

Macroeconomics of “Large Firms”

Lecture 1: The basics of firm dynamics

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CEMFI, PhD short course

- Hopenhayn H. 1992. “Entry, Exit, and Firm Dynamics in Long Run Equilibrium”. *Econometrica*. Vol. 60, No. 5, pp. 1127-1150
 - Workhorse model of industry dynamics
 - Partial equilibrium
 - Focus on steady-state: wages and prices are constant
 - Individual dynamics: firms enter, grow, decline, and exit
 - Competitive firms, no strategic interactions

- Time is discrete
- Wage is the model numeraire ($w = 1$)
- Output price p endogenous
- Endogenous measure of heterogeneous firms
 - DRS production function
 - Perfect competition in product and labor markets
 - No aggregate risk
 - Idiosyncratic risk: firm productivity follows a Markov process
 - Entry-exit dynamics
 - Fixed cost to enter
 - Fixed cost to operate each period
- Partial equilibrium: exogenous industry demand

- Firms differ in productivity z
- Firm-level output

$$f(z, n) = zn^\alpha \quad \alpha \in (0, 1)$$

- Static firm-level profits

$$\pi(z; p) = \max_{n \geq 0} pf(z, n) - n - pc_o$$

where c_o denotes per-period operating costs.

- Let $n(z; p)$ denote optimal employment

$$n(z; p) = (p\alpha z)^{\frac{1}{1-\alpha}}$$

- Let $y(z; p) = f(z, n(z; p))$ denote associated output

Problem of the incumbents

- Incumbents enter the period with states z_{-1}
- Exit decision:
 - if stay, firms draw new productivity level $z \sim \Gamma(z|z_{-1})$
- Employment decision (conditional on staying)
 - choose new employment level n conditional on z
 - pay operating costs c_o and produce $y(z; p)^*$

Problem of the incumbents

- $V(z; p)$: value function for firm in states z and aggregate price p

$$V(z, p) = \pi(z; p) + \frac{1}{1+r} \tilde{v}(z)$$

where

$$\tilde{v}(z) = \max \left\{ 0, \sum_{z'} V(z'; p) \Gamma(z'|z) \right\}$$

- Solution to this problem is policy function for optimal exit: $\mathbf{1}^x(z; p)$
- An exit productivity threshold z^* exists such that:

$$\sum_{z'} V(z'; p) \Gamma(z'|z^*) = 0$$

Problem of the entrants

- Potential entrants are ex-ante identical
- New entrants $M \geq 0$ pay c_e and enter
- Draw productivity level z from $\Gamma^e(z)$ (ergodic distribution obtained from $\Gamma(z|z_{-1})$)
- Start producing next period
- Free entry condition:

$$v^e(p) = \frac{1}{1+r} \sum_z V(z;p) \Gamma^e(z) \leq c_e$$

with equality if $M > 0$.

Evolution of distribution

- Let $\mu_t(z; p)$ be the measure of firms over individual state z when the goods price is p at time t
- Evolution of distribution over time:

$$\mu_{t+1}(z'; p_t) = T_t(\mu_t(z; p_t), M_t, p_t)$$

where

$$T_t(\mu_t(z; p_t), M_t, p_t) = \sum_z \psi_t(z'|z; p_t) d\mu_t(z; p_t) + M_t \Gamma^e(z)$$

and

$$\psi_t(z'|z; p_t) = \Gamma(z'|z)[\mathbf{1}_t^x(z; p_t) = 0]$$

denotes the transition function from the states z to z'

Industry demand and supply

- Industry demand curve exogenous: $Y^d(p) = \bar{Y}$
- Industry supply curve endogenous:

$$Y^s(p) = \sum_z y(z; p)\mu(z; p)$$

- Inelastic labor supply function: $N^d = 1$
- Industry labor demand, endogenous,

$$L^s(p) = \sum_z n(z; p)\mu(z; p) + Mc_e$$

A recursive stationary equilibrium for this economy is characterized by a measure of entrants M^* , a distribution of incumbent firms $\mu^*(z; p)$, a price p^* and a productivity threshold z^* such that the following four conditions hold:

- **Optimality:** $\forall z < z^*, \mathbf{1}^x(z) = 1$
- **Free-entry** $v^e = c_e$;
- **Goods market clearing:** $Y^s(p^*) = \bar{Y}$;
- **Aggregate consistency:** $\mu^*(z; p^*) = T(\mu^*(z; p^*), M^*, p^*)$.

