

# Incomplete markets as correlated distortions\*

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## Abstract

We argue that capital misallocation arises endogenously due to incomplete financial markets. We model risk-averse entrepreneurs with heterogeneous productivities who face idiosyncratic profitability shocks and make irreversible capital investment before uncertainty unfolds. We show that incomplete financial markets operate as *correlated distortions* — that is, they are isomorphic to a progressive output tax, leading to a reallocation of capital from more to less productive entrepreneurs relative to the complete markets benchmark. Using Portuguese administrative data, we test this hypothesis and document that capital misallocation is greater in locations and industries with higher shock volatility, consistent with the prediction of our framework. Completing financial markets would increase aggregate productivity by up to 27%.

**Keywords:** Insurance, volatility, misallocation, distortions, efficiency.

**JEL Classification:** E22, D61, L23, L26

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# 1 Introduction

Capital misallocation is a key driver of aggregate productivity losses. Yet, what ultimately drives poor allocative efficiency remains an open question. In this paper, we show that incomplete financial markets can act as a barrier to the efficient allocation of capital across firms.

We develop a parsimonious general equilibrium model of entrepreneurial finance and the capital market to study how financial market incompleteness affects the investment decisions of heterogeneous firms, as well as aggregate productivity. Our economy is populated by risk-averse entrepreneurs, each operating a firm with a decreasing-returns-to-scale production technology using capital as a factor of production. Entrepreneurs are heterogeneous in their permanent productivity levels, face idiosyncratic profitability shocks, and make irreversible capital investment decisions before uncertainty unfolds—that is, they face time-to-build constraints that prevent them from adjusting capital holdings after uncertainty resolves.

Financial market incompleteness ties entrepreneurs' utility to the profits generated by their firms, making them willing to forgo expected profits to reduce volatility. This distorts investment decisions away from expected profit maximization in an effort to avoid losses. As a result, the expected marginal products of capital are not equalized across firms—the hallmark of capital misallocation.

Specifically, we show that financial markets incompleteness operates as *correlated distortions*; that is, it introduces an endogenous wedge between the expected marginal product of capital and its price that is positively correlated with entrepreneurial productivity. As a result, capital reallocates from more to less productive entrepreneurs relative to the complete markets benchmark. This occurs because larger-scale operations amplify the sensitivity of expected utility to profitability volatility. The wedge disproportionately impacts high-productivity entrepreneurs, as they are the ones who would deploy more capital under complete risk sharing, and it becomes more severe as the shock volatility increases.

Our model provides a novel link between entrepreneurial risk and cap-

ital misallocation: a higher volatility of profitability shocks intensifies misallocation by weakening the cross-sectional correlation between entrepreneurial productivity and capital investment. We corroborate this prediction using administrative data on the quasi-universe of Portuguese firms, spanning more than a decade. We estimate production functions to recover firm-level permanent revenue productivity and idiosyncratic residuals in sales. We use these estimates to infer capital misallocation and the volatility of profitability shocks in each location-industry pair. The empirical evidence strongly supports our theoretical predictions: we find significantly lower correlations between firm productivity and capital holdings in locations and industries characterized by higher average idiosyncratic volatility.

Therefore, we leverage the structure of our model to quantify the aggregate cost of financial market incompleteness. To do so, we calibrate the model to reproduce the observed joint distribution of firm productivity and capital holdings from the data. Next, we conduct two counterfactual exercises. In the first one, we examine a counterfactual economy where profitability shocks are eliminated. This exercise allows us to quantify the extent to which uninsurable idiosyncratic shocks impact aggregate productivity when entrepreneurs lack access to a full set of state-contingent claims. In the second one, we complete financial markets while maintaining the volatility of profitability shocks to the baseline level. Our calibration implies that eliminating idiosyncratic risk increases aggregate output per capita would by 20%. If financial markets were complete, it would increase by 27%.

Finally, we check the robustness of our findings to i) incorporating firm-specific correlated distortions, capturing other frictions that disproportionately affect high-productivity entrepreneurs, and to ii) incorporating labor as a factor of production and allowing firms to adjust it after profitability shocks realize. We find that aggregate efficiency gains from completing financial markets remain substantial, ranging from 24% to 46% of baseline values.

