_{it}^L(\alpha,x;U)+\nu\max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U), S_{st}^L(\alpha,x,2;U),0\} \\
& +\Lambda S_{st}(\alpha,x,0)+\dot{S}_{st}^L(\alpha,x,0;U)
\end{aligned}
$$

while that for an unlocked match working away from home is

$$
\begin{aligned}
(r+\delta+\eta+(\lambda_0^L+\lambda_1)\ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,1;U) = {} & p(\alpha,x,1)-b_u \\
& -\phi_t \pi\beta \int\int \max\{S_{st}^L(\alpha,x,0;L), S_{st}^L(\alpha,x,1;L),0\} f(\alpha,x)d\alpha dx \\
& -\phi_t(1-\pi)\beta \int\int \max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U),0\} f(\alpha,x)d\alpha dx \\
& +(\lambda_0^L+\lambda_1)\ell_{it} S_{it}^L(\alpha,x;U)+\lambda_1\ell_{it}(U_{it}^L-U_{st}^L) \\
& +\nu\max\{S_{st}^L(\alpha,x,0;U), S_{st}^L(\alpha,x,1;U), S_{st}^L(\alpha,x,2;U),0\} \\
& +\Lambda S_{st}(\alpha,x,1)+\dot{S}_{st}^L(\alpha,x,1;U).
\end{aligned}
$$

Finally an unlocked furloughed match generates surplus

$$
\begin{aligned}
(r+\delta+\eta+\lambda_0^L\ell_{it}+\nu+\Lambda)S_{st}^L(\alpha,x,2;U) = {} & p(\alpha,x,2)-b_u \\
& -\phi_t\pi\beta\int\int \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),0\}f(\alpha,x)d\alpha dx \\
& -\phi_t(1-\pi)\beta\int\int \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),0\}f(\alpha,x)d\alpha dx \\
& +\lambda_0^L\ell_{it}S_{it}^L(\alpha,x;U) \\
& +\nu\max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),S_{st}^L(\alpha,x,2;U),0\} \\
& +\Lambda\max\{S_{st}(\alpha,x,0),S_{st}(\alpha,x,1)\}+\dot{S}_{st}^L(\alpha,x,2;U).
\end{aligned}
$$

In what follows we denote

$$
\begin{aligned}
S_{st}^L(\alpha,x;L) &= \max\{S_{st}^L(\alpha,x,0;L),S_{st}^L(\alpha,x,1;L),S_{st}^L(\alpha,x,2;L)\} \\
S_{st}^L(\alpha,x;U) &= \max\{S_{st}^L(\alpha,x,0;U),S_{st}^L(\alpha,x,1;U),S_{st}^L(\alpha,x,2;U)\}.
\end{aligned}
$$

O.4 Dynamics of the lockdown model with furlough

Similarly to earlier sections, measures with L superscripts here denote those under lockdown. Again assume that lockdown runs over time $[\underline{t}, \overline{t}]$, where $\overline{t} - \underline{t} = 1/\Lambda$. The initial allocations of unemployed and retired agents are the same as in the lockdown model without furlough. We assume that matches must wait until realising their renegotiation shock before furloughed contracts can be formed. The initial allocations for employed agents are

$$
\begin{aligned}
e^L_{0s\underline{t}}(\alpha, x; L) &= \pi e_{0s\underline{t}^-}(\alpha, x) \\
e^L_{0s\underline{t}}(\alpha, x; U) &= (1-\pi) e_{0s\underline{t}^-}(\alpha, x) \\
e^L_{1s\underline{t}}(\alpha, x; L) &= \pi e_{1s\underline{t}^-}(\alpha, x) \\
e^L_{1s\underline{t}}(\alpha, x; U) &= (1-\pi) e_{1s\underline{t}^-}(\alpha, x) \\
e^L_{1r\underline{t}}(\alpha, x; L) &= \pi e_{r\underline{t}^-}(\alpha, x) \\
e^L_{1r\underline{t}}(\alpha, x; U) &= (1-\pi) e_{r\underline{t}^-}(\alpha, x) \\
e^L_{0i\underline{t}}(\alpha, x; L) &= \pi e_{i\underline{t}^-}(\alpha, x) \\
e^L_{0i\underline{t}}(\alpha, x; U) &= (1-\pi) e_{i\underline{t}^-}(\alpha, x)
\end{aligned}
$$

