Incomplete markets as correlated distortions*

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February 17, 2025

Abstract

We argue that capital misallocation arises endogenously due to incomplete consumption insurance. We model risk-averse entrepreneurs with heterogeneous productivity who face idiosyncratic output shocks and choose how much capital to rent before uncertainty unfolds. We show that incomplete markets operate as *correlated distortions*, leading to a reallocation of capital from more to less productive firms relative to the complete markets benchmark. Using Portuguese administrative data, we document that capital misallocation is greater in locations and industries with higher output shock volatility, consistent with our framework. Leveraging the structure of the model, we show that completing insurance markets increases aggregate productivity and income by 64% and 97%, respectively.

Keywords: insurance, volatility, misallocation, distortions, efficiency.
JEL Classification: E22, D61, L23, L26

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1 Introduction

Resource misallocation is a key driver of aggregate productivity and income losses. Yet, what ultimately drives misallocation remains an open question. In this paper, we argue that insurance markets incompleteness can act as a barrier to the efficient allocation of factors of production across firms.

We develop a general equilibrium model of consumption insurance and the capital market to study how insurance markets incompleteness affects the allocation of capital to heterogeneous firms, as well as aggregate productivity and income. The model economy is populated by risk-averse entrepreneurs, each running a firm characterized by a span-of-control production function, which uses capital as a factor of production. Entrepreneurs are heterogeneous in their (fixed) productivity levels, face idiosyncratic output shocks, and choose how much capital to rent before uncertainty unfolds; that is, they are subject to time-to-build constraints.

Insurance markets incompleteness ties entrepreneurs' consumption to the profits generated by their enterprises, making them willing to forgo expected profits to reduce consumption volatility. This distorts entrepreneurs' demand for capital away from expected profit maximization in an effort to avoid losses. As a result, the expected marginal products of capital are not equalized across firms; i.e., capital is misallocated.

Specifically, we show that incomplete markets operate as *correlated distortions*, which introduce an endogenous wedge between the expected marginal product of capital and its price that is positively correlated with entrepreneurial productivity. Thus, capital is reallocated from more to less productive entrepreneurs relative to the complete markets benchmark. This happens because renting more capital implies a larger scale of production, which results in a higher sensitivity of expected utility to profit volatility. The wedge disproportionately impacts high-productivity entrepreneurs, as they are the ones who would employ more capital if they could fully insure their consumption. Moreover, these distortions become more severe as output shock volatility increases.

The model provides a novel link between entrepreneurial income risk and capital misallocation: a higher volatility of output shocks intensifies misallocation by weakening the cross-sectional correlation between entrepreneurial productivity and capital holdings. We corroborate this prediction using administrative data on the quasi-universe of Portuguese firms, which spans more than a decade. We estimate production functions to recover firm-level permanent revenue productivity and idiosyncratic residual volatility in sales. We use these estimates to infer capital misallocation and the volatility of output shocks in each location-industry pair. The empirical evidence aligns with our model's predictions: we find higher capital misallocation—measured by either the correlation between firm productivity and capital holdings, or by the dispersion of the expected marginal products of capital—in location and industries with higher average idiosyncratic residual volatility.

Finally, we leverage the structure of our model to quantify the aggregate cost of incomplete markets. To do so, we calibrate the model to reproduce the observed joint distribution of firm productivity and capital holdings from the data. Next, we conduct two counterfactual exercises. In the first one, we examine a counterfactual economy where each firm receives the expected value of the output shock with certainty; i.e., output shocks are eliminated. This exercise allows us to quantify the extent to which idiosyncratic output shocks impact aggregate productivity and income when entrepreneurs lack access to a full set of state-contingent claims spanning all possible realizations of output shocks. In the second one, we complete insurance markets while maintaining the volatility of output shocks to the baseline level. Our calibration implies that, in a world without output shocks, aggregate productivity and income would be 54% and 85% higher, respectively. If insurance markets were complete, they would increase by 64% and 97%.[1]

Our paper contributes to the literature examining the causes and consequences of factor misallocation (Hopenhayn, 2014; Restuccia and Rogerson, 2017). This body of work either explains misallocation as a result of specific policies or

[1]Removing production uncertainty leads to lower aggregate gains compared to completing markets for insurance because it also implies a reduction in the expected value of the output shock. Intuitively, in a deterministic world, entrepreneurs are precluded from enjoying very positive realizations of output shocks.

frictions—such as labor regulations (Gourio and Roys, 2014; Garicano et al., 2016), borrowing constraints (Banerjee and Moll, 2010; Moll, 2014), imperfect information (David et al., 2016), and corporate income taxes (Cisneros-Acevedo and Ruggieri, 2022; Ábrahám et al., 2023)—or reduced-form wedges distorting factor allocation across firms (Guner et al., 2008; Hsieh and Klenow, 2009; Bento and Restuccia, 2017; David and Venkateswaran, 2019). Building on this literature, we show that capital misallocation arises *endogenously* among risk-averse entrepreneurs as a consequence of missing insurance markets, generating aggregate productivity losses that exist independently of other market imperfections.

Within this line of work, our paper is mostly related to the literature that studies financial frictions and uncertainty as potential sources of misallocation (Moll, 2014; Midrigan and Xu, 2014; Buera et al., 2015; Karabarbounis and Macnamara, 2021). Most related to our work is Asker et al. (2014), who develop a model of capital investment with adjustment costs and show that industries exhibiting greater time-series volatility of productivity have greater cross-sectional dispersion of the marginal revenue product of capital. More recently, Boar et al. (2022) document that the average returns to private business wealth are dispersed and persistent, and interpret this evidence as mostly reflecting uninsurable entrepreneurial risk as opposed to collateral constraints. Similarly, David et al. (2022) develop a model of capital investments that links dispersion of the marginal products of capital to systematic investment risks, and show that capital misallocation induced by risk premium effects lowers aggregate productivity by as much as 7%. Robinson (2021) shows that uninsured idiosyncratic risk exacerbates misallocation by distorting the decision to become an entrepreneur. We innovate upon these papers by showing that, when risk-averse entrepreneurs differ in their productivity levels, insurance market incompleteness lowers the cross-sectional correlation between entrepreneurial productivity and capital holdings, giving rise to *correlated distortions*.

Finally, this paper is related to the literature that studies the aggregate implications of incomplete markets (Ríos-Rull, 1994; Angeletos and Calvet, 2006; Davila et al., 2012). We contribute to this body of work by showing that the inability of entrepreneurs to insure against idiosyncratic risks creates an additional and quantitatively significant margin of output losses through capital misallocation.

2 Model

We examine an economy with incomplete markets, inhabited by risk-averse heterogeneous entrepreneurs, each operating a firm and facing output shocks. In this section, we focus on a static version of the model where capital is the only factor of production. As shown in Appendix A, our theoretical results extend to an environment where entrepreneurs combine capital with other factors of production; they extend to settings where entrepreneurs face monopolistic competition and are subject to demand shocks, and are robust to allowing entrepreneurs to borrow and save through a risk-free bond.

There exists a unitary measure of entrepreneurs indexed by i. Let $u(\cdot)$ be a strictly increasing, strictly concave, and twice-continuously differentiable utility function representing their preferences over consumption. Entrepreneurs differ in their productivity $z \in \mathcal{Z} \subset \mathbb{R}_{++}$ and have access to a production technology $f(z,k)$ that depends on productivity and capital k. We assume that $f(z,k)$ is strictly increasing, strictly concave, supermodular, and twice-continuously differentiable. Production is subject to idiosyncratic multiplicate shocks $s \in \mathcal{S} \subset \mathbb{R}_{+}$, which are distributed according to a probability density function $\phi(s)$.[2] Each entrepreneur chooses capital before the realization of output shocks. Let r denote the rental rate of capital and $\overline{K}$ represent the exogenously given supply of capital.

2.1 Case 1: Complete markets

Under complete markets, entrepreneurs can trade a full set of state-contingent securities to insure against uncertainty in the realization of output shocks s. Hence, each entrepreneur i chooses capital k_i, state-contingent claims $\{\theta_i(s)\}_{s\in\mathcal{S}}$,

[2] In the empirical analysis we allow the production function f and distribution ϕ to vary across locations and industries. Our theoretical results remain valid regardless this dimension of heterogeneity. To streamline the exposition, we omit references to locations and industries at this stage. See Section 2.3 for a discussion.

and consumption in each state of the world $\{c_i(s)\}_{s\in\mathcal{S}}$ to maximize her expected utility. Let $q(s)$ denote the price of a contingent claim that pays a unit of consumption if state s realizes, and zero otherwise. Then, the problem of entrepreneur i can be written as follows:

$$\max_{k_i,\{\theta_i(s),c_i(s)\}_{s\in\mathcal{S}}} \quad \int_{\mathcal{S}} u\left(c_i(s)\right)\phi(s)ds$$
$$\text{s.t.} \quad c_i(s) + \int_{\mathcal{S}} q\left(s'\right)\theta_i\left(s'\right)ds' \leq sf\left(z_i,k_i\right) - rk_i + T + \theta_i(s), \quad \forall s\in\mathcal{S},$$

where $T = r\overline{K}$ denotes revenues from renting capital, assumed to be rebated lump-sum to each entrepreneur.

A competitive equilibrium is a list of capital choices $\left\{k_i^*\right\}_{i\in\mathcal{I}}$, state-contingent claims $\left\{\theta_i^*(s)\right\}_{s\in\mathcal{S},i\in\mathcal{I}}$, consumption plans $\left\{c_i^*(s)\right\}_{s\in\mathcal{S},i\in\mathcal{I}}$, a rental price of capital r^*, and a list of prices for contingent claims $\{q^*(s)\}_{s\in\mathcal{S}}$ such that

- $\{k_i^*\}_{i\in\mathcal{I}}$, $\{\theta_i^*(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ and $\{c_i^*(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ are the solution to the problem of each entrepreneur i;
- the price of contingent claims is fair; i.e., $q^*(s) = \phi(s)$;
- the market for capital clears; i.e.,

$$\int_i k_i^* di = \overline{K}.$$

Next, we characterize the competitive equilibrium under complete markets. Dropping index i for convenience and noting that entrepreneurs exhaust their budget constraints in equilibrium, we can write an entrepreneur's problem as follows:

$$\max_{k,\{\theta(s)\}_{s\in\mathcal{S}}} \quad \int_{s\in\mathcal{S}} u\left(sf(z,k) - rk + T - \int_{s\in\mathcal{S}} q(s)\theta(s)ds + \theta(s)\right)\phi(s)ds.$$

Letting u_c denote the marginal utility of consumption, the optimal choice of state-contingent claims under shock s, $\theta^*(s)$, satisfies the following first-order condition:

$$u_c(c^*(s))\phi(s) = q^*(s)\int_{s'\in\mathcal{S}} u_c(c^*(s'))\phi(s')ds', \quad \forall s \in \mathcal{S},$$

where the marginal benefit of an extra unit of an asset that pays off if shock s realizes equates its marginal cost before uncertainty is realized. When the price for insurance is fair, i.e., when $q^*(s) = \phi(s)$, we can rewrite the previous first-order condition as follows:

$$u_c(c^*(s)) = \int_{s'\in\mathcal{S}} u_c(c^*(s'))\phi(s')ds', \quad \forall s \in \mathcal{S},$$

which implies that consumption is constant across ouput shock realizations, i.e.,

$$c^*(s) = u_c^{-1}\left(\int_{s'\in\mathcal{S}} u_c(c^*(s'))\phi(s')ds'\right), \quad \forall s \in \mathcal{S}.$$

Perfect consumption smoothing implies that the expected marginal product of capital is equalized across firms. To see this, notice that an entrepreneur's optimal choice of capital k^* satisfies the following first-order condition:

$$\int_{s\in\mathcal{S}} u_c\left(c^*(s)\right)\left[sf_k(z,k^*) - r^*\right]\phi(s)ds = 0,$$

where f_k denotes the derivative of f with respect to k. Because $c^*(s) = c^*(s')$ for any two states $s, s' \in \mathcal{S}$, the condition above can be rewritten as:

$$\int_{s\in\mathcal{S}} [sf_k(z,k^*) - r^*]\phi(s)ds = 0. \tag{1}$$

That is, under complete markets, each entrepreneur chooses capital until the expected marginal product of capital equates its price. This leads to our first proposition.

