Chapter 6

Hypothesis testing

Exercise 1 The null and alternative hypotheses are given by

$$H_0 : \mu = 1500 \quad \text{versus} \quad H_0 : \mu \neq 1500$$

We are not told that earnings are normally distributed, but as n is large ($n > 30$), the central limit theorem applies and we can assume that the sample mean $\bar{x}$ is normally distributed. The test statistic is given by

$$T = \frac{\bar{x} - 1500}{\frac{s}{\sqrt{n}}} = \frac{1262 - 1500}{\frac{432}{\sqrt{150}}} = -6.747$$

The relevant critical value for a one-sided hypothesis testing with $n = 150$ observations and one parameter to estimate at 5% significance level is $-t_{n-1,\alpha} = -t_{149,0.05} = -1.655$. We reject the null hypothesis if $T < \text{cv}_\alpha$. Here, $T = -6.747 < -1.655$ thus we reject the null hypothesis. The sample provides evidence that the population mean earnings are less than 1500GBP.

Exercise 2

- The test statistic T has a sampling distribution. It is therefore possible that we do not always make the right decision from our hypothesis test. In fact, there are two possible types of error we can make. A Type I error occurs if we reject the null hypothesis H_0 when it is true (and therefore shouldn't be rejected). A Type II error occurs if we fail to reject H_0 when it is false (and therefore should be rejected).

- Power is the probability of rejecting the null hypothesis, H_0, given that it is false, i.e. the probability of correctly rejecting the null. This probability is equal to $P(T > \text{cv}_\alpha|\text{H}_1)$ for a one sided test and to $P(|T| > \text{cv}_{\frac{\alpha}{2}}|\text{H}_1)$ for a two sided test. If as n increases, the power approaches 1, we call the test consistent. This is a desirable property for a test to possess.

Exercise 3 The null and alternative hypotheses are given by

$$\mathrm{H}_0 : \mu = 50 \quad \text{versus} \quad \mathrm{H}_0 : \mu \neq 50$$

The test statistic is

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{43.9 - 50}{\frac{1.25}{\sqrt{20}}} = -\frac{-0.7}{0.280} = -2.504$$

With $n = 20$ observations and one parameter to estimate, the critical value for a two-sided test at 5% is given by

$$t_{\frac{0.05}{2},19} = t_{0.025,19} = 2.903$$

We reject the null hypothesis if $|T| > \mathrm{cv}_\alpha$. Since $|T| = 2.504 > 2.093$, we reject the null hypothesis. We have evidence to suggest the population mean number of matches in a box is not 50.

Exercise 4 The null and alternative hypotheses are given by

$$\mathrm{H}_0 : \mu = 50 \quad \text{versus} \quad \mathrm{H}_0 : \mu > 50$$

The test statistic is

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{50.4 - 50}{\frac{1.25}{\sqrt{20}}} = \frac{0.4}{0.280} = 1.429$$

With $n = 20$ observations and one parameter to estimate, the critical value for a one-sided test at 5% is given by

$$t_{0.05,19} = 1.729$$

We reject the null hypothesis if $T > \mathrm{cv}_\alpha$. Since $T = 1.429 > 1.729$, we fail to reject the null hypothesis. Evidence suggests that the population mean number of matches in a box is equal to 50.

Exercise 5 The null and alternative hypotheses are given by

$$\mathrm{H}_0 : \mu = 55 \quad \text{versus} \quad \mathrm{H}_0 : \mu < 55$$

The test statistic is

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{52 - 50}{\frac{1.5}{\sqrt{16}}} = \frac{-3}{0.375} = -8$$

With $n = 16$ observations and one parameter to estimate, the critical value for a one-sided test at 5% is given by

$$t_{0.05,15} = -1.753$$

We reject the null hypothesis if $T < -\text{cv}_\alpha$. Since $T = -8 < -1.753$, we reject the null hypothesis. There is evidence to suggest that the population mean number of matches in a box is less than 55.

Exercise 6 The null and alternative hypotheses are given by

$$\text{H}_0 : \mu_x - \mu_y = 0 \quad \text{versus} \quad \text{H}_0 : \mu_x - \mu_y \neq 0$$

The test statistic is

$$T = \frac{\bar{x} - \bar{y} - \mu_0}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}} = \frac{420 - 408 - 0}{\sqrt{\frac{12.5^2}{30} + \frac{18^2}{30}}} = \frac{12}{\sqrt{16.008}} = 2.999$$

Using the conservative approach that degrees of freedom v are equal to the smaller of $n_x - 1$ or $n_y - 1$, i.e. $v = 29$, the critical value for a two-sided test at 5% is given by

$$t_{\frac{\alpha}{2},v} t_{\frac{0.05}{2},29} = 2.045$$

We reject the null hypothesis if $|T| > \text{cv}_\alpha$. Since $|T| = 2.999 > 2.045$ we reject the null hypothesis that the number of cars produced in the two factories have the same population mean. There is evidence to suggest that the population mean number of cars produced is different for the different factories.