where notice that those infected at the commencement of lockdown remain on sick pay. The laws of motion governing retired individuals are

$$
\begin{aligned}
\dot{o}^L_{st} = \eta & \left(u^L_{st} + \int\int e^L_{0st}(\alpha, x; L) d\alpha dx + \int\int e^L_{0st}(\alpha, x; U) d\alpha dx \right) \\
& + \eta \left(\int\int e^L_{1st}(\alpha, x; L) d\alpha dx + \int\int e^L_{1st}(\alpha, x; U) d\alpha dx \right) \\
& + \eta \left(\int\int e^L_{2st}(\alpha, x; L) d\alpha dx + \int\int e^L_{2st}(\alpha, x; U) d\alpha dx \right) \\
& - (\lambda^L_0 \ell^L_{it} + \chi) o^L_{st}
\end{aligned}
$$

$$
\begin{aligned}
\dot{o}^L_{it} = \eta & \left(u^L_{it} + \int\int e^L_{0it}(\alpha, x; L) d\alpha dx + \int\int e^L_{0it}(\alpha, x; U) d\alpha dx \right) \\
& + \eta \left(\int\int e^L_{2it}(\alpha, x; L) d\alpha dx + \int\int e^L_{2it}(\alpha, x; U) d\alpha dx \right) \\
& + \lambda^L_0 \ell^L_{it} o^L_{st} - (\gamma_o + \chi + \rho_o) o^L_{it}
\end{aligned}
$$

$$
\begin{aligned}
\dot{o}_{rt}^L &= \eta \left(u_{rt}^L + \int \int e_{rt}^L(\alpha, x; L) d\alpha dx + \int \int e_{rt}^L(\alpha, x; U) d\alpha dx \right) \\
&+ \eta \left(\int \int e_{2rt}^L(\alpha, x; L) d\alpha dx + \int \int e_{2rt}^L(\alpha, x; U) d\alpha dx \right) \\
&+ \rho_o o_{it}^L - \chi o_{rt}^L.
\end{aligned}
$$

Those for unemployed agents are

$$
\begin{aligned}
\dot{u}_{st}^L &= \psi + \delta \int \int e_{0st}^L(\alpha, x; L) d\alpha dx + \delta \int \int e_{1st}^L(\alpha, x; L) d\alpha dx \\
&+ \delta \int \int e_{0st}^L(\alpha, x; U) d\alpha dx + \delta \int \int e_{1st}^L(\alpha, x; U) d\alpha dx \\
&+ \delta \int \int e_{2st}^L(\alpha, x; L) d\alpha dx + \delta \int \int e_{2st}^L(\alpha, x; U) d\alpha dx \\
&+ \nu \int \int e_{0st}^L(\alpha, x; L)\{S_{st}^L(\alpha, x; L) < 0\} d\alpha dx + \nu \int \int e_{1st}^L(\alpha, x; L)\{S_{st}^L(\alpha, x; L) < 0\} d\alpha dx \\
&+ \nu \int \int e_{0st}^L(\alpha, x; U)\{S_{st}^L(\alpha, x; U) < 0\} d\alpha dx + \nu \int \int e_{1st}^L(\alpha, x; U)\{S_{st}^L(\alpha, x; U) < 0\} d\alpha dx \\
&+ \nu \int \int e_{2st}^L(\alpha, x; L)\{S_{st}^L(\alpha, x; L) < 0\} d\alpha dx + \nu \int \int e_{2st}^L(\alpha, x; U)\{S_{st}^L(\alpha, x; U) < 0\} d\alpha dx \\
&- \phi_t^L u_{st}^L \pi \int \int \{S_{st}^L(\alpha, x, L) \geq 0\} f(\alpha, x) d\alpha dx \\
&- \phi_t^L u_{st}^L (1 - \pi) \int \int \{S_{st}^L(\alpha, x, U) \geq 0\} f(\alpha, x) d\alpha dx \\
&- \lambda_0^L \ell_{it}^L u_{st}^L - \eta u_{st}^L
\end{aligned}
$$