Proposition 1. *Under complete markets and an actuarially fair price of insurance, the expected marginal product of capital is equalized across firms, and aggregate expected output is maximized.*

Proof. See Appendix A. □

With a full set of state-contingent claims ensuring perfect consumption smoothing, risk-averse entrepreneurs behave as if they were risk-neutral. As a result, the separation theorem holds: production decisions are independent of individual preferences, and each entrepreneur chooses capital to maximize expected profits.

2.2 Case 2: Incomplete markets

Consider an economy with incomplete markets. Each entrepreneur i now lacks access to state-contingent securities, and chooses capital k_i and consumption in each state of the world $\{c_i(s)\}_{s\in\mathcal{S}}$ to maximize expected utility. Entrepreneur i's problem can be written as follows:

$$\max_{k_i,\{c_i(s)\}_{s\in\mathcal{S}}} \int_{s\in\mathcal{S}} u(c_i(s))\phi(s)ds$$
$$\text{s.t.} \quad c_i(s) \leq sf(z_i,k_i) - rk_i + T, \quad \forall s \in \mathcal{S},$$

where, again, $T = r\overline{K}$ are revenues from renting capital, assumed to be rebated lump-sum to each entrepreneur.

A competitive equilibrium for this economy is a list of capital choices $\{k_i^o\}_{i\in\mathcal{I}}$, consumption plans $\{c_i^o(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$, and a rental price of capital r^o such that

- $\{k_i^o\}_{i\in\mathcal{I}}$ and $\{c_i^o(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ solve the problem of the entrepreneurs;
- the market for capital clears; i.e.,

$$\int_i k_i^o di = \overline{K}.$$

Again, let us drop subscript i for economy of notation. To characterize the equilibrium for this economy, we consider a CRRA utility function; i.e., $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 1$; and assume that $s \sim \log\mathcal{N}(0,\sigma^2)$. Denote the expected value and variance of s as $\bar{s}$ and σ_s^2, respectively.[3] Noting that entrepreneurs exhaust their

[3]Using the properties of the log-normal distribution, $\bar{s} = e^{\frac{1}{2}\sigma^2}$ and $\sigma_s^2 = e^{\sigma^2}(e^{\sigma^2} - 1)$.

budget constraints in equilibrium, we can write an entrepreneur's problem as follows:

$$\max_{k} \quad \int_{s\in\mathcal{S}} \frac{(sf(z,k) - rk + T)^{1-\gamma}}{1-\gamma}\phi(s)ds.$$

Given the log-normality of output shocks s, consumption follows a three-parameter log-normal distribution (Aitchison and Brown, 1957; Singh, 1998), with an expected value of $\bar{c} = \bar{s}f(z,k) - rk + T$, variance $\sigma_c^2 = f(z,k)^2\sigma_s^2$, and a threshold of $-rk + T$.[4] Therefore, an entrepreneur's problem can be rewritten in terms of consumption equivalent as follows:

$$\max_{k} \quad \bar{s}f(z,k) - rk + T - \frac{(\gamma-1)}{2}f(z,k)^2\sigma_s^2.$$

Taking the first-order condition with respect to capital, we obtain:

$$f_k(z,k^\circ)[\bar{s} - (\gamma-1)\sigma_s^2 f(z,k^\circ)] - r^o = 0, \tag{2}$$

which leads to our second proposition.

Proposition 2. *Under incomplete markets, when $\gamma > 1$, there is misallocation along the intensive margin; i.e., the expected marginal product of capital is not equalized across entrepreneurs with different productivity.*

Proof. Obvious from Equation (1) and (2). □

When ex-post instruments for consumption smoothing fail to exist—i.e., when a full set of state-contingent claims is unavailable—risk-averse entrepreneurs choose capital to balance maximizing expected income and minimizing income volatility. As a result, capital allocation across entrepreneurs deviates from the allocation that maximizes aggregate expected output. In particular, lack of insurance introduces an endogenous *wedge* $\tau(z,k^\circ)$ between the marginal product

[4] The three-parameter log-normal distribution is similar to the standard two-parameter log-normal distribution, except that its support is shifted by an amount representing a lower bound, known as the threshold. In this context, if aggregate transfers net of capital payments are relatively small (i.e., $T - rK \approx 0$), then consumption would approximately follow a standard two-parameter log-normal distribution.

of capital and its marginal cost; i.e., under incomplete markets, the equilibrium allocation of capital satisfies

$$\bar{s} f_k(z, k^\circ)(1 - \tau(z, k^\circ)) = r^o,$$

where

$$\tau(z, k^\circ) = (\gamma - 1)\frac{\sigma_s^2}{\bar{s}} f(z, k^\circ).$$

This leads to our third proposition.

Proposition 3. *When $\gamma > 1$, insurance markets incompleteness operates as a correlated distortion, leading to a reallocation of capital from more to less productive firms relative to the complete markets benchmark.*

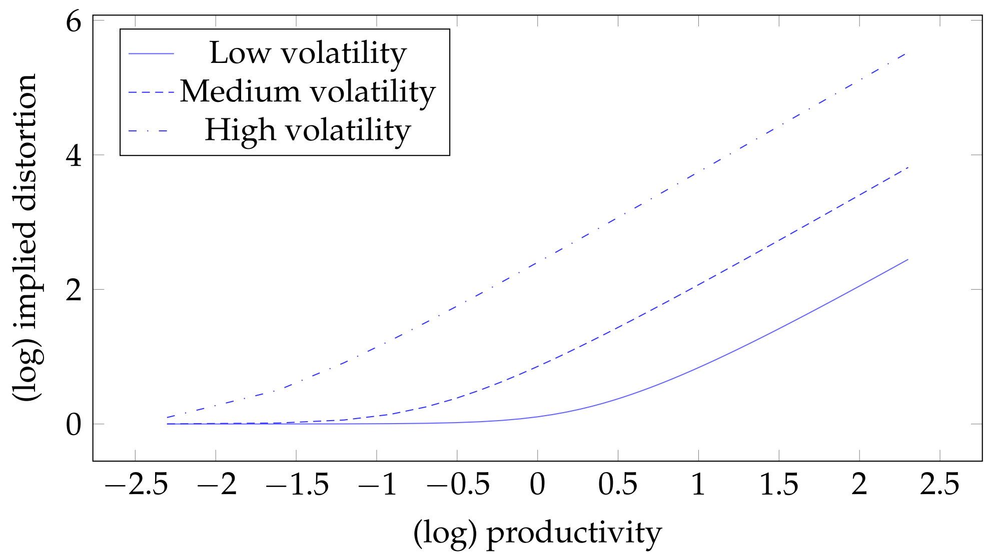
Proof. See Appendix A. □

When entrepreneurs are sufficiently risk-averse, market incompleteness acts as a tax on output that is positively correlated with entrepreneurial productivity. Figure 1 illustrates how the distortions implied by our model endogenously vary with entrepreneurial productivity. Under incomplete markets, all else equal, a larger scale of production makes utility more sensitive to output shocks. Due to complementarities in production, this distortion is particularly pronounced for high-productivity entrepreneurs, who optimally reduce their scale and forgo expected profits to secure a less volatile income.[5]

The figure also shows that this distortion becomes more pronounced when the volatility of output shocks is higher. This occurs because the wedge between the marginal product of capital and its price, $\tau(z, k^\circ)$ increases with the volatility of idiosyncratic output shocks σ. This observation leads to our fourth proposition.

[5]A second force operates in general equilibrium: capital being in fixed supply, the reduced demand for capital from high-productivity entrepreneurs lowers the rental price relative to the complete markets benchmark. This, in turn, incentivizes low-productivity entrepreneurs—those whose utility is less sensitive to output shocks—to rent more capital. See Section 4 for further discussion.

Figure 1: Correlated distortions and entrepreneurial productivity



NOTES: This figure reports the endogenous distortion, $\frac{1}{1-\tau(z,k^{\circ})}$, implied by the estimated model as a function of the estimated entrepreneurial (log) productivity, z.

Proposition 4. *Under incomplete markets, when $\gamma > 1$, a higher volatility of output shocks σ reduces the correlation between capital k and productivity z.*

Proof. See Appendix A. □

This is a central property of the model. Under incomplete markets, the distribution of marginal utilities of consumption across states of the world increasingly reflects the impact of output shocks. Amplifying production uncertainty is isomorphic to imposing larger distortions on the marginal product of capital for high-productivity entrepreneurs. As the marginal utilities of high-productivity entrepreneurs become more closely tied to their realized output shocks, their capital allocation decisions deviate even further from the complete markets benchmark. This result implies that, all else being equal, markets with higher volatility should also exhibit greater capital misallocation.

2.3 Discussion

Our model shows that incomplete insurance markets are sufficient to cause capital misallocation across entrepreneurs, above and beyond that which is caused by other distortions in the capital market. Lack of insurance generates an endogenous wedge between the marginal product of capital and its price, which is positively correlated with firm productivity. As a result, incomplete markets lead to a reallocation of capital from high- to low-productivity firms relative to the complete markets benchmark.

In Appendix A, we show that the predictions of our model are robust to the inclusion of additional factors of production, such as labor, which can be adjusted ex-post (i.e., after the realization of output shocks), and a risk-free bond that entrepreneurs can use to smooth consumption over time. To the extent that insurance markets remain incomplete—that is, no set of contingent claims spans the entire set of states of the world—the equilibrium capital allocation does not maximize aggregate expected output.

Our results do not depend on the source of uncertainty or the nature of heterogeneity across entrepreneur types. Whether the former arises from supply uncertainty due to output shocks or demand uncertainty due to taste shocks, and whether the latter reflects differences in firm productivity or permanent taste heterogeneity for firm-specific products, the allocation of capital across entrepreneurs is still distorted by the lack of insurance, with higher uncertainty amplifying the magnitude of this distortion.

Finally, none of our theoretical results are affected by allowing the production function f and the distribution ϕ to be heterogeneous across firms. Specifically, all propositions continue to hold when technology and shock volatility vary by locations and industries, and regardless of whether factor and insurance markets are integrated into a unique market or segmented into smaller local markets.[6]

[6] In such model, for each local market $j \in \mathcal{J}$, there exists a continuum $\mathcal{I}^j$ of entrepreneurs, with $\cup_{j=1}^{\mathcal{J}} \mathcal{I}^j = 1$. Entrepreneurs in local market j have access to a technology $f^j(z,k)$ and are subject to output shocks that follow a probability density function $\phi^j(s)$.

3 Empirics

Our model links the lack of insurance markets to capital misallocation among heterogeneous entrepreneurs through the degree of idiosyncratic production uncertainty. It predicts that, under incomplete markets, greater idiosyncratic production uncertainty results in increased capital misallocation. In this section, we test this prediction.

3.1 Data

We use firm-level data from Portugal's *Sistema de Contas Integradas das Empresas* (SCIE, henceforth) for the period 2010-2021.[7] The dataset is an annual panel covering all firms engaged in the production of goods and services in Portugal, excluding financial and insurance companies as well as state-owned enterprises.[8] For each firm-year pair, we observe sales, number of employees, payroll, material expenditures, current and non-current fixed assets, firm age, 4-digit industry classification, and location at the NUTS-2 level, which corresponds to one of Portugal's seven main administrative regions.[9] All nominal variables are deflated using the national CPI and expressed in 2010 constant prices. Since, in our data, labor is only recorded for firms with at least one employee, we exclude sole proprietors from our analysis.

In our analysis, we relate variation in idiosyncratic production uncertainty to capital misallocation across local markets, defined as the combination of a 2-digit industry classification and a NUTS-2 region. While the raw data includes 543 local markets, we restrict our focus to those with at least 100 firm-year observations to ensure sufficient power for estimating firm-level permanent productivity and output shock volatility. The resulting panel covers 397 local markets and consists of approximately 2.2 million firm-year observations, with around 190,000 firms observed annually.

[7] In English: Integrated Business Accounts System.