Computation

- **Step 1:** guess a price p_0
- **Step 2:** solve for the value of the incumbent, $v(z; p^0)$. A solution to this problem implies an optimal exit rule, $\mathbf{1}^x(z; p^0)$
- **Step 3:** compute the value of entry, v^e and check if free-entry condition is satisfied:
 - if no, make a new guess p_1 and go back to step 2 till convergence
 - if yes, store $p^* = p_0$
- **Step 4:** given p^* , solve for the stationary distribution of incumbents $\mu(z; p^*)$ and measure of entrants M^e
 - guess measure of entrants M_0^e
 - calculate the stationary distribution $\mu(z; p^*, M_0^e)$
 - given $\mu(z; p^*, M_0^e)$, compute total industry supply and check the if market clearing condition is satisfied:
 - if no, make a new guess M_1^e and go back to step 4 till convergence
 - if yes, store $M^{e*} = M^e$

Computation

- Steps 4 can be speeded up exploiting the property of linear homogeneity of the function $\mu(\cdot)$ with respect to M
- Re-write in matrix notation the function μ , i.e.

$$\mu = \Psi\mu + M\Gamma$$

- Solving for μ

$$\mu = M(\mathbf{I} - \Psi)^{-1}\Gamma$$

- No need to use simulations to find stationary distribution μ
- No need to iterate on M . Solve for μ imposing $M = 1$
- Compute equilibrium M using market clearing condition

$$M^* = \frac{\bar{Y}}{\sum_z y(z; p^*)\mu(z; p^*)}$$

Computation

- What if corner solution, i.e. $M = 0$? No entry/exit dynamics
- The stationary distribution of firms just given by stationary distribution of Markov chain, i.e.

$$\mu(z; p) = \sum_{z'} \Gamma(z'|z) = \mu(z)$$

- Stationary distribution independent of p - no need to use free entry condition!
- Solve for p^* using the market clearing condition, i.e.

$$Y^s(p^*) = \bar{Y}$$

Productivity state-space:

- AR(1) process in logs:

$$\log z' = \mu + \rho \log z' + \epsilon' \quad \epsilon' \sim \mathcal{N}(0, \sigma^2)$$

- Markov chain approximation using Tauchen method on 201 nodes

Baseline parameters (period model = 1 year)

$$r = 0.01, \alpha = 0.78, \mu = 0, \rho = 0.995, \sigma = 0.14$$

$$c_o = 10, c_e = 80, Y^d = 500$$

Comparative statics

What happens if we increase the entry cost c_e ?

Entry costs: c_e/Y	0.36	0.41	0.46	0.50
Price	1	1.162	1.247	1.316
Output	1	1.448	1.728	1.989
Profits	1	1.453	1.732	1.993
# firms	1	0.676	0.5620	0.485
Firm Size	14.429	24.295	31.081	37.799
Exit rate	47.711	44.422	43.072	42.191
Productivity threshold	1	0.741	0.652	0.598

Increase in entry costs c_e :

- decreases exit productivity threshold — **less selection!**
- decreases number of firms and exit rate
- increases prices

Two contrasting effects on firm-size

- *price/value effects*: higher c_e increases prices, which leads to higher output and employment
- *selection effects*: higher c_e reduces productivity thresholds, keeping low-productivity firms in the industry

If the density of firms near the exit point is very small, the first effect will dominate and the average value and profits increase with c_e .

Comparative statics

What happens if we increase the operating cost c_o ?

Operating costs: c_o/Y	0.04	0.08	0.13	0.20
Price	1	1.027	1.086	1.158
Output	1	1.165	1.532	1.8160
Profits	1	1.125	1.369	1.526
# firms	1	0.881	0.696	0.547
Firm size	14.429	17.270	24.010	33.337
Exit rate	47.711	51.761	57.402	61.217
Productivity threshold	1	1.408	2.159	2.791

Increase in operating costs, c_o :

- increases exit productivity threshold — **more selection!**
- decreases number of firms
- increases entry/exit rate
- increases prices

As the c_o increases, the profit function changes, pushing profits down for every state. Productivity thresholds need to increase to maintain free-entry.