$$
\begin{aligned}
\dot{u}_{it}^L &= \delta \int \int e_{0it}^L(\alpha, x; L) d\alpha dx + \delta \int \int e_{0it}^L(\alpha, x; U) d\alpha dx \\
&+ \delta \int \int e_{2it}^L(\alpha, x; L) d\alpha dx + \delta \int \int e_{2it}^L(\alpha, x; U) d\alpha dx \\
&+ \nu \int \int e_{0it}^L(\alpha, x; L)\{S_{it}^L(\alpha, x; L) < 0\} d\alpha dx + \nu \int \int e_{0it}^L(\alpha, x; U)\{S_{it}^L(\alpha, x; U) < 0\} d\alpha dx \\
&+ \nu \int \int e_{2it}^L(\alpha, x; L)\{S_{it}^L(\alpha, x; L) < 0\} d\alpha dx + \nu \int \int e_{2it}^L(\alpha, x; U)\{S_{it}^L(\alpha, x; U) < 0\} d\alpha dx \\
&+ \lambda_0^L \ell_{it}^L u_{st}^L - (\rho_y + \gamma(\ell_{it}^L) + \eta) u_{it}^L
\end{aligned}
$$

$$
\begin{aligned}
\dot{u}^L_{rt} &= \delta \int\int e^L_{rt}(\alpha,x;L)d\alpha dx + \delta \int\int e^L_{rt}(\alpha,x;U)d\alpha dx \\
&+ \delta \int\int e^L_{2rt}(\alpha,x;L)d\alpha dx + \delta \int\int e^L_{2rt}(\alpha,x;U)d\alpha dx \\
&+ \nu \int\int e^L_{rt}(\alpha,x;L)\{S^L_{rt}(\alpha,x;L) < 0\}d\alpha dx + \nu \int\int e^L_{rt}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) < 0\}d\alpha dx \\
&+ \nu \int\int e^L_{2rt}(\alpha,x;L)\{S^L_{rt}(\alpha,x;L) < 0\}d\alpha dx + \nu \int\int e^L_{2rt}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) < 0\}d\alpha dx \\
&+ \rho_y u^L_{it} - \phi^L_t u^L_{rt}\pi \int\int \{S^L_{rt}(\alpha,x,L) \geq 0\}f(\alpha,x)d\alpha dx \\
&- \phi^L_t u^L_{rt}(1-\pi) \int\int \{S^L_{rt}(\alpha,x,U) \geq 0\}f(\alpha,x)d\alpha dx.
\end{aligned}
$$

The measures of employed that are recovered evolve as follows

$$
\begin{aligned}
\dot{e}^L_{1rt}(\alpha,x;U) &= (1-\pi)u^L_{rt}\phi^L_t\{S^L_{rt}(\alpha,x;U) \geq 0\}f(\alpha,x) \\
&+ \rho_y e^L_{0it}(\alpha,x;U)\{S^L_{rt}(\alpha,x,1;U) \geq S^L_{rt}(\alpha,x,2;U)\} \\
&+ \rho_y e^L_{2it}(\alpha,x;U)\{S^L_{rt}(\alpha,x,1;U) \geq S^L_{rt}(\alpha,x,2;U)\} \\
&- (\delta+\eta)e^L_{1rt}(\alpha,x;U) \\
&- \nu e^L_{1rt}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) < 0\} \\
&- \nu e^L_{1rt}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) \geq 0\}\{S^L_{rt}(\alpha,x,1;U) < S^L_{rt}(\alpha,x,2;U)\} \\
&+ \nu e^L_{2rt}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) \geq 0\}\{S^L_{rt}(\alpha,x,1;U) \geq S^L_{rt}(\alpha,x,2;U)\}
\end{aligned}
$$