[8] The dataset includes all firms classified under sections A to S (except sections K and O) of the third revision of the Portuguese Classification of Economic Activities (CAE).

[9] The NUTS-2 regions in Portugal are Norte, Centro, Área Metropolitana de Lisboa, Alentejo, Algarve, Região Autónoma dos Açores, and Região Autónoma da Madeira.

Table B1 in Appendix B presents selected summary statistics for our sample. On average, firms are 15 years old and employ about 12 workers. Their average annual sales amount to roughly 1 million euros, corresponding to an average of 56 thousand euros per employee. The value of their fixed assets is approximately twice this amount. On average, firms have an annual payroll of about 230 thousand euros, implying an average salary of 14 thousand euros per employee. The vast majority of these firms are small and privately owned. Figure B.1 in Appendix B shows that approximately 80% of firms have 10 or fewer employees and that 90% operate a single establishment. Additionally, Table B2 indicates that over 95% of firms are privately held ("Sociedade por quotas").

3.2 Volatility and misallocation

We use our data to estimate production functions and recover two key statistics. The first is firm productivity, which we use to calculate measures of capital misallocation at the local market level. The second is the volatility of output shocks in each local market. Firm productivity is derived from estimates of permanent unobserved heterogeneity in sales across firms, after controlling for differences in the use of factors of production and aggregate shocks. To estimate the volatility of output shocks, we rely on the residuals from the regression of sales on the quantity of inputs used in production, firm dummies, and year dummies.

Our empirical strategy involves estimating production functions separately for each local market. This serves two key purposes. First, it provides greater flexibility, allowing input shares in the production function to vary across local markets.[10] Second, and most importantly, it enables us to account for heterogeneity in the variance of the error term across local markets. Uncovering this heterogeneity is crucial for our analysis, as our empirical strategy aims to link volatility to capital misallocation across location-industry pairs.

Thus, for each location-industry pair j, consider the following revenue production function:

$$\log y_{it} = \beta^{0j} + \beta^{1j} \log k_{it} + \beta^{2j} \log(w_{it}\ell_{it}) + \beta^{3j} \log m_{it} + \mu_i + \mu_t + \epsilon_{it}, \qquad (3)$$

[10] See Asker et al. (2014) for a similar approach.

where y_{it} represents revenues (sales) for firm i in year t, k_{it} denotes the capital stock, ℓ_{it} is the number of employees, w_{it} is the average wage paid by the firm, which is a proxy for its employees' average productivity, m_{it} denotes intermediate inputs, μ_i and μ_t represent firm and year fixed effects, respectively, and ϵ_{it} is an error term with mean 0 and variance σ^j.[11]

The estimated firm fixed effects $\widehat{\mu}_i$ capture all unobserved factors that permanently affect revenue of different firms. In our framework, they map into firm productivity z_i, i.e.,

$$\log z_i = \widehat{\mu}_i.$$

The estimated residuals $\widehat{\epsilon}_{it}$ represent instead a sample of innovations to sales that cannot be explained by variations in input use, firm-level permanent unobserved heterogeneity, or common factors affecting revenues. In our framework, they correspond to realized output shocks to output. These residuals contain information that can be used to estimate the volatility of output shocks. Our preferred approach consists of first estimating the volatility of output shocks at the firm level, $\widehat{\sigma}_i$, and then averaging these estimates across firms within the same location. Specifically, we begin by computing the within-firm standard deviation of the estimated residuals $\widehat{\epsilon}_{it}$ over time:

$$\widehat{\sigma}_i = \sqrt{\sum_{t=1}^{T} (\widehat{\epsilon}_{it} - \overline{\widehat{\epsilon}}_{it})^2},$$

where $\overline{\widehat{\epsilon}}_{it} = \frac{1}{T}\sum_{t=1}^{T} \widehat{\epsilon}_{it}$ is the time-average firm-level residual. Next, we compute the within-local-market average standard deviation of the estimated residuals:

$$\widehat{\sigma}_j = \sum_{i \in \mathcal{I}^j} \frac{\widehat{\sigma}_i}{|\mathcal{I}^j|},$$

where $\mathcal{I}^j$ denotes the set of firms in local market j. Figure B.2 and Table B3 in

[11] In equation 3 we include labor and intermediate inputs, in addition to capital, to account for the observed heterogeneity in labor and material usage across firms. In Appendix A.2, we discuss a version of the model where entrepreneurs are allowed to choose multiple factors of production.

Appendix B.1 present the distribution of estimated firm productivity and output shock volatility. Firm productivity exhibits substantial dispersion across firms, with the standard deviation of $\widehat{\mu}_i$ approximately 0.656. The average standard deviation of output shocks is around 0.347.

Next, we explore the relationship between output shock volatility and capital misallocation across local markets. To measure capital misallocation within each location-industry pair, we employ two distinct strategies. First, we compute the correlation between firm productivity and average (log) capital holdings across firms within each local market. Specifically, for each local market j, we estimate the following regression:

$$\overline{\log k}_{it} = \alpha^j + \eta^j \widehat{\mu}_i + \varepsilon_i,$$

where $\overline{\log k}_{it} = \frac{1}{T}\sum_{t=1}^{T} \log k_{it}$. A higher η^j indicates a higher correlation between firm productivity and capital holdings, and hence a lower capital misallocation, in local market j. Second, leveraging the production function estimates of equation (3), we construct an estimate of the (log) marginal product of capital for each firm i and year t:

$$\log \text{MPK}_{it} = \widehat{\mu}_i + \widehat{\beta}^{1j} \log k_{it}.$$

Then, for each firm i, we compute the average within-firm marginal product of capital as

$$\overline{\log \text{MPK}^j_{it}} = \frac{1}{T}\sum_{t=1}^{T} \log \text{MPK}_{it},$$

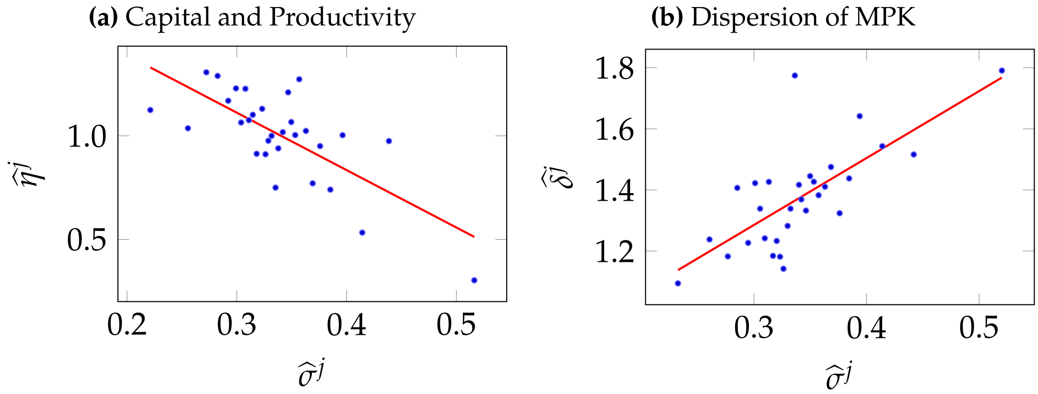
and finally we calculate its standard deviation within each local market:

$$\delta^j = \text{s.d.}\left(\overline{\log \text{MPK}_{it}}\right),$$

Figure 2 scatters both measures of capital misallocation on the average firm-level volatility across local markets.[12] In the figure, each dot represents an average local market within a certain bin in the distribution of volatility. The

[12] In Table B4 Appendix B, we report point estimates (and standard errors) from regressing our measures of capital misallocation on the average firm-level volatility across local markets,

Figure 2: Volatility and capital misallocation across local markets



NOTES:: This figure binscatters the relationship between capital misallocation and output shock volatility across local markets. In Panel A, capital misallocation is measured as the correlation between firm productivity and the average (log) capital holdings of firms within each local market. In Panel B, capital misallocation is measured as the standard deviation of the average within-firm marginal product of capital across firms in each local market. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations.

patterns that emerge are striking: local markets with higher firm-level volatility are also those with lower correlation between firm-level capital and productivity (Panel A). This pattern implies that the marginal product of capital is not equalized across firms located in high-volatility markets as it would be the case in efficient allocation (Panel B).[13]

Figure 3 telescopes into local markets with different levels of average idiosyncratic volatility. Panel A presents a binscatter plot of capital holdings against estimated firm productivity for firms operating in low-volatility markets, while Panel B replicates this analysis for firms in high-volatility markets, where low- (high-) volatility markets are defined as location-industry pairs that fall within

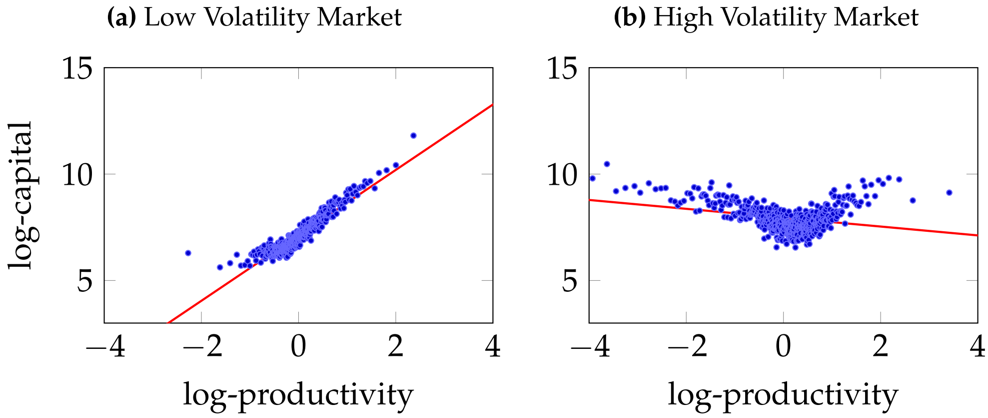
conditional on 2-digit CAE industry and NUTS-2 region fixed effects. That is, we estimate,

$$\widehat{\omega}^j = \gamma_0 + \gamma_1 \widehat{\sigma}^j + \mu^{s(j)} + \mu^{\ell(j)} + \varepsilon^j,$$

where $\widehat{\omega}^j$ is either $\widehat{\eta}^j$ or $\widehat{\delta}^j$, $\mu^{s(j)}$ are 2-digit CAE industry fixed effects and $\mu^{\ell(j)}$ are NUTS-2 region fixed effects, and ε^j is an error term.

[13]Figure B.3 and Table B5 in Appendix B show that a similar relationship holds for labor misallocation and output shock volatility across local markets, though the magnitudes are approximately half as large.

Figure 3: Capital allocation in low- and high-volatility markets



NOTES:: This figure binscatters firm-level (log) capital, $\log k_i$, against the estimated firm-level (log) productivity, $\widehat{\mu}_i$, across firms located in low-volatility (Panel A) and high-volatility (Panel B) local markets. Low- (high-) volatility markets are defined as a tuple of 2-digit CAE industry classification and NUTS-2 region with at least 100 firm-year observations and with an estimated average volatility at the bottom (top) 1 percent of the distribution.

the bottom (top) 1% of the estimated distribution of output shock volatilities. Consistent with our theory, capital holdings and firm productivity show a strong positive correlation of 0.49 in low-volatility markets. However, in high-volatility markets, this relationship breaks down, with capital allocation deviating from efficiency and becoming essentially uncorrelated with productivity, as reflected in a point estimate of -0.09.