Our paper contributes to the literature examining the causes and consequences of capital misallocation (Hopenhayn, 2014; Restuccia and Roger-

son, 2017). This body of work either explains it as a result of specific policies or frictions—such as borrowing constraints (Gilchrist et al., 2012; Moll, 2014; Midrigan and Xu, 2014), imperfect information (David et al., 2016), corporate income taxes (Ábrahám et al., 2023) and firm-level subsidies (Cong et al., 2019)—or reduced-form wedges (David and Venkateswaran, 2019; Whited and Zhao, 2021). Building on this literature, we show that capital misallocation arises *endogenously* among risk-averse entrepreneurs as a consequence of missing financial markets, generating aggregate productivity losses that exist *independently* of other market imperfections.

Within this line of work, our paper is mostly related to the macro-finance literature that studies aggregate and distributional consequence of uninsurable entrepreneurial risk for entrepreneurial activities. Chen et al. (2010) build a dynamic incomplete-markets model of risk-averse entrepreneurial firms to show that non-diversifiable business risks cause them to underinvest (relative to otherwise identical public firms), and to allocate resources to projects with low idiosyncratic volatility. Boar et al. (2022) document that the average returns to private business wealth are dispersed and persistent, and interpret this evidence as mostly reflecting uninsurable entrepreneurial risk as opposed to collateral constraints. David et al. (2022) link dispersion in the marginal product of capital across firms to exposure to aggregate shocks, showing that firm-level risk premia in capital contribute to capital misallocation and aggregate productivity. Our paper contribute to this literature by providing a novel mechanism that links uninsurable entrepreneurial risks to capital investment and the dispersion in the marginal revenue product of capital. With incomplete financial markets, higher business risk weakens the cross-sectional correlation between entrepreneurial productivity and capital holdings relative to the efficient benchmark, thus leading to capital misallocation.

Recent contributions highlight that uninsurable idiosyncratic risk also distorts the decision to become an entrepreneur (Robinson, 2021), affects investment decisions and portfolio allocations (Wang et al., 2012), induces over-accumulation of wealth to self-insure (Di Tella et al., 2025), and might create inefficient recessions (Di Tella and Hall, 2022). Keeping aside the focus on aggregate risk, it remains poorly understood how the

absence of insurance differentially affects entrepreneurs based on their permanent productivity. This paper fills this gap by showing that, when firms are heterogeneous, missing insurance against idiosyncratic shocks alters the allocation of capital across firms and behave as if they were *correlated distortions*, leading to under-investment of high-productivity entrepreneurs and over-investment of low-productivity ones.

## 2 Model

We examine an economy with incomplete financial markets, populated by risk-averse heterogeneous entrepreneurs, each operating a firm and facing profitability shocks. In this section, we focus on a static model where capital is the only factor of production.

There exists a unitary measure of entrepreneurs indexed by  $i$ . Let  $u(\cdot)$  be a strictly increasing, strictly concave, and twice-continuously differentiable utility function representing their preferences over consumption. Entrepreneurs differ in their productivity  $z \in \mathcal{Z} \subset \mathbb{R}_{++}$  and have access to a production technology  $f(z, k)$  that depends on productivity and capital  $k$ . We assume that  $f(z, k)$  is strictly increasing, strictly concave, supermodular, and twice continuously differentiable. Production is subject to an idiosyncratic multiplicative shock,  $s \in \mathcal{S} = [\underline{s}, \bar{s}] \subset \mathbb{R}_+$ , which is distributed according to a probability density function  $\phi(s)$ . Each entrepreneur commits to a capital investment before the realization of profitability shocks. Let  $r$  denote the rental rate of capital and  $\bar{K}$  represent the exogenously given supply of capital.