$$
\begin{aligned}
\dot{e}^L_{2rt}(\alpha,x;U) &= \rho_y e^L_{0it}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) \geq 0\}\{S^L_{rt}(\alpha,x,1;U) < S^L_{rt}(\alpha,x,2;U)\} \\
&+ \rho_y e^L_{2it}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) \geq 0\}\{S^L_{rt}(\alpha,x,1;U) < S^L_{rt}(\alpha,x,2;U)\} \\
&- (\delta+\eta)e^L_{2rt}(\alpha,x;U) \\
&- \nu e^L_{2rt}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) < 0\} \\
&- \nu e^L_{2rt}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) \geq 0\}\{S^L_{rt}(\alpha,x,1;U) \geq S^L_{rt}(\alpha,x,2;U)\} \\
&+ \nu e^L_{1rt}(\alpha,x;U)\{S^L_{rt}(\alpha,x;U) \geq 0\}\{S^L_{rt}(\alpha,x,1;U) < S^L_{rt}(\alpha,x,2;U)\}
\end{aligned}
$$

$$
\begin{aligned}
\dot{e}^L_{1rt}(\alpha, x, L) = {} & \pi u^L_{rt}\phi_t\{S^L_{rt}(\alpha, x; L) \geq 0\} f(\alpha, x) \\
& + \rho_y e^L_{0it}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) \geq 0\}\{S^L_{rt}(\alpha, x, 1; L) \geq S^L_{rt}(\alpha, x, 2; L)\} \\
& + \rho_y e^L_{2it}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) \geq 0\}\{S^L_{rt}(\alpha, x, 1; L) \geq S^L_{rt}(\alpha, x, 2; L)\} \\
& - (\delta + \eta) e^L_{1rt}(\alpha, x; L) \\
& - \nu e^L_{1rt}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) < 0\} \\
& - \nu e^L_{1rt}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) \geq 0\}\{S^L_{rt}(\alpha, x, 1; L) < S^L_{rt}(\alpha, x, 2; L)\} \\
& + \nu e^L_{2rt}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) \geq 0\}\{S^L_{rt}(\alpha, x, 1; L) \geq S^L_{rt}(\alpha, x, 2; L)\}
\end{aligned}
$$

$$
\begin{aligned}
\dot{e}^L_{2rt}(\alpha, x; L) = {} & \rho_y e^L_{0it}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) \geq 0\}\{S^L_{rt}(\alpha, x, 1; L) < S^L_{rt}(\alpha, x, 2; L)\} \\
& + \rho_y e^L_{2it}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) \geq 0\}\{S^L_{rt}(\alpha, x, 1; L) < S^L_{rt}(\alpha, x, 2; L)\} \\
& - (\delta + \eta) e^L_{2rt}(\alpha, x; L) \\
& - \nu e^L_{2rt}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) < 0\} \\
& - \nu e^L_{2rt}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) \geq 0\}\{S^L_{rt}(\alpha, x, 1; L) \geq S^L_{rt}(\alpha, x, 2; L)\} \\
& + \nu e^L_{1rt}(\alpha, x; L)\{S^L_{rt}(\alpha, x; L) \geq 0\}\{S^L_{rt}(\alpha, x, 1; L) < S^L_{rt}(\alpha, x, 2; L)\}
\end{aligned}
$$