3.3 Robustness checks

Shock persistence. In our baseline specification, given by equation (3), the error terms represent output shocks, which our framework assumes to be independently and identically distributed over time. We test the robustness of our results to allowing the shocks to be persistent over time. To do it, we extend the production function in equation (3) as follows:

$$\log y_{it} = \beta^{0j} + \beta^{1j} \log k_{it} + \beta^{2j} \log(w_{it}\ell_{it}) + \beta^{3j} \log m_{it} + \mu_i + \nu_{it},$$

where ν_{it} follows an autoregressive process of order one; i.e.,

$$\nu_{it} = \rho \nu_{it-1} + \epsilon_{it},$$

$\rho \in (0,1)$ captures the persistence of ν_{it}, and ϵ_{it} is an i.i.d. mean-zero error term. To obtain consistent estimates of β^{1j}, β^{2j}, and β^{3j}, we follow Blundell and Bond (2000) and employ a dynamic panel data approach while treating capital as a predetermined variable. Further details are provided in Appendix B. This estimation strategy yields firm-level estimates of productivity and output shock volatility, which we use to construct measures of capital misallocation and output shock volatility at the local market level. In Tables B6 and B7, we report the correlations between these measures of capital misallocation and output shock volatility across location-industry pairs. Our results are robust to the inclusion of shock persistence. As we move from markets with an average firm-level volatility of 20 percent to those with twice the volatility, the correlation between firm productivity and capital decreases by 0.53, while the dispersion of marginal products of capital increases by 0.99. The relationship between uncertainty and capital misallocation remains significant, with magnitudes comparable to or exceeding those in our baseline specification.

Firm exit. Each year, approximately 6.5 percent of firms exit the sample. Since exiting firms are likely to be non-randomly selected from the population of operating firms — for instance, they tend to be smaller than surviving firms, ignoring this attrition could introduce bias in our estimates of the production function parameters. To address this concern, we consider the following binary selection equation for firm exit:

$$d^*_{it} = \alpha_t \eta_i + \epsilon_{it}, \quad d_{it} = \mathbf{1}[d^*_{it} > 0] \tag{4}$$

where $\mathbf{1}[.]$ is an indicator function equal to one if its argument is true, and zero otherwise, η_i are unobserved individual-specific effects, with a possibly time-varying effect, α_t, and ϵ_{it} is an unobserved disturbance term. We jointly estimate equations (3) and (4) separately for every period t following the semi-parametric approach proposed by Wooldridge (1995), and using panel-level average of time-varying covariates in equation (3) to parametrize η_i. We report details in Appendix B.

Tables B8 and B9 present the conditional correlation between output shock volatility and our two measures of misallocation, both constructed using production function estimates that account for self-selection in firm exit. As we move from markets with an average output shock volatility of 20 percent to those with twice the volatility, the correlation between firm productivity and capital holdings decreases by 0.46, while the dispersion of marginal products of capital increases by 0.31. Although accounting for endogenous firm exit slightly reduces the correlation, it remains large and statistically significant.

4 The aggregate cost of incomplete markets

In this section, we leverage the structure of our model to answer the following question: What is the aggregate cost of incomplete consumption insurance? We begin by specifying a production function in which capital is the only factor of production:

$$f(z, k) = zk^{\alpha},$$

where $\alpha \in (0, 1)$.[14] We assume that productivity z follows a gamma distribution with shape and scale parameters κ and θ, respectively:

$$z \sim \Gamma(\kappa, \theta).$$

Finally, we maintain the assumption that output shocks s follow a log-normal distribution. Since our focus is on aggregate outcomes, we abstract from the heterogeneity in capital share α and output shock volatility σ across local markets, focusing instead on their average values.

Table 1 summarizes the model parameters and their respective sources. The output elasticity of capital is externally calibrated to 0.73 Erosa et al. (2023). The aggregate volatility of output shocks σ is calibrated at 0.347, a value directly es-

[14]In Section 4.1, we extend the model to include labor as a factor of production, allowing entrepreneurs to adjust it after output shocks realize.

timated from the data. The shape and scale parameters of the firm productivity distribution are obtained by fitting a gamma distribution to the estimated firm productivity and set to 1.404 and 1.060, respectively. Finally, we estimate the coefficient of relative risk aversion, γ, to match the observed correlation of 0.494 between (log) capital holdings and (log) permanent productivity across firms. The estimation yields $\widehat{\gamma} = 4.024$, which is close to the estimate of 4.2 found in Chiappori and Paiella (2011).

Table 1: Parameters

Parameters	Description	Value	Source/Target
	A. Parameters calibrated		
α	Output elasticity of capital	0.730	Erosa et al. (2023)
σ	Volatility of shocks	0.347	Data
(κ, θ)	Distribution of permanent productivity	(1.404,1.060)	Data
	B. Parameters estimated		
γ	Relative risk aversion	4.024	corr. $[\log k, \log z] = 0.494$

NOTES: This table reports the values and sources of the calibrated and estimated parameters.

Equipped with our estimates, we perform two counterfactual exercises. In the first, we eliminate production uncertainty by setting σ to zero; i.e., we simulate the aggregate behavior of a deterministic economy where entrepreneurs face no output shocks. In the second, we complete consumption insurance markets while maintaining output shock volatility at its baseline level.

Table 2 presents the results of these counterfactual exercises. Transitioning from the baseline to a deterministic economy removes the distortions in capital allocation caused by production uncertainty under incomplete markets. As a consequence, the correlation between firm productivity and capital holdings increases, leading to greater dispersion of capital holdings across firms. This reallocation enhances aggregate productivity, measured using output per unit of capital, by 54.1%, resulting in aggregate income gains of 85.3%.

A counterfactual economy with complete markets exhibits the same capital allocation across firms as the economy without production uncertainty, but achieves higher aggregate productivity and income gains, reaching 63.7% and 96.8%, respectively. This difference arises because eliminating production uncertainty, that is, setting σ to zero, affects higher moments of the output distribution as well as the expected value of the output shock. In contrast, the availability of state-contingent claims when markets are complete enables entrepreneurs to fully insure against consumption uncertainty, effectively eliminating the distortionary effects of the higher moments of the output distribution without reducing the expected value of output shocks. Intuitively, completing insurance markets eliminates misallocation without reducing the likelihood of favorable output shock realizations.[15]

Table 2: The aggregate cost of incomplete markets

		Counterfactual	
	Baseline (1)	No Uncertainty (2)	Complete markets (3)
Volatility, σ	0.347	0	0.347
$\text{corr}[\log z, \log k]$	0.494	1.000	1.000
$\eta[\log z, \log k]$	1.114	3.700	3.700
$\text{sd}[\log k]$	1.141	3.445	3.445
Rental rate, r	0.294	1.952	2.073
Output per capital	1.000	1.541	1.637
Income	1.000	1.853	1.968

NOTES: This table reports selected outcomes for the baseline economy (column 1), a counterfactual economy with no idiosyncratic production uncertainty (column 2), and a counterfactual economy with complete insurance markets (column 3).

4.1 Robustness checks

Factor shares. We assess the robustness of our findings to different values of the output elasticity of capital, α. In Appendix C, Table C1, we report the

[15] Effectively, the gains from completing insurance markets could be decomposed into the gains from efficiently allocating capital to entrepreneurs and those from allowing them to fully exploit the prospect of favorable shock realizations. In other words, completing insurance markets generates aggregate gains that extend beyond pure allocative efficiency.

percentage gains in aggregate productivity and income from eliminating production uncertainty or completing consumption insurance markets while varying α from 0.5 to 0.9. In each counterfactual economy, all parameters remain as in the baseline, except for the relative risk aversion coefficient, γ, which is re-calibrated to match the observed correlation between firm productivity and capital holdings. We find that increasing the output elasticity of capital raises the estimated RRA coefficient, from 3.51 when $\alpha = 0.5$ to 4.72 when $\alpha = 0.9$. This occurs because, holding all else constant, a higher α increases the expected marginal product of capital more significantly for high-productivity entrepreneurs. Consequently, a higher value of γ is required to reduce the correlation between firm productivity and capital holdings to the targeted moment. As before, the counterfactual gains in aggregate productivity and income from completing insurance markets consistently exceed those from eliminating production uncertainty. Moreover, higher values of the output elasticity of capital α amplify both types of gains. Eliminating production uncertainty increases aggregate efficiency and income by 4% and 30% when $\alpha = 0.5$, rising to 187% and 251% when $\alpha = 0.9$, respectively. Similarly, completing markets generates larger gains, ranging from 10% and 39% for $\alpha = 0.5$ to 205% and 270% for $\alpha = 0.9$, respectively.

Technology. Finally, we assess the robustness of our findings to incorporating labor as a factor of production and allowing firms to adjust it after output shocks realize. To do so, we solve and estimate the model introduced in Section C.2. Table C2 reports the values of the calibrated parameters. We set the output elasticity of labor, α, to 0.7 and the span of control, η, to 0.9, following Erosa et al. (2023). As in the benchmark model, we estimate the relative risk aversion coefficient to match the observed correlation between firm productivity and capital holdings, obtaining a value of 1.631.

Table C3 presents the counterfactual outcomes. Allowing firms to adjust labor ex post significantly reduces the gains from completing markets, cutting them by half: total factor productivity and aggregate income increase by 34.8% and 53.9%, respectively, compared to 63.7% and 96.8% in the benchmark model. While labor demand adjustments do provide firms with a mechanism to mitigate negative output shocks, they are insufficient to fully eliminate the aggregate cost of market incompleteness.

5 Conclusion

We study the allocation of capital to risk averse entrepreneurs with heterogeneous productivity in an economy where insurance markets are incomplete and output requires time to build. We demonstrate that insurance markets incompleteness acts as a correlated distortion, reallocating capital from more productive to less productive entrepreneurs relative to the complete markets benchmark. Using Portuguese administrative data, we show that capital misallocation is greater in locations and industries where output is subject to higher idiosyncratic production uncertainty, consistent with our framework. By closing the model in general equilibrium, we quantify the misallocation cost of incomplete markets. Our findings suggest that completing insurance markets would result in substantial gains, raising aggregate productivity by 64% and income by 97%.

Our findings speak to the link between insurance markets incompleteness and the aggregate costs of resource misallocation. Our results highlight how imperfect insurance endogenously generates capital misallocation among risk-averse entrepreneurs with heterogeneous productivity. In doing so, our paper introduces a complementary channel for misallocation, which can operate alongside the well-known drivers examined in the classic misallocation literature, such as policy distortions and factor market frictions.

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Online appendix (not intended for publication)

A Theory appendix

A.1 Proofs

Proof of Proposition 1. Recall that an equilibrium capital allocation is such that the marginal product of capital is constant across firms and equal to the rental price of capital; i.e.,

$$\int_{s \in \mathcal{S}} [s f_k(z_i, k_i^*) - r^*] \phi(s) ds = 0, \quad \forall i \in \mathcal{I}.$$

Next, consider the problem of a planner that allocates capital to entrepreneurs to maximize aggregate expected output subject to an aggregate feasibility constraint. The problem can be stated as follows:

$$\max_{\{k_i\}_{i \in \mathcal{I}}} \quad \int_{s \in \mathcal{S}} s f(z_i, k_i) \phi(s) ds$$
$$\text{s.t.} \quad \sum_i k_i di \leq \overline{K},$$

where $\overline{K}$ is the economy-wide capital stock. A solution to this problem is characterized by the following first-order conditions:

$$\int_{s \in \mathcal{S}} s f_k(z_i, k_i^\star) \phi(s) ds - \lambda = 0, \quad \forall i \in \mathcal{I},$$

where λ is the Lagrange multiplier attached to the feasibility constraint. That is, to maximize output, the planner allocates capital so that each entrepreneur's expected marginal product equals its shadow price, λ. □

Proof of Proposition 3. Let $\{k_i^*\}_{i \in \mathcal{I}}$ and $\{k_i^o\}_{i \in \mathcal{I}}$ be the equilibrium capital allocations under complete and incomplete markets. Let us express both allocations as policy functions mapping entrepreneurial productivity to equilibrium capital choices; i.e., $k^*(z)$ and $k^o(z)$, respectively.

To prove that market incompleteness results in the reallocation of capital from more to less productive firms, we show that

- there is exists a unique value for the entrepreneurial productivity such that the optimal capital chosen under complete markets equates the one under incomplete markets, i.e.,

$$\exists ! \tilde{z} \in \mathcal{Z} \quad \text{s.t.} \quad k^*(\tilde{z}) = k^o(\tilde{z});$$

- at the crossing point, $\tilde{z}$, the slope of the policy function for capital under complete markets is larger than the slope of the policy function under incomplete markets , i.e.,

$$\frac{\partial k^*(z)}{\partial z}\Big|_{z=\tilde{z}} > \frac{\partial k^o(z)}{\partial z}\Big|_{z=\tilde{z}}.$$

We start by proving the second bullet point. Let $\tilde{z}$ be a point in $\mathcal{Z}$ such that $k^*(\tilde{z}) = k^o(\tilde{z}) = \tilde{k}$. We show later that this point exists and is unique.