### 2.1 Case 1: Complete markets

Under complete financial markets, entrepreneurs can trade a full set of state-contingent securities to hedge against uncertainty in the realization of profitability shocks  $s$ . Hence, each entrepreneur  $i$  chooses capital  $k_i$ , state-contingent claims  $\{\theta_i(s)\}_{s \in \mathcal{S}}$ , and consumption in each state of the world  $\{c_i(s)\}_{s \in \mathcal{S}}$  to maximize her expected utility. Let  $q(s)$  denote the price of a contingent claim that pays a unit of consumption if state  $s$  realizes, and zero otherwise. Then, the problem of entrepreneur  $i$  can be

written as follows:

$$\begin{aligned} & \max_{k_i, \{\theta_i(s), c_i(s)\}_{s \in \mathcal{S}}} \int_{\mathcal{S}} u(c_i(s)) \phi(s) ds \\ \text{s.t. } & c_i(s) + \int_{\mathcal{S}} q(s') \theta_i(s') ds' \leq sf(z_i, k_i) - rk_i + T + \theta_i(s), \quad \forall s \in \mathcal{S}, \end{aligned}$$

where  $T = r\bar{K}$  denotes revenues from renting capital, assumed to be rebated lump sum to each entrepreneur. A competitive equilibrium for an economy is defined in Appendix A.

Dropping index  $i$  for convenience and noting that entrepreneurs exhaust their budget constraints in equilibrium, we can write an entrepreneur's problem as follows:

$$\max_{k, \{\theta(s)\}_{s \in \mathcal{S}}} \int_{s \in \mathcal{S}} u \left( sf(z, k) - rk + T - \int_{s' \in \mathcal{S}} q(s') \theta(s') ds' + \theta(s) \right) \phi(s) ds.$$

Let the subscript  $*$  denote an interior solution to this problem. Letting  $u_c$  denote the marginal utility of consumption, the optimal choice of state-contingent claims under shock  $s$ ,  $\theta^*(s)$ , satisfies the following first-order condition:

$$u_c(c^*(s))\phi(s) = q(s) \int_{s' \in \mathcal{S}} u_c(c^*(s'))\phi(s') ds', \quad \forall s \in \mathcal{S},$$

where the marginal benefit of an extra unit of an asset that pays off if shock  $s$  realizes equates its marginal cost before uncertainty is realized. When the price for insurance is actuarially fair—i.e.,  $q(s) = \phi(s)$ —we can rewrite the previous first-order condition as follows:

$$u_c(c^*(s)) = \int_{s' \in \mathcal{S}} u_c(c^*(s'))\phi(s') ds', \quad \forall s \in \mathcal{S},$$

which implies that consumption is constant across output shock realizations; i.e.,

$$c^*(s) = u_c^{-1} \left( \int_{s' \in \mathcal{S}} u_c(c^*(s'))\phi(s') ds' \right), \quad \forall s \in \mathcal{S}.$$

Perfect consumption smoothing implies that the expected marginal product of capital is equalized across firms. To see this, notice that an en-

trepreneur's optimal choice of capital  $k^*$  satisfies the following first-order condition:

$$\int_{s \in \mathcal{S}} u_c(c^*(s)) [sf_k(z, k^*) - r^*] \phi(s) ds = 0,$$

where  $f_k$  denotes the derivative of  $f$  with respect to  $k$ . Because  $c^*(s) = c^*(s')$  for any two states  $s, s' \in \mathcal{S}$ , the condition above can be rewritten as:

$$\int_{s \in \mathcal{S}} [sf_k(z, k^*) - r^*] \phi(s) ds = 0. \quad (1)$$

That is, under complete markets, each entrepreneur chooses capital until the expected marginal product of capital equates its price. This leads to our first proposition.

**Proposition 1.** *Under complete financial markets and an actuarially fair price of insurance, the expected marginal product of capital is equalized across firms, and aggregate expected output is maximized.*

*Proof.* See Appendix A. □

With a full set of state-contingent claims ensuring perfect consumption smoothing, risk-averse entrepreneurs behave as if they were risk-neutral. As a result, the separation theorem holds: production decisions are independent of individual preferences, and each entrepreneur chooses capital to maximize expected profits.