Extension: Adjustment Costs

Firm dynamics with adjustment costs

- Hopenhayn H. and Rogerson R. 1993. “Job Turnover and Policy Evaluation: A General Equilibrium Analysis”. *Journal of Political Economy*. Vol.101, N.5, pp. 915-938
 - General equilibrium version of Hopenhayn (1992)
 - Non-convex adjustment cost (firing costs)
 - No aggregate shocks
 - Optimal employment policy characterized by *inaction* region
 - *Misallocation* of resources across heterogeneous plants

- Large volume of job creation/destruction at firm-level
- Policies that make more costly to adjust employment level, i.e.
 - legislated severance payments
 - advance notice
 - plant closing legislation
- What are the effects of such policies on
 - employment
 - aggregate output
 - productivity
- Can labor market regulations explain heterogeneity in labor market performance across countries?

- Time is discrete
- Wage is the model numeraire ($w = 1$)
- Output price p endogenous
- Representative household
 - Consumption/labor supply decision
 - No savings
- Endogenous measure of heterogeneous firms
 - Perfect competition in product and labor markets
 - Time-varying productivity
 - Entry-exit dynamics
 - Employment adjustment costs (firing tax)
 - Free-entry

- Utility function of consumption C and labor supply N

$$U(C, N) = \log C - AN$$

where A denotes disutility from supplying labor.

- Discount factor: $\beta = 1/1 + r$
- Budget constraint:

$$pC \leq N + \Pi \quad (w = 1 \text{ is the numeraire})$$

where Π are aggregate profits, re-distributed to HH lump-sum

- Problem of the HH:

$$\begin{aligned}
 U(C, N) &= \max_{C, N} \log C - AN \\
 \text{s.t. } & pC \leq N + \Pi
 \end{aligned}$$

- First order conditions (at the interior) imply:

$$\begin{aligned}
 C &= \frac{1}{Ap} \quad (: \text{consumption demand}) \\
 N^s &= \frac{1}{A} - \Pi \quad (: \text{labor supply})
 \end{aligned}$$

- Firms differ in productivity z and employment n
- Firm-level output

$$f(z, n) = zn^\alpha \quad \alpha \in (0, 1)$$

- Static firm-level profits

$$\pi(z, n, n_{-1}) = pf(z, n) - n - pc_o - g(n, n_{-1})$$

where c_o denotes per-period operating costs.

- Adjustment costs (expressed in units of labor)

$$g(n, n_{-1}) = \tau \max\{0, n_{-1} - n\}$$

Problem of the incumbents

- Incumbents enter the period with states (z_{-1}, n_{-1})
- Exit decision:
 - if exit, firms pay $g(0, n_{-1})$
 - if stay, firms draw new productivity level $z \sim \Gamma(z|z_{-1})$
- Employment decision (conditional on staying)
 - choose new employment level n conditional on (z, n_{-1})
 - expanding firms ($n > n_{-1}$) subject to no cost
 - shrinking firms ($n < n_{-1}$) subject to firing costs
 - pay operating costs pc_o and produce $f(z, n)$

Problem of the incumbents

- $V(z, n_{-1}; p)$: value function for firm in states (z, n_{-1}) and aggregate price p

$$V(z, n_{-1}; p) = \max_{n \geq 0} \pi(z, n, n_{-1}) + \frac{1}{1+r} \tilde{v}(z, n)$$

where

$$\tilde{v}(z, n) = \max \left\{ -g(0, n), \sum_{z'} V(z', n; p) \Gamma(z'|z) \right\}$$

- Solution to this problem:
 - Policy function for optimal employment policy: $n = g_n(z, n_{-1}; p)$
 - Policy policy for optimal exit: $\mathbf{1}^x(z, n_{-1}; p)$

Problem of the entrants

- Potential entrants are ex-ante identical
- New entrants $M \geq 0$ pay c_e and enter
- Draw productivity level z from $\Gamma^e(z)$ (ergodic distribution obtained from $\Gamma(z|z_{-1})$)
- Hire n workers and produce
- Free entry condition:

$$v^e(p) = \frac{1}{1+r} \sum_z V(z, 0; p) \Gamma^e(z) \leq c_e$$

with equality if $M > 0$.