Let $H(z,k) = \int_{s\in\mathcal{S}} s f_k(z,k)\Gamma(s)ds - r$. By the implicit function theorem, under complete market it must be the case that:

$$\frac{\partial k^*(z)}{\partial z}\Big|_{z=\tilde{z}} = -\frac{\frac{\partial H(z,k^*(z))}{\partial z}\Big|_{z=\tilde{z}}}{\frac{\partial H(z,k^*(z))}{\partial k}\Big|_{z=\tilde{z}}} = -\frac{f_{kz}(\tilde{z},k^*(\tilde{z}))}{f_{kk}(\tilde{z},k^*(\tilde{z}))} = -\frac{f_{kz}(\tilde{z},\tilde{k})}{f_{kk}(\tilde{z},\tilde{k})} > 0$$

Let $G(z,k) = f_k(z,k)[\bar{s} - (\gamma-1)\sigma_s^2 f(z,k)] - r$. By the implicit function theorem, under no insurance it must be the case that:

$$\frac{\partial k^o(z)}{\partial z}\Big|_{z=\tilde{z}} = -\frac{\frac{\partial G(z,k^o(z))}{\partial z}\Big|_{z=\tilde{z}}}{\frac{\partial G(z,k^o(z))}{\partial k}\Big|_{z=\tilde{z}}} = -\frac{f_{kz}(\tilde{z},k^o(\tilde{z}))[\bar{s}-(\gamma-1)\sigma_s^2 f(\tilde{z},k^o(\tilde{z}))]-f_k(\tilde{z},k^o(\tilde{z}))[(\gamma-1)\sigma_s^2 f_z(\tilde{z},k^o(\tilde{z}))]}{f_{kk}(\tilde{z},k^o(\tilde{z}))[\bar{s}-(\gamma-1)\sigma_s^2 f(\tilde{z},k^o(\tilde{z}))]-f_k(\tilde{z},k^o(\tilde{z}))[(\gamma-1)\sigma_s^2 f_k(\tilde{z},k^o(\tilde{z}))]}$$

$$= -\frac{\bar{s}f_{kz}(\tilde{z},\tilde{k})-(\gamma-1)\sigma_s^2 f(\tilde{z},\tilde{k})f_{kz}(\tilde{z},\tilde{k})-(\gamma-1)\sigma_s^2 f_z(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})}{\bar{s}f_{kk}(\tilde{z},\tilde{k})-(\gamma-1)\sigma_s^2 f(\tilde{z},\tilde{k})f_{kk}(\tilde{z},\tilde{k})-(\gamma-1)\sigma_s^2 f_k(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})}.$$

We need to prove that:

$$\frac{\partial k^*(z)}{\partial z}\Big|_{z=\tilde{z}} > \frac{\partial k^o(z)}{\partial z}\Big|_{z=\tilde{z}}$$

or, equivalently,

$$-\frac{f_{kz}(z,k)}{f_{kk}(\tilde{z},\tilde{k})} > -\frac{\bar{s}f_{kz}(\tilde{z},\tilde{k}) - (\gamma-1)\sigma_s^2[f(\tilde{z},\tilde{k})f_{kz}(\tilde{z},\tilde{k}) + f_z(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})]}{\bar{s}f_{kk}(\tilde{z},\tilde{k}) - (\gamma-1)\sigma_s^2[f(\tilde{z},\tilde{k})f_{kk}(\tilde{z},\tilde{k}) + f_k(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})]}$$

This condition is equivalent to:

$$\begin{aligned} f_{kz}(\tilde{z},\tilde{k}) \left[\bar{s}f_{kk}(\tilde{z},\tilde{k}) - (\gamma-1)\sigma_s^2[f(\tilde{z},\tilde{k})f_{kk}(\tilde{z},\tilde{k}) + f_k(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})]\right] < \\ f_{kk}(\tilde{z},\tilde{k}) \left[\bar{s}f_{kz}(\tilde{z},\tilde{k}) - (\gamma-1)\sigma_s^2[f(\tilde{z},\tilde{k})f_{kz}(\tilde{z},\tilde{k}) + f_z(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})]\right] \end{aligned}$$

which can be re-arranged as follows:

$$\begin{aligned} f_{kz}(\tilde{z},\tilde{k})\bar{s}f_{kk}(\tilde{z},\tilde{k}) - (\gamma-1)\sigma_s^2 f_{kz}(\tilde{z},\tilde{k})[f(\tilde{z},\tilde{k})f_{kk}(\tilde{z},\tilde{k}) + f_k(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})] < \\ f_{kk}(\tilde{z},\tilde{k})\bar{s}f_{kz}(\tilde{z},\tilde{k}) - (\gamma-1)\sigma_s^2 f_{kk}(\tilde{z},\tilde{k})[f(\tilde{z},\tilde{k})f_{kz}(\tilde{z},\tilde{k}) + f_z(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})] \end{aligned}$$

Simplifying terms, we obtain:

$$\begin{aligned} -(\gamma-1)\sigma_s^2 f_{kz}(\tilde{z},\tilde{k})[f(\tilde{z},\tilde{k})f_{kk}(\tilde{z},\tilde{k}) + f_k(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})] < \\ -(\gamma-1)\sigma_s^2 f_{kk}(\tilde{z},\tilde{k})[f(\tilde{z},\tilde{k})f_{kz}(\tilde{z},\tilde{k}) + f_z(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})] \end{aligned}$$

Because $\gamma > 1$ and $\sigma_s^2 > 0$, then we can further simplify the condition above as follows:

$$f_{kz}(\tilde{z},\tilde{k})[f(\tilde{z},\tilde{k})f_{kk}(\tilde{z},\tilde{k}) + f_k(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})] > f_{kk}(\tilde{z},\tilde{k})[f(\tilde{z},\tilde{k})f_{kz}(\tilde{z},\tilde{k}) + f_z(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})]$$

This condition is true if and only if

$$\underbrace{f_{kz}(\tilde{z},\tilde{k})f_k(\tilde{z},\tilde{k})}_{>0} > \underbrace{f_{kk}(\tilde{z},\tilde{k})}_{<0}\underbrace{f_z(\tilde{z},\tilde{k})}_{>0}$$

which is always the case because $f_{kz}(z,k) > 0$, $f_k(z,k) > 0$, $f_z(z,k) > 0$ and $f_{kk}(z,k) < 0$, $\forall z$ and $\forall k$. This completes the first part of the proof.

We now move to prove existence and uniqueness of a crossing point between the capital policy function under complete market and the one under incomplete markets.

Let us start by assuming there were no crossing points. Without loss of generality, let us assume that $k^*(z) > k^o(z), \forall z \in \mathcal{Z}$. If this is the case, then it must be true that:

$$\sum_{i \in \mathcal{I}} k_i^* di > \sum_{i \in \mathcal{I}} k_i^o di$$

On the other hand, in equilibrium the aggregate demand for capital under both scenarios (i.e., complete and incomplete markets) has to equal to an exogenously given supply $\bar{K}$, i.e.,

$$\sum_{i \in \mathcal{I}} k_i^* di = \sum_{i \in \mathcal{I}} k_i^o di = \bar{K}$$

which contradicts the previous condition. By contradiction, this implies that there must exist at least one point $\tilde{z}$ such that $k^*(\tilde{z}) = k^o(\tilde{z})$.

The crossing point must also be unique. Suppose it was not, i.e., suppose $\exists \tilde{z}_1, .., \tilde{z}_n \in \mathcal{Z}$ such that $k^*(\tilde{z}_j) = k^o(\tilde{z}_j), \forall j = 1, ...n$. Following what we proved in the first part of the proof, it must be the case that

$$\frac{\partial k^*(z)}{\partial z}\Big|_{z=\tilde{z}_j} > \frac{\partial k^o(z)}{\partial z}\Big|_{z=\tilde{z}_j} \quad \forall j = 1, ...n.$$

On the other hand, because both policy functions $k^*(z)$ and $k^o(z)$ are continuous in $\mathcal{Z}$, and because $k^*(z)$ is monotonically increasing in z, i.e., $\frac{\partial k^*(z)}{\partial z} > 0$, the only scenario where two (or more) crossing points exist is a scenario where

$$\frac{\partial k^*(z)}{\partial z}\Big|_{z=\tilde{z}_{\widehat{j}}} \leq \frac{\partial k^o(z)}{\partial z}\Big|_{z=\tilde{z}_{\widehat{j}}}$$

for some $\widehat{j}$, contradicting what we proved in the first part of the proof. This completes the second part of the proof.

Proof of Proposition 4. Consider a decentralized equilibrium under incomplete markets. We need to prove that a higher volatility of output shocks σ_s^2 reduces the correlation between capital holdings k^o and productivity z. To do so, notice that the derivative of $(\partial z)^{-1}/\partial k^o$ with respect to σ^2 is negative if and only if

$$\left(\frac{\partial \bar{s}}{\partial \sigma^2}\sigma_s^2 - \bar{s}\frac{\partial \sigma_s^2}{\partial \sigma^2}\right)(\gamma-1)f_k(z,k^o)\left[f_{kz}(z,k^o)f_k(z,k^o) - f_{kk}(z,k^o)f_z(z,k^o)\right] < 0.$$

Since $f_{kk}(z,k^o) < 0$, $f_k(z,\kappa^o) > 0$, $f_{kz}(z,k^o) > 0$ and $f_z(z,k^o) > 0$,

$$f_k(z,k^o)\left[f_{kz}(z,k^o)f_k(z,k^o) - f_{kk}(z,k^o)f_z(z,k^o)\right] > 0.$$

Thus, if $\gamma > 1$, the derivative of $(\partial z)^{-1}/\partial k^o$ with respect to σ^2 is negative if and only if

$$\left(\frac{\partial \bar{s}}{\partial \sigma^2}\sigma_s^2 - \bar{s}\frac{\partial \sigma_s^2}{\partial \sigma^2}\right) < 0. \tag{5}$$

Recall that $s \sim \log\mathcal{N}(0,\sigma^2)$; thus,

$$\bar{s} = \exp\left(\frac{\sigma^2}{2}\right)$$

and

$$\sigma_s^2 = (\exp(\sigma^2) - 1)\exp(\sigma^2).$$

Substituting these expressions into Equation (5), we have that

$$\left(\frac{\partial\left(\exp\left(\frac{\sigma^2}{2}\right)\right)}{\partial\sigma^2}\left[(\exp(\sigma^2)-1)\exp(\sigma^2)\right] - \exp\left(\frac{\sigma^2}{2}\right)\frac{\partial\left[(\exp(\sigma^2)-1)\exp(\sigma^2)\right]}{\partial\sigma^2}\right) < 0;$$

i.e.,

$$\exp(\sigma^2) + \frac{1}{2}(\exp(\sigma^2) - 1) > 0,$$

which is always the case because $\sigma^2 > 0$. □

A.2 An extension of the model that allows for other factors of production, monopolistic competition in the product market, and demand shocks

We consider a version of the model outlined in Section 2.2 that allows for other factors of production, monopolistic competition in the product market, and demand shocks. For simplicity, we focus on a model where labor is the only additional input besides capital; however, it is clear from the discussion that there is no conceptual difficulty in including other factors of production as well. Entrepreneurs have access to a production technology $f(z, k, \ell)$ that depends on productivity z, capital k, and labor ℓ. As before, output produced is subject to idiosyncratic, multiplicate shocks s distributed according to a probability density function $\phi(s)$. Differently from the previous model, where entrepreneurs produce a homogeneous commodity, each entrepreneur now produces a differentiated variety of final goods or services, and faces a downward-sloping demand curve given by

$$y_i(\nu, \omega_i, p_i) = \Delta(\nu\omega_i)^{\eta-1} p_i^{-\eta},$$

where η denotes the price elasticity of demand, Δ is a parameter governing aggregate demand for final goods and services, $\omega_i \in \mathcal{W}$ is a parameter capturing permanent taste heterogeneity for firm-specific products, and $\nu \in \mathcal{V}$ is an idiosyncratic demand shock, distributed with a pdf $\Lambda(\nu)$.