## 2.2 Case 2: Incomplete markets

Consider now an economy with incomplete markets. Each entrepreneur  $i$  lacks access to state-contingent securities, and chooses capital  $k_i$  and consumption in each state of the world  $\{c_i(s)\}_{s \in \mathcal{S}}$  to maximize expected utility. An entrepreneur  $i$ 's problem can be written as follows:

$$\begin{aligned} \max_{k_i, \{c_i(s)\}_{s \in \mathcal{S}}} & \int_{s \in \mathcal{S}} u(c_i(s)) \phi(s) ds \\ \text{s.t.} & c_i(s) \leq sf(z_i, k_i) - rk_i + T, \quad \forall s \in \mathcal{S}, \end{aligned}$$

where, again,  $T = r\bar{K}$  represents capital rental income, assumed to be rebated lump sum to each entrepreneur. A definition of competitive equilibrium for this economy is reported in Appendix A.

Again, let us drop subscript  $i$ . To characterize the equilibrium for this economy, we consider a CRRA utility function; i.e.,  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ , with  $\gamma \neq 1$ ; and assume that  $s \sim \log \mathcal{N}(\underline{s}, \sigma^2)$ . Alternatively, one can assume that entrepreneurs have a strictly concave utility function and interpret our mean–variance analysis below as an approximate case, which becomes exact under specific assumptions such as CRRA utility with log-normal shocks or CARA utility with normal shocks (Markowitz, 2014). Denote the expected value and variance of  $s$  as  $\mu_s$  and  $\sigma_s^2$ , respectively.<sup>1</sup> Noting that entrepreneurs exhaust their budget constraints in equilibrium, we can write an entrepreneur’s problem as follows:

$$\max_k \int_{s \in \mathcal{S}} \frac{(sf(z, k) - rk + T)^{1-\gamma}}{1-\gamma} \phi(s) ds.$$

Given the log-normality of shocks  $s$ , consumption follows a three-parameter log-normal distribution (Aitchison and Brown, 1957; Singh, 1998), with an expected value of  $\bar{c} = \mu_s f(z, k) - rk + T$ , variance  $\sigma_c^2 = f(z, k)^2 \sigma_s^2$ , and a threshold of  $-rk + T$ .<sup>2</sup> Therefore, an entrepreneur’s problem can be rewritten in terms of consumption equivalent as follows:

$$\max_k \mu_s f(z, k) - rk + T - \frac{(\gamma - 1)}{2} f(z, k)^2 \sigma_s^2.$$

Let the subscript  $^o$  denote an interior solution to this problem. Taking the first-order condition with respect to capital, we obtain:

$$f_k(z, k^o) [\mu_s - (\gamma - 1) \sigma_s^2 f(z, k^o)] - r^o = 0, \quad (2)$$

which leads to our second proposition.

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<sup>1</sup>Using the properties of the log-normal distribution,  $\mu_s = e^{\underline{s} + \frac{1}{2}\sigma^2}$  and  $\sigma_s^2 = e^{2\mu_s + \sigma^2} (e^{\sigma^2} - 1)$ .

<sup>2</sup>The three-parameter log-normal distribution is similar to the standard two-parameter log-normal distribution, except that its support is shifted by an amount representing a lower bound, known as the threshold. In this context, if aggregate transfers net of capital payments are relatively small (i.e.,  $T - rK \approx 0$ ), then consumption would approximately follow a standard two-parameter log-normal distribution.

**Proposition 2.** *Under incomplete financial markets, when  $\gamma \neq 1$ , there is misallocation along the intensive margin; i.e., the expected marginal product of capital is not equalized across entrepreneurs with different productivity.*

*Proof.* Obvious from Equation (1) and (2). □

When ex-post instruments for consumption smoothing fail to exist—i.e., when a full set of state-contingent claims is unavailable—risk-averse entrepreneurs choose capital to balance maximizing expected income and minimizing income volatility. As a result, capital allocation across entrepreneurs deviates from the allocation that maximizes aggregate expected output. In particular, lack of insurance introduces an endogenous wedge  $\tau(z, k^\circ)$  between the marginal product of capital and its marginal cost; i.e., under incomplete markets, the equilibrium allocation of capital satisfies