Stationary distributions

- Let $\mu(z, n; p)$ be the measure of firms over individual states z and n when the goods price is p
- Solution of the following linear system:

$$\mu(z', n'; p) = T(\mu(z, n; p), M, p)$$

where

$$T(\mu(z, n; p), M, p) = \sum_z \int_n \psi(z', n'|z, n; p) d\mu(z, n; p) \\ + M\Gamma^e(z)\mathbf{1}[g_n(0, z; p) = n']$$

and

$$\psi(z', n'|z, n; p) = \mathbf{1}[g_n(n, z'; p) = n']\Gamma(z'|z)[\mathbf{1}^x(z, n) = 0]$$

denotes the transition function from the states (z, n) to z', n'

- Aggregate output:

$$Y = \sum_z \int_{n_{-1}} [f(z, g_n(z, n_{-1}) - c_f] d\mu(z, n_{-1}; p) \\ + M \sum_z f(z, g_n(z, 0)) \Gamma^e(z)$$

- Labor demand:

$$N^d = \sum_z \int_{n_{-1}} g_n(z, n_{-1}) d\mu(z, n_{-1}; p) + M \sum_z g_n(z, 0) \Gamma^e(z) + M c^e$$

- Aggregate profits:

$$\Pi = pY - N^d - p c_o$$

A recursive stationary equilibrium for this economy is characterized by a measure of entrants M^* , a distribution of incumbent firms $\mu^*(z, n; p)$, and a price p^* such that the following three conditions hold:

- **Free-entry** $v^e = c_e$;
- **Labor market clearing:** $N^s = N^d$;
- **Aggregate consistency:** $\mu^*(z, n; p^*) = T(\mu^*(z, n; p^*), M^*, p^*)$.

Computation

- **Step 1:** guess a price p_0
- **Step 2:** solve for the value of the incumbent, $v(z, n; p^0)$
- **Step 3:** compute the value of entry, v^e and check if free-entry condition is satisfied:
 - if no, make a new guess p_1 and go back to step 2 till convergence
 - if yes, store $p^* = p_0$
- **Step 4:** given p^* , solve for the stationary distribution of incumbents $\mu(z, n; p^*)$ associated with $M = 1$
 - Exploit linear homogeneity of T
$$M\mu^*(z, n; p^*) = MT(\mu^*(z, n; p^*), M, Mp^*)$$
 - Fixed point/MC simulation
- **Step 5:** given $\mu^*(z, n; p^*)$, find M^* that makes the labor market clear

Productivity state-space:

- AR(1) process in logs:

$$\log z' = \mu + \rho \log z + \epsilon' \quad \epsilon' \sim \mathcal{N}(0, \sigma^2)$$

- Markov chain approximation using Tauchen method on 51 nodes

Employment state-space:

- maximum number of employees: 3000
- 500 points (400 points between 1 and 200 employees)

Baseline parameters (period model = 5 years)

$$r = 0.25, \alpha = 0.64, \mu = 0.25, \rho = 93, \sigma = 0.17$$

$$c_o = 20, c_e = 40, A = 0.45, \tau = 0$$

Employment policy function

- Without adjustment costs ($\tau = 0$), optimal employment decision given by:

$$n' = (\alpha pz)^{\frac{1}{1-\alpha}}$$

- With adjustment costs ($\tau > 0$), employment decision characterized by two reservation thresholds, $z_F(n)$ and $z_H(n)$, such that:

$$\begin{aligned}n' &= n_F(z) & \text{if } z < z_F(n) \\n' &= n_{-1} & \text{if } z \in [z_F(n), z_H(n)] \\n' &= n_H(z) & \text{if } z > z_H(n)\end{aligned}$$