Entrepreneurs choose capital before the realization of output and taste shocks, but decide how much labor to employ after these shocks are realized. Let w be the wage rate (normalized to 1), r be the rental rate for capital, and $\overline{K}$ be the exogenous supply of capital. For simplicity, we adopt a partial equilibrium framework for the labor market, taking wages as given.[16] Entrepreneur i's problem

[16]Alternatively, we may assume that labor is supplied at no utility cost by a unit mass of identical workers.

can be written as follows:

$$\max_{k_i,\{c_i(s,\nu)\}_{s\in\mathcal{S}}} \int_{s\in\mathcal{S}}\int_{\nu\in\mathcal{V}} u(c_i(s,\nu))\phi(s)\Lambda(\nu)dsd\nu$$
$$\text{s.t.}\quad c_i(s,\nu)) \leq \pi(s,\nu,z_i,\omega_i,k_i) - rk_i + T \quad \forall s\in\mathcal{S},\forall \nu\in\mathcal{V},$$

where $T = r\overline{K}$ are revenues from renting capital, assumed to be rebated lump-sum to each entrepreneur, while $\pi(s,\nu,z_i,\omega_i,k_i)$ denotes per-period maximized revenues net of labor costs (henceforth, the profit function); i.e.,

$$\begin{aligned}\pi(s,\nu,z_i,\omega_i,k_i) = \max_{\ell_i,p_i} \quad & y_i(s,z_i,k_i,\ell_i)p_i - \ell_i \\ \text{s.t.}\quad & y_i(s,z_i,k_i,\ell_i) = sf(z_i,k_i,\ell_i) \\ \text{and}\quad & y_i(\nu,\omega_i,p_i) = \Delta(\nu\omega_i)^{\eta-1}p_i^{-\eta}.\end{aligned} \tag{6}$$

Substituting price p_i from the demand function into the profit function, we can rewrite the problem in Equation (6) as follows:

$$\pi(s,\nu,z_i,\omega_i,k_i) = \max_{\ell_i} \quad \Delta^{1-\zeta}(\nu\omega_i)^{\zeta}(sf(z_i,k_i,\ell_i))^{\zeta} - \ell_i,$$

where $\zeta = 1 - 1/\eta$.

For a given Δ, a competitive equilibrium for this economy is a list of capital choices, $\{k_i^o\}_{i\in\mathcal{I}}$, labor choices, $\{\ell_i^o(s,\nu)\}_{s\in\mathcal{S},\nu\in\mathcal{V},i\in\mathcal{I}}$, product prices $\{p_i^o(s,\nu)\}_{s\in\mathcal{S},\nu\in\mathcal{V},i\in\mathcal{I}}$, consumption plans, $\{c_i^o(s,\nu)\}_{s\in\mathcal{S},\nu\in\mathcal{V},i\in\mathcal{I}}$, and a rental price of capital r^o such that

- $\{k_i^o\}_{i\in\mathcal{I}}$, $\{\ell_i^o(s,\nu)\}_{s\in\mathcal{S},\nu\in\mathcal{V},i\in\mathcal{I}}$, $\{p_i^o\}_{s\in\mathcal{S},\nu\in\mathcal{V},i\in\mathcal{I}}$ and $\{c_i^o(s,\nu)\}_{s\in\mathcal{S},\nu\in\mathcal{V},i\in\mathcal{I}}$ solve entrepreneur i's problem;
- the market for capital clears; i.e.,

$$\int_i k_i^o di = \overline{K}.$$

To characterize the competitive equilibrium, we make the following assumptions:

1. production displays constant return to scale in capital and labor; i.e.,

$$f(z,k,\ell) = zk^{1-\alpha}\ell^{\alpha}, \quad \alpha \in (0,1);$$

2. the utility function exhibits constant degree of relative risk aversion; i.e.,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1;$$

3. the shocks s and ν are two independent log-normal random variables; i.e.,

$$s \sim \log \mathcal{N}(0,\sigma_s^2) \quad \text{and} \quad \nu \sim \log \mathcal{N}(0,\sigma_\nu^2).$$

We start by characterizing the solution to the problem in Equation (6). Dropping index i for convenience, and taking the first-order condition with respect to labor, we obtain

$$\alpha\zeta\Delta^{1-\zeta}(s\nu z\omega k^{1-\alpha}\ell^{\alpha})^{\zeta} = \ell,$$

which implies the following labor demand:

$$\ell = \left(\alpha\zeta\Delta^{1-\zeta}[s\nu z\omega k^{1-\alpha}]^{\zeta}\right)^{\frac{1}{1-\zeta\alpha}}.$$

Plugging the demand for labor into the profit function, we obtain:

$$\pi(s,\nu,z,\omega,k) = (1-\alpha\zeta)\Delta^{1-\zeta}[s\nu z\omega k^{1-\alpha}]^{\zeta}\left(\alpha\zeta\Delta^{1-\zeta}[s\nu z\omega k^{1-\alpha}]^{\zeta}\right)^{\frac{\zeta\alpha}{1-\zeta\alpha}},$$

or, equivalently,

$$\pi(s,\nu,z,\omega,k) = (1-\alpha\zeta)\,(\alpha\zeta)^{\frac{\zeta\alpha}{1-\zeta\alpha}}\left(\Delta^{1-\zeta}[s\nu z\omega k^{1-\alpha}]^{\zeta}\right)^{\frac{1}{1-\zeta\alpha}}.$$

Notice the profit function exhibits decreasing returns to scale in capital; i.e.,

$$\frac{\zeta(1-\alpha)}{1-\zeta\alpha} < 1.$$

This is because $\alpha \in (0,1)$ and $\eta > 1$, which implies that $\zeta < 1$.

Let

$$\Omega = (1-\alpha\zeta)\,(\alpha\zeta)^{\frac{\zeta\alpha}{1-\zeta\alpha}}\,\Delta^{\frac{1-\zeta}{1-\zeta\alpha}}.$$

Denote by x be the product of firm-idiosyncratic permanent productivity and firm-idiosyncratic permanent taste; i.e., $x = z\omega$. Let χ be the product of firm-idiosyncratic productivity shocks and firm-idiosyncratic taste shocks; i.e., $\chi = s\nu$. Then, we can rewrite the profit function as follows:

$$\pi(\chi, x, k) = \Omega\left(\chi^{\zeta} x^{\zeta} k^{1-\alpha}\right)^{\frac{1}{1-\zeta\alpha}}$$

Because s and ν are log-normally distributed, χ is also log-normally distributed; i.e.,

$$\chi \sim \log\mathcal{N}(0, \sigma^2),$$

where $\sigma^2 = \sigma_s^2 + \sigma_\nu^2$. Given the log-normality of χ, consumption is a three-parameter log-normal random random variable with expected value and variance equal to

$$\bar{c} = \Omega\left(x^{\zeta} k^{\zeta(1-\alpha)}\right)^{\frac{1}{1-\zeta\alpha}} \mathrm{E}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] - rk + T$$

and

$$\sigma_c^2 = \left(\Omega\left(x^{\zeta} k^{\zeta(1-\alpha)}\right)^{\frac{1}{1-\zeta\alpha}}\right)^2 \mathrm{var}[\chi^{\frac{\zeta}{1-\zeta\alpha}}],$$

respectively (and threshold equal to $-rk + T$). Since $\chi \sim \log\mathcal{N}(0, \sigma^2)$, $\log\chi \sim \mathcal{N}(0, \sigma^2)$. Thus,

$$\frac{\zeta}{1-\zeta\alpha}\log\chi \sim \mathcal{N}\left(0, \left(\frac{\zeta}{1-\zeta\alpha}\right)^2 \sigma^2\right),$$

and

$$\chi^{\frac{\zeta}{1-\zeta\alpha}} \sim \log\mathcal{N}\left(0, \left(\frac{\zeta}{1-\zeta\alpha}\right)^2 \sigma^2\right),$$

which implies that

$$\mathrm{E}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] = \exp\left(\frac{\left(\frac{\zeta}{1-\zeta\alpha}\right)^2 \sigma^2}{2}\right)$$

and

$$\mathrm{var}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] = \left(\exp\left(\left(\frac{\zeta}{1-\zeta\alpha}\right)^2 \sigma^2\right) - 1\right) \exp\left(\left(\frac{\zeta}{1-\zeta\alpha}\right)^2 \sigma^2\right).$$

Therefore, an entrepreneur's problem can be rewritten in terms of consumption equivalent as follows:

$$\max_k \quad \Omega x^{\frac{\zeta}{1-\zeta\alpha}} k^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}} \mathrm{E}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] - rk + T - \frac{(\gamma-1)}{2}\left(\Omega x^{\frac{\zeta}{1-\zeta\alpha}} k^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}}\right)^2 \mathrm{var}[\chi^{\frac{\zeta}{1-\zeta\alpha}}].$$

Let $g(x,k) = \Omega x^{\frac{\zeta}{1-\zeta\alpha}} k^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}}$. Taking the first-order condition with respect to capital, we obtain

$$g_k(x,k^o)\left[\mathrm{E}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] - (\gamma-1)\mathrm{var}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] g(x,k^o)\right] - r^o = 0,$$

which resembles Equation (2) in the main text.

A.3 A two-period model with a risk-free asset

We consider a two-period version of the model in Section 2.2, which allows entrepreneurs to self-insure through borrowing and saving in a risk-free asset. Unlike before, entrepreneurs are born with different initial wealth levels $\omega \in \mathcal{W} \subseteq \mathbb{R}_+$. Moreover, they can transfer resources over time using a risk-free asset a that costs q units of consumption. Entrepreneur i's problem can be written as follows:

$$\max_{k_i, a_i, c_i, \{c_i'(s)\}_{s\in\mathcal{S}}} \quad u(c_i) + \int_{s\in\mathcal{S}} u(c_i'(s))\phi(s)ds$$
$$\text{s.t.} \quad c_i \leq \omega_i - qa_i$$
$$\text{and} \quad c_i'(s) \leq sf(z_i, k_i) - rk_i + a_i + T, \quad \forall s \in \mathcal{S}.$$

A competitive equilibrium for this economy is a list of capital choices, $\{k_i^o\}_{i\in\mathcal{I}}$, asset choices $\{a_i^o\}_{i\in\mathcal{I}}$, consumption plans, $\{c_i^o(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$, a rental price of capital r^o, and a price of the asset q^o such that:

- $\{k_i^o\}_{i\in\mathcal{I}}$, $\{a_i^o\}_{i\in\mathcal{I}}$ and $\{c_i^o(s)\}_{s\in\mathcal{S},i\in\mathcal{I}}$ solve entrepreneur i's problem;
- the market for capital clears; i.e.,

$$\int_i k_i^o di = \overline{K}.$$

- the asset market clears; i.e.,

$$\int_i a_i^o di = 0.$$

Drop index i for convenience. Notice that, given the log-normality of s, consumption is a three-parameter log-normal random variable with expected value and variance equal to

$$\overline{c}' = \overline{s}f(z,k) - rk + a + T$$

and

$$\sigma_c^2 = f(z,k)^2\sigma_s^2,$$

respectively (and threshold equal to $-rk + a + T$). Therefore, an entrepreneur's problem can be rewritten in terms of consumption equivalent as follows:

$$\max_{k,a} \quad u(\omega - qa) + u\left(\bar{c}' - \frac{\gamma - 1}{2}\sigma^2_{c'}\right).$$

Taking the first-order conditions with respect to a and k, we obtain

$$0 = u_c\left(\bar{s}f(z,k^o) - r^o k^o + a^o + T - \frac{\gamma-1}{2}f(z,k^o)^2\sigma_s^2\right)\left(f_k(z,k^o)[\bar{s} - (\gamma-1)f(z,k^o\sigma_s^2] - r^o\right),$$

and

$$q^o u_c(\omega - q^o a^o) = u_c\left(\bar{s}f(z,k^o) - r^o k^o + a^o + T - \frac{\gamma-1}{2}f(z,k^o)^2\sigma_s^2\right),$$

Because $u_c > 0$, the first order conditions can be rewritten as follows:

$$f_k(z,k^o)\left[\bar{s} - \frac{\gamma-1}{2}f(z,k^o)^2\sigma_s^2\right] = r^o$$

and

$$[1 + (q^o)^{(1-\frac{1}{\gamma})}]a^o = (q^o)^{-\frac{1}{\gamma}}\omega - \left(\bar{s}f(z,k^o) - r^o k^o + T - \frac{\gamma-1}{2}f(z,k^o)^2\sigma_s^2\right).$$

Notice that the first-order condition for capital allocation mirrors the one derived in a Section 2.2. While the presence of a risk-free asset enables entrepreneurs to smooth consumption over time, it is not sufficient to complete insurance markets, leaving second-period utilities uncertain.