Then the dynamics for the infected are given by

$$
\dot{e}^L_{0it}(\alpha, x; L) = -\nu e^L_{0it}(\alpha, x; L) - (\delta + \rho_y + \gamma(\ell^L_{it}) + \eta) e^L_{0it}(\alpha, x; L)
$$

$$
\begin{aligned}
\dot{e}^L_{2it}(\alpha, x; L) = {} & e^L_{0st}(\alpha, x; L)\lambda^L_0 \ell^L_{it} + e^L_{1st}(\alpha, x; L)\lambda^L_0 \ell^L_{it} + e^L_{2st}(\alpha, x; L)\lambda^L_0 \ell^L_{it} \\
& + \nu e^L_{0it}(\alpha, x; L)\{S^L_{it}(\alpha, x; L) \geq 0\} - \nu e^L_{2it}(\alpha, x; L)\{S^L_{it}(\alpha, x; L) < 0\} \\
& - (\delta + \rho_y + \gamma(\ell^L_{it}) + \eta) e^L_{2it}(\alpha, x; L)
\end{aligned}
$$

$$
\dot{e}^L_{0it}(\alpha, x; U) = -\nu e^L_{0it}(\alpha, x; U) - (\delta + \rho_y + \gamma(\ell^L_{it}) + \eta) e^L_{0it}(\alpha, x; U)
$$

$$
\begin{aligned}
\dot{e}^L_{2it}(\alpha, x; U) = {} & e^L_{0st}(\alpha, x; U)\lambda^L_0 \ell^L_{it} + e^L_{1st}(\alpha, x; U)[\lambda^L_0 + \lambda_1]\ell^L_{it} + e^L_{2st}(\alpha, x; U)\lambda^L_0 \ell^L_{it} \\
& + \nu e^L_{0it}(\alpha, x; U)\{S^L_{it}(\alpha, x; U) \geq 0\} - \nu e^L_{2it}(\alpha, x; U)\{S^L_{it}(\alpha, x; U) < 0\} \\
& - (\delta + \rho_y + \gamma(\ell^L_{it}) + \eta) e^L_{2it}(\alpha, x; U).
\end{aligned}
$$

Finally, the measures of the locked employed susceptible evolve as follows

$$
\begin{aligned}
&\dot{e}^L_{0st}(\alpha, x; L) = \pi u^L_{st}\phi^L_t\{S^L_{st}(\alpha, x; L) > 0\}\{S^L_{st}(\alpha, x, 1; L) < S^L_{st}(\alpha, x, 0; L)\}f(\alpha, x) \\
&\quad - (\delta + \eta)e^L_{0st}(\alpha, x; L) - \nu e^L_{0st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) < 0\} \\
&\quad - \nu e^L_{0st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 0; L) < S^L_{st}(\alpha, x, 2; L) \mid S^L_{st}(\alpha, x, 1; L) \geq S^L_{st}(\alpha, x, 0; L)\} \\
&\quad + \nu e^L_{1st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 0; L) \geq S^L_{st}(\alpha, x, 2; L) \,\&\, S^L_{st}(\alpha, x, 1; L) < S^L_{st}(\alpha, x, 0; L)\} \\
&\quad + \nu e^L_{2st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 0; L) \geq S^L_{st}(\alpha, x, 2; L) \,\&\, S^L_{st}(\alpha, x, 1; L) < S^L_{st}(\alpha, x, 0; L)\} \\
&\quad - e^L_{0st}(\alpha, x; L)\lambda^L_0\ell^L_{it}
\end{aligned}
$$

$$
\begin{aligned}
&\dot{e}^L_{1st}(\alpha, x; L) = \pi u^L_{st}\phi^L_t\{S^L_{st}(\alpha, x, 1; L) \geq 0\}\{S^L_{st}(\alpha, x, 1; L) \geq S^L_{st}(\alpha, x, 0; L)\}f(\alpha, x) \\
&\quad - (\delta + \eta)e^L_{1st}(\alpha, x; L) - \nu e^L_{1st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) < 0\} \\
&\quad - \nu e^L_{1st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 1; L) < S^L_{st}(\alpha, x, 0; L) \mid S^L_{st}(\alpha, x, 1; L) < S^L_{st}(\alpha, x, 2; L)\} \\
&\quad + \nu e_{0st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 1; L) \geq S^L_{st}(\alpha, x, 0; L) \,\&\, S^L_{st}(\alpha, x, 1; L) \geq S^L_{st}(\alpha, x, 2; L)\} \\
&\quad + \nu e_{2st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 1; L) \geq S^L_{st}(\alpha, x, 0; L) \,\&\, S^L_{st}(\alpha, x, 1; L) \geq S^L_{st}(\alpha, x, 2; L)\} \\
&\quad - e^L_{1st}(\alpha, x; L)\lambda^L_0\ell^L_{it}
\end{aligned}
$$