$$\mu_s f_k(z, k^\circ)(1 - \tau(z, k^\circ)) = r^o,$$

where

$$\tau(z, k^\circ) = (\gamma - 1) \frac{\sigma_s^2}{\mu_s} f(z, k^\circ).$$

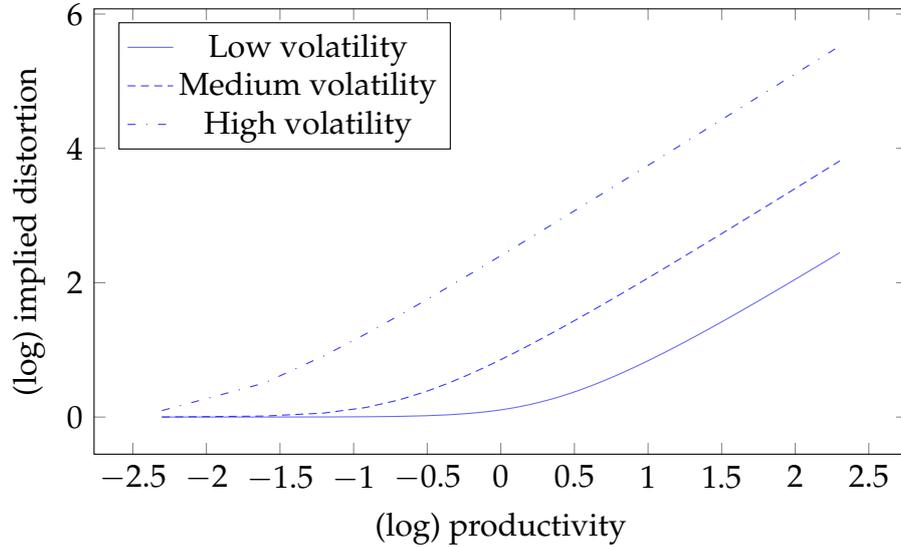
This leads to our third proposition.

**Proposition 3.** *When  $\gamma > 1$ , financial markets' incompleteness operates as a correlated distortion, leading to a reallocation of capital from more to less productive firms relative to the complete markets benchmark.*

*Proof.* See Appendix A. □

When entrepreneurs are sufficiently risk-averse, market incompleteness acts as a tax on output that is positively correlated with entrepreneurial productivity. Figure 1 illustrates how the distortions implied by our model endogenously vary with entrepreneurial productivity. Under incomplete markets, all else equal, a larger scale of production makes utility more sensitive to profitability shocks. Due to complementarities in production, this distortion is particularly pronounced for high-productivity

**Figure 1:** Correlated distortions and entrepreneurial productivity



NOTES: This figure reports the endogenous tax on output,  $\frac{1}{1-\tau(z,k^\circ)}$ , implied by the estimated model as a function of the estimated entrepreneurial (log) productivity,  $z$ .

entrepreneurs, who optimally reduce their scale and forgo expected profits to secure lower volatility.<sup>3</sup>

The figure also shows that the implied tax distortion becomes more pronounced when the volatility of profitability shocks is higher. This occurs because the wedge between the marginal product of capital and its price,  $\tau(z, k^\circ)$  increases with the volatility of the idiosyncratic shock,  $\sigma$ . This observation leads to our fourth proposition.

**Proposition 4.** *Under incomplete financial markets, when  $\gamma > 1$ , a higher volatility of profitability shocks  $\sigma$  reduces the correlation between capital  $k$  and productivity  $z$ .*

*Proof.* See Appendix A. □

This is a central property of the model. Under incomplete markets,

<sup>3</sup>A second force operates in general equilibrium: capital being in fixed supply, the reduced demand for capital from high-productivity entrepreneurs lowers the rental price relative to the complete markets benchmark. This, in turn, incentivizes low-productivity entrepreneurs—those whose expected utility is less sensitive to profitability shocks—to rent more capital.

the distribution of marginal utilities of consumption across states of the world increasingly reflects the impact of profitability shocks. Amplifying production uncertainty is isomorphic to imposing a larger tax on the marginal product of capital for high-productivity entrepreneurs. As the marginal utilities of high-productivity entrepreneurs become more closely tied to their realized profitability shocks, their capital allocation decisions deviate even further from the complete markets benchmark. This result implies that, all else being equal, markets with higher volatility should also exhibit greater capital misallocation.