B Data appendix

B.1 Summary statistics

Table B1 presents selected summary statistics for our sample.

Table B1: Summary Statistics

	Mean (1)	S.d. (2)	p10 (3)	Median (4)	p90 (5)	Observations (6)
Age	15.3	13.3	2	12	32	2195287
N. of Employees	12.5	123.1	1	4	18	2195287
Fixed Assets	2188449.6	67542046.5	23074.4	168697.4	1772260.8	2195287
Sales	1032526.8	22591464.4	21683.9	115972	981823.7	2195287
Sales per employee	56227.1	780720.9	10878.5	30355.9	92847.9	2195287
Material expenditure	9462.3	194044.9	190	1816	13597	2195287
Payroll	233563.6	2615366.9	10041.9	44258.2	307049.9	2195287
Wage per employee	14031.1	12311.8	6941.6	11667.1	22786.5	2195287
Value added	1023064.5	22561743.2	20782.8	113089.6	966400.6	2195287
Value added per employee	55256.6	780490.3	10479.2	29600.5	91097.4	2195287

NOTES: Nominal variables are deflated using the national CPI and expressed in 2010 price levels. The number of employees includes both full-time and part-time workers. Fixed assets encompass both current and non-current assets.

Figure B.1 shows the distributions of firm size and establishments per firm.

Table B2 shows the share of firms by type of ownership. Over 90% of firms are held privately by Portuguese nationals. Around 29% of firms have a single proprietor.

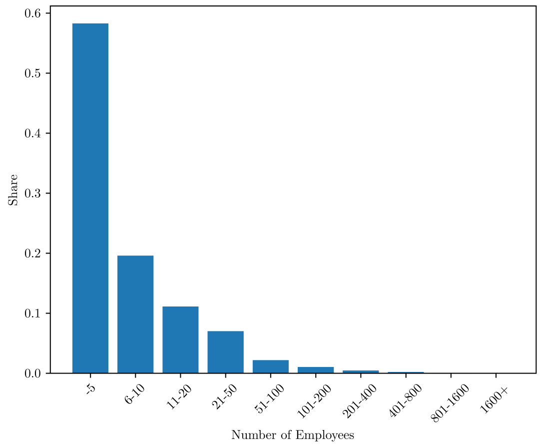
Table B2: Firm distribution by ownership

Type	Share
Sociedade por quotas	0.642
Sociedade unipessoal por quotas	0.289
Sociedade anonima	0.052
Other	0.016

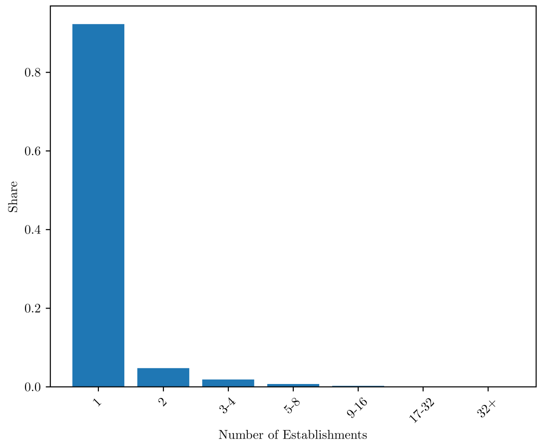
Figure B.2 shows the distribution of firm productivity μ_i and firm-level volatility of output shocks σ_i.

Figure B.1: Firm size distributions

(a) Employees



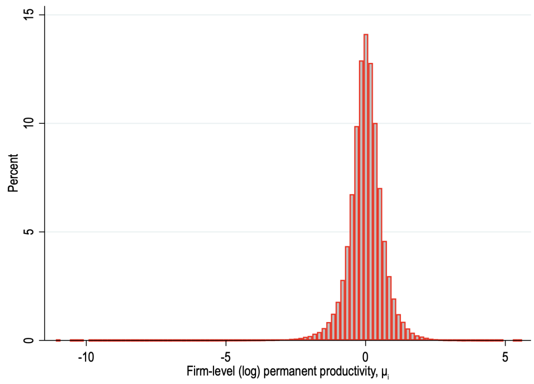
(b) Establishments



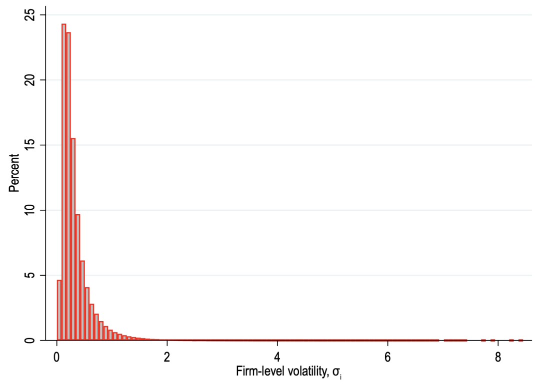
NOTES: This figure reports the empirical distribution of firm size by employees (Panel A), and number of establishments (Panel B).

Figure B.2: Production function estimates—distributions

(a) Firm (log) productivity $\widehat{\mu}_i$



(b) Firm level volatility of output shocks $\widehat{\sigma}_i$



NOTES: This figure reports the empirical distribution of firm productivity estimates $\widehat{\mu}_i$ (Panel A), and the estimate volatilty of output shocks at the firm level $\widehat{\sigma}_i$ (Panel B) across firms in the sample.

Table B3 reports selected statistics for the distributions of firm productivity and firm-level volatility of output shocks. (Log) permanent productivity is largely dispersed across firms: the standard deviation is 65.6%. Output shocks are volatile: the average firm-level volatility in the sample is about 34.7%.

Table B3: Production function estimates—summary statistics

	Mean (1)	S.d. (2)	p10 (3)	Median (4)	p90 (5)	Observations (6)
Firm-level (log) permanent productivity μ_i	0	0.656	-0.691	0.011	0.706	2,170,852
Firm-level volatility of output shocks σ_i	0.347	0.327	0.108	0.243	0.646	2,173,951

Table B4 presents the estimated correlation between output shock volatility and capital misallocation, controlling for 2-digit CAE industry and NUTS-2 region fixed effects. Based on the estimates in columns (2) and (4), transitioning from local markets with an average volatility of 20 percent to those with twice the volatility reduces the correlation between firm productivity and capital holdings by 0.51, while increasing the dispersion of marginal products of capital by 0.33.

Table B4: Output shock volatility and capital misallocation across local markets

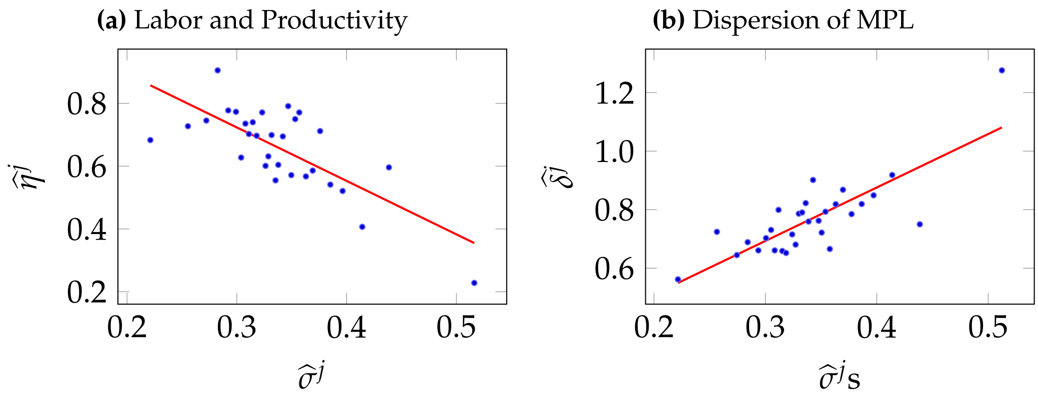
	$\widehat{\eta}^j$		$\widehat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\widehat{\sigma}^j$	-3.999***	-2.530**	2.096***	1.649***
	(0.677)	(0.730)	(0.435)	(0.134)
R^2	0.433	0.809	0.534	0.902
Observations	397	397	397	397
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of capital misallocation on output shock volatility across local markets. In Columns 1 and 2, capital misallocation is measured as the correlation between firm productivity and the average (log) capital holdings of firms within each local market. In Columns 3 and 4, capital misallocation is measured as the standard deviation of the average within-firm marginal product of capital within each local market. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parenthesis) are clustered at the location-industry level. *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$.

B.2 Labor misallocation

Figure B.3 scatters two measures of labor misallocation against the volatility of output shocks across local markets.

Figure B.3: Volatility and labor misallocation across local markets



NOTES: This figure binscatters the relationship between labor misallocation and output shock volatility across local markets. In Panel A, labor misallocation is measured as the correlation between firm productivity and the average (log) employment of firms within each local market. In Panel B, labor misallocation is measured as the standard deviation of the average within-firm marginal product of labor across firms in each local market. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations.

Table B5 presents the estimated correlation between the two measures of labor misallocation and the volatility of output shocks across local markets, controlling for 2-digit CAE industry and NUTS-2 region fixed effects.

Although firm-level volatility exhibits a stronger correlation with capital misallocation, labor allocation follows similar patterns across local markets. Based on estimates from columns (2) and (4), transitioning from local markets with an average volatility of 20% to those with twice the volatility reduces the correlation between firm productivity and employee count by 0.25, while increasing the dispersion of log marginal product of labor by 0.20.

Table B5: Output shock volatility and labor misallocation across local markets

	$\widehat{\eta}^j$		$\widehat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\widehat{\sigma}^j$	-2.017**	-1.251*	1.952***	1.304***
	(0.586)	(0.582)	(0.493)	(0.244)
R^2	0.451	0.744	0.550	0.818
Observations	397	397	397	397
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of labor misallocation on output shock volatility across local markets. In Columns 1 and 2, labor misallocation is measured as the correlation between firm productivity and the average (log) employment of firms within each local market. In Columns 3 and 4, labor misallocation is measured as the standard deviation of the average within-firm marginal product of labor within each local market. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the industry and location level. *$p < 0.1$, $p < 0.05$, *$p < 0.01$.