$$
\begin{aligned}
&\dot{e}^L_{2st}(\alpha, x; L) = -(\delta + \eta)e^L_{2st}(\alpha, x; L) - \nu e^L_{2st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) < 0\} \\
&\quad - \nu e^L_{2st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 1; L) \geq S^L_{st}(\alpha, x, 2; L) \mid S^L_{st}(\alpha, x, 0; L) \geq S^L_{st}(\alpha, x, 2; L)\} \\
&\quad + \nu e_{0st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 1; L) < S^L_{st}(\alpha, x, 2; L) \,\&\, S^L_{st}(\alpha, x, 0; L) < S^L_{st}(\alpha, x, 2; L)\} \\
&\quad + \nu e_{1st}(\alpha, x; L)\{S^L_{st}(\alpha, x; L) \geq 0\}\{S^L_{st}(\alpha, x, 1; L) < S^L_{st}(\alpha, x, 2; L) \,\&\, S^L_{st}(\alpha, x, 0; L) < S^L_{st}(\alpha, x, 2; L)\} \\
&\quad - e^L_{2st}(\alpha, x; L)\lambda^L_0\ell^L_{it}
\end{aligned}
$$

while those for unlocked employed susceptible agents are

$$
\begin{aligned}
&\dot{e}^L_{0st}(\alpha, x; U) = (1 - \pi)u^L_{st}\phi^L_t\{S^L_{st}(\alpha, x; U) > 0\}\{S^L_{st}(\alpha, x, 1; U) < S^L_{st}(\alpha, x, 0; U)\}f(\alpha, x) \\
&\quad - (\delta + \eta)e^L_{0st}(\alpha, x; U) - \nu e^L_{0st}(\alpha, x; U)\{S^L_{st}(\alpha, x; U) < 0\} \\
&\quad - \nu e^L_{0st}(\alpha, x; U)\{S^L_{st}(\alpha, x; U) \geq 0\}\{S^L_{st}(\alpha, x, 0; U) < S^L_{st}(\alpha, x, 2; U) \mid S^L_{st}(\alpha, x, 1; U) \geq S^L_{st}(\alpha, x, 0; U)\} \\
&\quad + \nu e^L_{1st}(\alpha, x; U)\{S^L_{st}(\alpha, x; U) \geq 0\}\{S^L_{st}(\alpha, x, 0; U) \geq S^L_{st}(\alpha, x, 2; U) \,\&\, S^L_{st}(\alpha, x, 1; U) < S^L_{st}(\alpha, x, 0; U)\} \\
&\quad + \nu e^L_{2st}(\alpha, x; U)\{S^L_{st}(\alpha, x; U) \geq 0\}\{S^L_{st}(\alpha, x, 0; U) \geq S^L_{st}(\alpha, x, 2; U) \,\&\, S^L_{st}(\alpha, x, 1; U) < S^L_{st}(\alpha, x, 0; U)\} \\
&\quad - e^L_{0st}(\alpha, x; U)\lambda^L_0\ell^L_{it}
\end{aligned}
$$

$$
\begin{aligned}
&\dot{e}^L_{1st}(\alpha,x;U) = (1-\pi)u^L_{st}\phi^L_t\{S^L_{st}(\alpha,x,1;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U)\}f(\alpha,x)\\
&\quad -(\delta+\eta)e^L_{1st}(\alpha,x;U) - \nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)<0\}\\
&\quad -\nu e^L_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,0;U) \mid S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,2;U)\}\\
&\quad +\nu e_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U) \;\&\; S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,2;U)\}\\
&\quad +\nu e_{2st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,0;U) \;\&\; S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,2;U)\}\\
&\quad -e^L_{1st}(\alpha,x;U)[\lambda^L_0+\lambda_1]\ell^L_{it}
\end{aligned}
$$