- Inaction region wider with higher τ

Inaction regions: $\tau = 0$

Inaction regions: $\tau = 0.1$

Inaction regions: $\tau = 0.2$

Inaction regions: $\tau = 0.3$

Misallocation

- Without adjustment costs ($\tau = 0$), MPL equalized across firms:

$$\frac{\partial f(z, n)}{\partial n} = \frac{1}{p}$$

- With adjustment costs ($\tau > 0$), MPL is the solution of two *necessary but not sufficient* conditions:

$$p \frac{\partial f(z, n)}{\partial n} + \frac{1}{1+r} \frac{\partial \tilde{v}(z, n)}{\partial n} = 1 \quad \text{if } n' > n$$
$$p \frac{\partial f(z, n)}{\partial n} + \frac{1}{1+r} \frac{\partial \tilde{v}(z, n)}{\partial n} = \left[1 + \underbrace{\frac{\partial g(n, n_{-1})}{\partial n}}_{\tau} \right] \quad \text{if } n' < n$$

Distribution of MPL

Marginal product of labor: $\frac{\partial f(z,n)}{\partial n}$

Firing cost: τ	Dispersion	Percentiles			
	St.Dev.	20th	40th	60th	80th
0	0	1	1	1	1
0.1	0.0439	0.9485	0.9546	1.0171	1.0349
0.2	0.0720	0.9097	0.9349	1.0156	1.0568
0.3	0.0911	0.8735	0.9199	1.0134	1.0715

Firing costs increase the dispersion of the MPL

Counterfactual outcomes

Firing cost: τ	0	0.1	0.2	0.3
Price	1	1.0085	1.0145	1.0193
Consumption (output)	1	0.9915	0.9856	0.9810
Employment	0.6000	0.5988	0.5986	0.5988
Profits	1	1.1561	1.2758	1.4127
# firms	1	1.1350	1.2432	1.3702
Firm size	20.055	20.333	20.655	21.152
Labor productivity	1	0.9914	0.9818	0.9718
JC rate	0.1879	0.1403	0.1108	0.0953

Firing costs reduce aggregate productivity and aggregate output

Extension: Policy Distortions

Firm dynamics with policy distortions

- Restuccia D. and Rogerson R. 2008. “Policy Distortions and Aggregate Productivity with Heterogeneous Plants.” Review of Economic Dynamics. Vol 11, No. 4, pp. 707-72
 - Neoclassical model of growth
 - Firms heterogeneity in productivity
 - Exogenous idiosyncratic distortions
 - General equilibrium
 - Productivity losses of *misallocation*

- Large cross-country differences in income p.c.
- Evidence of heterogeneous distortions across countries:
 - financial markets (e.g., Parente and Prescott 1999)
 - labour regulation (e.g., Lagos 2006)
 - gov. subsidies (e.g., Guner et al. 2008)
- Assess quantitative significance of firm-level distortions on output, productivity and employment
 - misallocation across productive units
- Reduced form representation of idiosyncratic distortions to producer prices: tax/subsidy τ on output
- Results: heterogeneity in prices faced by producers can lead to decrease in TFP and output of up 50%

- Time is discrete, focus on steady-state
- Output price is the model numeraire ($p = 1$)
- Wage w and interest rate r determined endogenously
- Representative household
 - Consumption/savings decision
 - Inelastic labor supply
- Endogenous measure of heterogeneous firms
 - Perfect competition in product and labor markets
 - Innate differences in firm-level productivity
 - Entry-exit dynamics
 - Free-entry

- Utility function of consumption C_t

$$U = \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \beta \in (0, 1)$$

where β is the discount factor

- Budget constraint:

$$C_t + I_t \leq w_t + r_t K_t + \Pi_t - T_t$$

where Π_t are aggregate profits and T_t are net taxes

- Capital depreciates at rate δ

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- Problem of the HH:

$$\begin{aligned} \mathcal{U} &= \max_{C_t, K_{t+1}} \sum_{t=0}^{\infty} \beta^t U(C_t) \\ \text{s.t. } C_t + K_{t+1} &\leq w_t + (1 + r_t - \delta)K_t + \Pi_t - T_t \\ K_0 &> 0 \quad \text{given} \end{aligned}$$

- First order condition (at the interior) imply:

$$U'(C_t) = \beta U'(C_{t+1})(1 + r_t - \delta)$$

- In steady-state, $C_t = C_{t+1}$ and

$$r^* = \frac{1}{\beta} - 1 + \delta$$

- Firms differ in productivity, z and distortion, τ
- Firm-level output

$$f(z, n, k) = zn^\alpha k^\gamma \quad \alpha, \gamma \in (0, 1)$$

where $\alpha + \gamma < 1$: decreasing return to scale.