B.3 Shock persistence

For each local market j, we estimate the following production function:

$$\log y_{it} = \beta^{0j} + \beta^{1j} \log k_{it} + \beta^{2j} \log(w_{it}\ell_{it}) + \beta^{3j} \log m_{it} + \mu_i + \nu_{it}$$

where y_{it} represents revenues (sales) for firm i in year t, k_{it} denotes the capital stock, ℓ_{it} is the number of employees, w_{it} is the average wage paid by the firm, which is a proxy for its employees' average productivity, m_{it} denotes intermediate inputs, and μ_i and μ_t represent firm and year fixed effects, respectively. Finally, ν_{it} is an error term that follows an autoregressive process of order 1; i.e.,

$$\nu_{it} = \rho \nu_{it-1} + \epsilon_{it}$$

where ϵ_{it} is an i.i.d. error term. Following Blundell and Bond (2000), we can express the model using a dynamic (common factor) representation; i.e.,

$$\begin{aligned}\log y_{it} = \beta^{0j} &+ \beta^{1j}\log k_{it} - \rho\beta^{1j}\log k_{it-1} \\ &+ \beta^{2j}\log(w_{it}\ell_{it}) - \rho\beta^{2j}\log(q_{it-1}\ell_{it-1}) \\ &+ \beta^{3j}\log m_{it} - \rho\beta^{3j}\log m_{it-1} \\ &+ \rho\log y_{it-1} + \mu_i(1-\rho) + \epsilon_{it}\end{aligned}$$

Therefore, we can estimate the following unrestricted equation:

$$\begin{aligned}\log y_{it} = \pi_{0j} &+ \pi_{1j}\log k_{it} + \pi_{2j}\log k_{it-1} \\ &+ \pi_{3j}\log(w_{it}\ell_{it}) + \pi_{4j}\log(q_{it-1}\ell_{it-1}) \\ &+ \pi_{5j}\log m_{it} + \pi_{6j}\log m_{it-1} \\ &+ \pi_{7j}\log y_{it-1} + \mu_i^* + \epsilon_{it},\end{aligned}$$

using suitably lagged levels of the variables as instruments after first differences to control for firm-level permanent unobserved heterogeneity. This allows us to obtain an estimate of our coefficients of interest, $\beta^{0j}, \beta^{1j}, \beta^{2j}, \beta^{3j}$, and ρ, as given by

$$\widehat{\beta}^{0j} = \widehat{\pi}_{0j}, \quad \widehat{\beta}^{1j} = \frac{\widehat{\pi}_{1j} + \widehat{\pi}_{2j}}{1 - \widehat{\pi}_{7j}}, \quad \widehat{\beta}^{2j} = \frac{\widehat{\pi}_{3j} + \widehat{\pi}_{4j}}{1 - \widehat{\pi}_{7j}}, \quad \widehat{\beta}^{3j} = \frac{\widehat{\pi}_{5j} + \widehat{\pi}_{6j}}{1 - \widehat{\pi}_{7j}}, \quad \widehat{\rho} = \widehat{\pi}_{7j}.$$

We use $\widehat{\beta}^{0j}, \widehat{\beta}^{1j}, \widehat{\beta}^{2j}$, and $\widehat{\beta}^{3j}$ to compute the following residuals:

$$\widehat{r}_{it} = \log y_{it} - \widehat{\beta}^{0j} - \widehat{\beta}^{1j}\log k_{it} - \widehat{\beta}^{2j}\log(w_{it}\ell_{it}) - \widehat{\beta}^{3j}\log m_{it}.$$

We obtain an estimate of firm productivity $\widehat{\mu}_i$ by taking the time average of $\widehat{r}_{it}$ for every firm i; i.e.,

$$\widehat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T}\widehat{r}_{it}.$$

Finally, notice that $\widehat{r}_{it} - \widehat{\mu}_i = \widehat{v}_{it}$. Hence we can estimate the volatility of output shocks at the firm level, $\widehat{\sigma}_i$, as the standard deviation of the demeaned residual, adjusted by the estimated persistence of the error term process; i.e.,

$$\widehat{\sigma}_i = \sqrt{(1-\widehat{\pi}_{7j}^2)\sum_{t=1}^{T}(\widehat{v}_{it}-\overline{\widehat{v}}_{it})^2}$$

where $\overline{\widehat{v}}_{it} = \frac{1}{T}\sum_{t=1}^{T}\widehat{v}_{it}$.

With $\widehat{\beta}^{1j}$, $\widehat{\beta}^{2j}$, $\widehat{\mu}_i$, and $\widehat{\sigma}_i$ at hand, we construct the measures of average volatility, $\widehat{\sigma}^j$, and capital (labor) misallocation, $\widehat{\eta}^j$ and $\widehat{\delta}^j$, for each local market j, as described in the main text, and proceed to estimate the following regression:

$$\widehat{\omega}^j = \gamma_0 + \gamma_1\widehat{\sigma}^j + \varepsilon^j,$$

where $\widehat{\omega}^j$ is either $\widehat{\gamma}^j$ or $\widehat{\delta}^j$.

Tables B6 and B7 present the estimated correlation between capital misallocation and the volatility of output shocks across local markets, controlling for 2-digit CAE industry and NUTS-2 region fixed effects. Using estimates from columns (2) and (4), an increase in local market volatility from an average of 20% to twice that level reduces the correlation between firm productivity and capital holdings by 0.53, while the dispersion of log marginal product of capital rises by 1.00. Similarly, the correlation between firm productivity and number of employees decreases by 0.31, while the dispersion of log marginal product of labor increases by 0.35. Consistent with the main specification, variations in output shock volatility across local markets exhibit a stronger correlation with capital misallocation than with labor misallocation.

Table B6: Volatility and capital misallocation across local markets with shock persistence

	$\widehat{\eta}^j$		$\widehat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\widehat{\sigma}^j$	-2.826***	-2.672*	1.974***	4.978**
	(0.728)	(1.113)	(0.369)	(1.717)
R^2	0.410	0.575	0.397	0.611
Observations	397	397	389	389
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of capital misallocation on output shock volatility across local markets. In Columns 1 and 2, capital misallocation is measured as the correlation between firm productivity and the average (log) capital holdings of firms within each local market. In Columns 3 and 4, capital misallocation is measured as the standard deviation of the average within-firm marginal product of capital within each local market. Estimates of firm productivity, the average within-firm marginal product of capital, and firm-level output shock volatility—used to construct our measures of capital misallocation and volatility of output shocks at the local market level—are obtained from regression models in which the error term follows an autoregressive process of order one. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions in this table include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the location-industry level. *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$.

Table B7: Volatility and labor misallocation across local markets with shock persistence

	$\widehat{\eta}^j$		$\widehat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\widehat{\sigma}^j$	-1.705**	-1.530**	2.079***	1.726***
	(0.578)	(0.538)	(0.380)	(0.368)
R^2	0.443	0.675	0.627	0.812
Observations	397	397	395	395
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of labor misallocation on output shock volatility across local markets. In Columns 1 and 2, labor misallocation is measured as the correlation between firm productivity and the average (log) employment of firms within each local market. In Columns 3 and 4, labor misallocation is measured as the standard deviation of the average within-firm marginal product of labor within each local market. Estimates of firm productivity, the average within-firm marginal product of labor, and firm-level output shock volatility—used to construct our measures of labor misallocation and volatility of output shocks at the local market level—are obtained from regression models in which the error term follows an autoregressive process of order one. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions in this table include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the location-industry level. *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$.

B.4 Firm exit

We correct for firm-level exit selection following the approach proposed by **?**. We rely on on a full parameterisation of the sample selection mechanism, and assume the conditional expectation of the individual effects in the selection equation, η_i to be a function of the firm-level average of the time-varying covariates used in equation 3, i.e.

$$\eta_i = f(\overline{\log k_{it}}, \overline{\log w_{it}\ell_{it}}, \overline{\log m_{it}}) \tag{7}$$

We choose a fully interacted third-order polinomial for $f(\cdot)$. Therefore we follow Wooldridge (1995) and implement a two-step procedure. In the first step, we estimate a probit on equations (4) and (7) separately for every year t, and obtain a selection correction terms λ_{it}, which is equal to the inverse Mills ratio. In the second step we estimate equation (3) including λ_{it} as a control.

Tables B8 and B9 report the estimated correlation of firm-level volatility and capital misallocation conditional on 2-digit CAE industry and NUTS-2 region fixed effects, and accounting for self-selection in firm exit.

Using estimates from columns (2) and (4), as we move from lo- cal markets with an average volatility of 20% to those with volatility twice as large, the correlation between firm-level capital and productivity reduces by 0.45, while the dispersion of log MPK increases by 0.30.

Correcting for exit selection of firms only marginally affects the magnitude of our estimates: while it amplify the estimated correlations in the unweighted specifications (columns 1 and 3), it hampers them in the weighted ones (columns 2 and 4), suggesting the selection introduces a larger bias as we move towards markets with a higher number of firms.

Table B8: Volatility and capital misallocation across local markets —Heckman correction for firm exit

	$\widehat{\eta}^j$		$\widehat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\widehat{\sigma}^j$	-3.891***	-2.288**	2.496***	1.547***
	(0.707)	(0.646)	(0.501)	(0.103)
R^2	0.434	0.786	0.533	0.883
Observations	397	397	389	389
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of capital misallocation on output shock volatility across local markets. In Columns 1 and 2, capital misallocation is measured as the correlation between firm productivity and the average (log) capital holdings of firms within each local market. In Columns 3 and 4, capital misallocation is measured as the standard deviation of the average within-firm marginal product of capital within each local market. Estimates of firm productivity, the average within-firm marginal product of capital, and firm-level output shock volatility—used to construct our measures of capital misallocation and volatility of output shocks at the local market level—are obtained from regression models that account for firm exit as in Wooldridge (1995). Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions in this table include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the location-industry level. *$p < 0.1$, **$p < 0.05$, ***$p < 0.01$.

Table B9: Volatility and labor misallocation across local markets — Heckman correction for exit

	$\widehat{\eta}^j$		$\widehat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\widehat{\sigma}^j$	-2.089**	-1.279*	2.066***	1.263***
	(0.614)	(0.529)	(0.490)	(0.226)
R^2	0.455	0.721	0.557	0.822
Observations	397	397	395	395
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of labor misallocation on output shock volatility across local markets. In Columns 1 and 2, labor misallocation is measured as the correlation between firm productivity and the average (log) employment of firms within each local market. In Columns 3 and 4, labor misallocation is measured as the standard deviation of the average within-firm marginal product of labor within each local market. Estimates of firm productivity, the average within-firm marginal product of labor, and firm-level output shock volatility—used to construct our measures of labor misallocation and volatility of output shocks at the local market level—are obtained from regression models that account for firm exit as in Wooldridge (1995). Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions in this table include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the location-industry level. $^*p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

C Counterfactual appendix

C.1 Alternative elasticity of capital

Table C1 reports the estimated coefficient of relative risk aversion (RRA) for different values of capital elasticity, along with the corresponding output per unit of capital and income gains from the counterfactual exercises under under different capital share values.

Table C1: Sensitivity to alternative capital shares

Capital share α	0.5	0.6	0.7	0.8	0.9
Estimated γ	3.507	3.715	4.024	4.235	4.717
Output per capital gains %					
from no uncertainty	4.121	20.11	54.06	87.95	187.0
from complete markets	10.04	27.15	63.63	98.76	204.8
Income gains %					
from no uncertainty	29.94	47.05	85.37	126.0	251.9
from complete markets	38.86	57.22	96.85	138.3	270.6

NOTES: This table reports output per unit of capital and aggregate income gains (in %) from two counterfactual scenarios: one with no idiosyncratic production uncertainty and another with complete markets, for alternative economies that differ in the capital share α. In each of these economies, the coefficient of relative risk aversion γ is re-estimated to match the observed correlation between productivity and capital holdings across firms.

C.2 Alternative technology

Table C2 shows the calibrated parameters for the model that includes labor as a factor of production.

Table C2: Parameters

Parameters	Description	Value	Source/Target
	A. Parameters calibrated		
α	Output elasticity of labor	0.7	Standard
η	Span on control	0.9	Standard
σ	Volatility of shocks	0.347	Data
(κ, θ)	Distribution of permanent productivity	(1.404,1.060)	Data
	B. Parameters estimated		
γ	Relative risk aversion	1.631	corr. $[\log k, \log z] = 0.494$

NOTES: This table reports the values and source of the calibrated and estimated parameters for a version of the model that includes labor as a factor of production.

Table C3 shows the counterfactual results when using the calibrated model that includes labor as a factor of production.

Table C3: The aggregate cost of incomplete markets

		Counterfactual	
	Baseline	No uncertainty	Complete markets
	(1)	(2)	(3)
Volatility, σ	0.347	0	0.347
corr$[\log z, \log k]$	0.494	1.000	1.000
$\omega[\log z, \log k]$	0.934	3.700	3.700
sd$[\log k]$	2.603	6.816	6.816
Rental rate, r	0.743	24.11	25.61
Output per capital	1.000	1.343	1.348
Income	1.000	1.376	1.539

NOTES: This table reports selected outcomes for the baseline economy (column 1), a counterfactual economy with no idiosyncratic production uncertainty (column 2), and a counterfactual economy with complete insurance markets (column 3), obtained from a version of the model that includes labor as a factor of production.