$$
\begin{aligned}
&\dot{e}^L_{2st}(\alpha,x;U) = -(\delta+\eta)e^L_{2st}(\alpha,x;U) - \nu e^L_{2st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)<0\}\\
&\quad -\nu e^L_{2st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)\geq S^L_{st}(\alpha,x,2;U) \mid S^L_{st}(\alpha,x,0;U)\geq S^L_{st}(\alpha,x,2;U)\}\\
&\quad +\nu e_{0st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,2;U) \;\&\; S^L_{st}(\alpha,x,0;U)<S^L_{st}(\alpha,x,2;U)\}\\
&\quad +\nu e_{1st}(\alpha,x;U)\{S^L_{st}(\alpha,x;U)\geq 0\}\{S^L_{st}(\alpha,x,1;U)<S^L_{st}(\alpha,x,2;U) \;\&\; S^L_{st}(\alpha,x,0;U)<S^L_{st}(\alpha,x,2;U)\}\\
&\quad -e^L_{2st}(\alpha,x;U)\lambda^L_0\ell^L_{it}
\end{aligned}
$$

O.5 Waning immunity

In this section we extend our baseline model assuming recovered individuals can again become susceptible to the virus. We assume this happens at a rate φ. Under the new specification, being recovered is no longer an absorbing state and the health dynamics of the young and the old now includes the terms below underlined.

$$\begin{aligned}
\dot{n}^y_{st} &= \psi - \lambda^y_t \ell_{it} n^y_{st} - \eta n^y_{st} + \underline{\varphi n^y_{rt}} \\
\dot{n}^y_{it} &= \lambda^y_t \ell_{it} n^y_{st} - (\rho + \gamma_y) n^y_{it} - \eta n^y_{it} \\
\dot{n}^y_{rt} &= \rho n^y_{it} - \eta n^y_{rt} - \underline{\varphi n^y_{rt}},
\end{aligned}$$

and

$$\begin{aligned}
\dot{n}^o_{st} &= \eta n^y_{st} - (\lambda_0 \ell_{it} + \chi) n^o_{st} + \underline{\varphi n^o_{rt}} \\
\dot{n}^o_{it} &= \eta n^y_{it} + \lambda_0 \ell_{it} n^o_{st} - (\rho + \gamma_o + \chi) n^o_{it} \\
\dot{n}^o_{rt} &= \eta n^y_{rt} + \rho n^o_{it} - \chi n^o_{rt} - \underline{\varphi n^o_{rt}}
\end{aligned}$$

To solve the baseline model (absent any lockdown policy) is conceptually very similar with or without waning immunity. The complication is the model loses its recursive structure. Previously, one can compute value functions of the recovered independently of the infected and susceptible, since being recovered is an absorbing state. Under this specification, we instead rely on an iterative solution where we jointly solve for all value functions simultaneously. Below we provide the surplus function of a match involving a recovered worker. Underlined are the changes relative to the model in the paper. One can see that equation below depends on the value of the surplus of the susceptible. The value function for the susceptible and the infected remain the same.

$$\begin{aligned}
(r + \delta + \eta + \underline{\varphi}) S_{rt}(\alpha, x, m) = {} & p(\alpha, x, m) - b_u \\
& - \phi_t \beta \int \int \max\{S_{rt}(\alpha, x, 1), S_{rt}(\alpha, x, 0), 0\} f(\alpha, x) d\alpha dx \\
& + \nu \left(\max\{S_{rt}(\alpha, x, 1), S_{rt}(\alpha, x, 0), 0\} - S_{rt}(\alpha, x, m) \right) \\
& + \underline{\varphi \max\{S_{st}(\alpha, x, 0), S_{st}(\alpha, x, 1)\}} + \dot{S}_{rt}(\alpha, x, m)
\end{aligned}$$

The only additional parameter to calibrate is φ. $1/\varphi$ describes the average length of antibody immunity, hence we calibrate it such that immunity would last on average one year, following Phillips (2021). This gives a value equal to 0.01918. Keeping all the other parameters fixed at their calibrated values, we conduct the same counterfactual experiments reported in Section 4 of the main text.