- Static firm-level profits

$$\pi(z, \tau; w, r) = \max_{n \geq 0, k \geq 0} (1 - \tau)f(z, n, k) - wn - rk - c_o$$

where c_o denotes per-period operating costs.

- Let $n(z, \tau; w, r)$ and $k(z, \tau; w, r)$ be the optimal employment and capital demand function

Value of the incumbents

- $V(z, \tau; w, r)$: value function for firm in states (z, τ) and prices (w, r)

$$V(z, \tau; w, r) = \pi(z, \tau; w, r) + \frac{(1 - \lambda)}{1 + r - \delta} V(z, \tau; w, r)$$

where λ is an exogenous probability of exit

- Expected discounted value of per-period profits:

$$V(z, \tau; w, r) = \frac{\pi(z, \tau; w, r)}{1 - \frac{(1-\lambda)}{1+r-\delta}}$$

Problem of the entrants

- Large measure of identical entrants pay c_e and draw productivity z and distortion τ from a joint distribution

$$\Gamma(z, \tau) = P(\tau|z)H(z)$$

- Entry decision: $V^e(z, \tau; w, r) = \max\{0, V(z, \tau; w, r)\}$. Solution to this problem is a policy for optimal entry: $\mathbf{1}^e(z, \tau; w, r)$
- Measure of entry: $M \geq 0$
- Free entry condition:

$$v^e(w, r) = \sum_z \sum_\tau V^e(z, \tau; w, r)\Gamma(z, \tau) \leq c_e$$

with equality if $M > 0$.

Evolution of distribution

- Let $\mu_t(z, \tau; w_t, r_t)$ be the measure of firms over individual state (z, τ) when wage and interest rate rate (w_t, r_t) at time t
- Evolution of distribution over time:

$$\mu_{t+1}(z, \tau; w_{t+1}, r_{t+1}) = T_t(\mu_t(z, \tau; w_t, r_t), M_t, w_t, r_t)$$

where

$$T_t(\mu_t(z, \tau; w_t, r_t), M_t, w_t, r_t), M_t, p_t) = \\ (1 - \lambda)\mu_t(z, \tau; w_t, r_t) + M\mathbf{1}^e(z, \tau; w_t, r_t)P(\tau|z)H(z)$$

- Endogenous labor and capital demand

$$L^d = \sum_{z,\tau} n(z, \tau; w, r) d\mu(z, \tau; w, r)$$

$$K^d = \sum_{z,\tau} k(z, \tau; w, r) d\mu(z, \tau; w, r)$$

- Aggregate output

$$Y^s = \sum_{z,\tau} [zn(z, \tau; w, r)^\alpha k(z, \tau; w, r)^\gamma - c_o] d\mu(z, \tau; w, r)$$

- Goods demand: $Y^d = C + \delta K + Mc_e$
- Aggregate taxes: $\sum_{z,\tau} \tau f(z, n(z, \tau), k(z, \tau)) d\mu(z, \tau; w, r)$

Equilibrium

A steady-state competitive equilibrium is a wage rate w , a rental rate r , a lump-sum tax T , a policy function $\mathbf{1}^e(z, \tau)$, a distribution $\mu(z, \tau)$, and a mass of entry M such that:

- **Consumers optimality:** $r = 1/\beta - 1 + \delta$
- **Firms optimality:** $\mathbf{1}^e(z, \tau)$ solves the problem of the entrants
- **Free-entry:** $v^e(w, r) = c_e$
- **Markets clearing:** $L^d = 1 \quad K^d = \bar{K} \quad Y^s = Y^d$
- **Balanced budget:**

$$T = \sum_{z, \tau} \tau f(z, n(z, \tau), k(z, \tau)) d\mu(z, \tau; w, r)$$

- **Time-invariance:** $\mu(z, \tau; w, r) = M \frac{\mathbf{1}^e(z, \tau; w, r) P(\tau|z) H(z)}{\lambda}$

Computation

- **Step 0:** fix interest rate to steady-state value: r^*
- **Step 1:** guess a wage rate w_0
- **Step 2:** solve for the value of the incumbent, $v(z, \tau; w_0, r^*)$
- **Step 3:** solve the problem of the potential entrant, $1^e(z, \tau; w_0, r^*)$
- **Step 4:** compute the value of entry, $v^e(w_0, r^*)$ and check if free-entry condition is satisfied:
 - if no, make a new guess w_1 and go back to step 2 till convergence
 - if yes, store $w^* = w_0$
- **Step 5:** compute the invariant distribution of plants (normalized $M = 1$)
- **Step 6:** find mass of firms such that the labor market clears

Calibration

- Distortions: U.S treated as un-distorted benchmark, $\tau = 0$
- Period model: 1 Year $\implies r = 0.04, \beta = 0.96$
- Decreasing return to scale: $\alpha + \gamma = 0.85$. Two-third assigned to labor return $\implies \alpha = 0.57, \gamma = 0.28$
- Depreciation to match capital/output ratio of 2.3 $\implies \delta = 0.08$
- No operating costs, $c_o = 0$.
- Entry cost normalized to 1, $c_e = 1$ (identification issues: changes to c_e isomorphic to changes in establishment-level productivity)

Productivity state-space:

- 100 nodes
- Lowest productivity normalized to 1
- Range of values chosen to match the range of employment across establishments

$$\frac{n_i}{n_j} = \left(\frac{z_i}{z_j} \right)^{\frac{1}{1-\gamma-\alpha}}$$

In US data, biggest firms are 10000 times larger than smallest.
Given α and γ , largest productivity equal to 3.98

- Distribution of productivity $H(z)$ to match observed firm-size distribution

Calibration

Benchmark economy

- Two main experiments:
 - Uncorrelated distortions, τ independent of z
 - half producers taxed, half subsidized
 - resources flow from taxed to subsidized, but no systematic effect across productivity classes
 - Correlated distortions, either positively or negatively
 - lowest half producers subsidized, top half taxed
 - systematic reallocation across productivity classes, not just within productivity class
- Size of the subsidy so that the net effect on steady-state capital accumulation is zero.

Uncorrelated distortions

Variable	Description	τ_t			
		0.1	0.2	0.3	0.4
Y	Relative Output	0.98	0.96	0.93	0.92
TFP	Relative TFP	0.98	0.96	0.93	0.92
E	Relative employment	1.00	1.00	1.00	1.00
Y_s/Y	Output of subsidized	0.72	0.85	0.93	0.97
S/Y	Subsidy share of output	0.05	0.08	0.09	0.10
τ_s	Subsidy rate	0.06	0.09	0.10	0.11

- comparatively small effect on TFP and output, no effect on E
- subsidies to undo effects on capital accumulation are smaller
- as tax increases, larger TFP-effect and larger subsidies

Correlated distortions

Variable	Description	τ_t			
		0.1	0.2	0.3	0.4
Y	Relative Output	0.90	0.80	0.73	0.69
TFP	Relative TFP	0.90	0.80	0.73	0.69
E	Relative employment	1.00	1.00	1.00	1.00
Y_s/Y	Output of subsidized	0.42	0.67	0.83	0.92
S/Y	Subsidy share of output	0.17	0.32	0.43	0.49
τ_s	Subsidy rate	0.40	0.48	0.52	0.53

- qualitatively similar to uncorrelated case
- larger negative effect on TFP and output
- also more costly to finance (higher subsidies)

- Non-constant capital stock
 - taxing all but some exempt producers at 40% rate and no subsidy
 - lower capital stock, wages and entry rate also fall in proportion
 - amplifies effects on TFP
- Taxes on capital and labor