Figure 1: Dynamics of the Pandemic and Economy with Waning Immunity

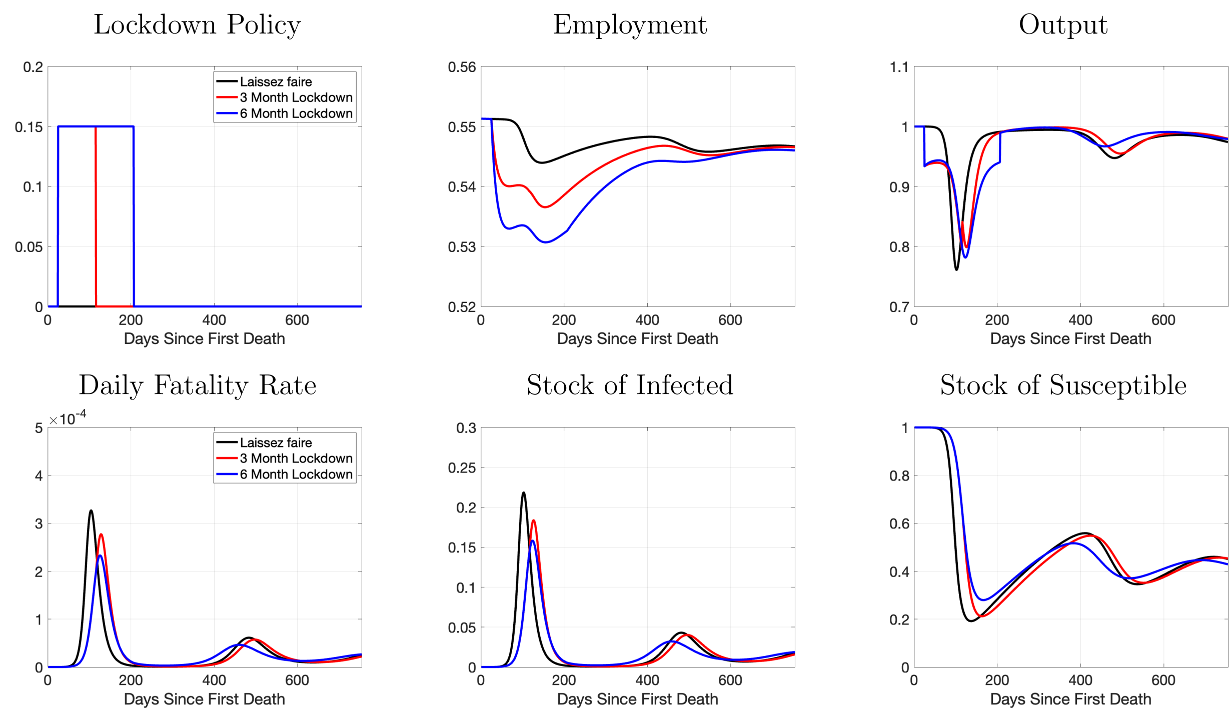

Figure 1 displays dynamics over two years post-lockdown. The dynamics of pandemics and the economy are now characterized by dampened oscillations. As in the baseline model, the stock of infected peaks after few months from the first registered death. If you compare the first year with Figure 2 in the paper, the plots are indistinguishable. So for shorter term comparisons all results across the two specifications are close to identical. Although not explicitly modeled, the presence of a vaccine arriving in a year to eighteen months would make the two approximately equivalent.

In the short run, the baseline model matches well quantitatively the dynamics presented here. The longer run are qualitatively quite different, in that the virus here is an ever present. That said, because of the long duration of antibody resistance (relative to infection and recovery rates) subsequent waves are an order of magnitude less than the first wave. Hence, a single lockdown, absent waning immunity acts as a good first order approximation.

References

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