### **2.3 Discussion**

Our model shows that incomplete financial markets are sufficient to cause capital misallocation across entrepreneurs, above and beyond what is caused by other distortions in the capital market. Lack of insurance generates an endogenous wedge between the marginal product of capital and its price, which is positively correlated with firm productivity. As a result, incomplete markets lead to a reallocation of capital from high- to low-productivity firms relative to the complete markets benchmark.

In Appendix A, we show that the predictions of our model are robust to the inclusion of additional factors of production, such as labor, which can be adjusted ex-post (i.e., after the realization of profitability shocks), and a risk-free bond that entrepreneurs can use to smooth payoffs over time. To the extent that financial markets remain incomplete—that is, no set of contingent claims spans the entire set of states of the world—the equilibrium capital allocation does not maximize aggregate expected output.

Our results do not depend on the source of uncertainty or the nature of heterogeneity across entrepreneur types. Whether the former arises from supply uncertainty due to productivity shocks or demand uncertainty due to taste shocks, and whether the latter reflects differences in firm productivity or permanent taste heterogeneity for firm-specific products, the allocation of capital across entrepreneurs is still distorted by the lack of insurance, with higher uncertainty amplifying the magnitude of this

distortion. In Appendix A, we also show that our mechanisms might operate alongside with financial frictions, when borrowing constraints are binding for *some firms*, eventually amplifying existing firm-level distortions, as long as entrepreneurs are enough risk averse.

Finally, none of our theoretical results are affected by allowing the production function  $f$  and the distribution  $\phi$  to be heterogeneous across firms. Specifically, all propositions continue to hold when technology and shock volatility vary by locations and industries, and regardless of whether factor and insurance markets are integrated into a unique market or segmented into smaller local markets.<sup>4</sup>

### 3 Empirics

Our model predicts that, under incomplete financial markets, greater idiosyncratic production uncertainty results in increased capital misallocation. In this section, we test this prediction.

#### 3.1 Data

We use firm-level data from Portugal’s *Sistema de Contas Integradas das Empresas* (SCIE, henceforth) for the period 2010-2021.<sup>5</sup> The dataset is an annual panel covering all firms engaged in the production of goods and services in Portugal, excluding financial and insurance companies as well as state-owned enterprises. For each firm-year pair, we observe sales, number of employees, payroll, material expenditures, current and non-current fixed assets, firm age, 4-digit industry classification, and location at the NUTS-2 level, which corresponds to one of Portugal’s seven main administrative regions.<sup>6</sup> All nominal variables are deflated using the national CPI and expressed in 2010 constant prices. Since, in our data, labor is only recorded for firms with at least one employee, we ex-

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<sup>4</sup>In such model, for each local market  $j \in \mathcal{J}$ , there exists a continuum  $\mathcal{I}^j$  of entrepreneurs, with  $\cup_{j=1}^{\mathcal{J}} \mathcal{I}^j = 1$ . Entrepreneurs in local market  $j$  have access to a technology  $f^j(z, k)$  and are subject to shocks with pdf  $\phi^j(s)$ .

<sup>5</sup>In English: Integrated Business Accounts System.

<sup>6</sup>The NUTS-2 regions in Portugal are Norte, Centro, Área Metropolitana de Lisboa, Alentejo, Algarve, Região Autónoma dos Açores, and Região Autónoma da Madeira.

clude sole proprietors (“Empresas Individuais”) from our analysis.

In our analysis, we relate variation in idiosyncratic production uncertainty to capital misallocation across local markets, defined as the combination of a 2-digit industry classification and a NUTS-2 region. While the raw data includes 543 local markets, we restrict our focus to those with at least 100 firm-year observations to ensure sufficient power for estimating firm-level permanent productivity and shock volatility. The resulting panel covers 397 local markets and consists of approximately 2.2 million firm-year observations, with around 190,000 firms observed annually. Table B1 in Appendix B presents selected summary statistics for our sample.

### **3.2 Volatility and misallocation**

We use our data to estimate production functions and recover two key statistics. The first is firm productivity, which we use to calculate measures of capital misallocation at the local market level. The second is the volatility of profitability shocks in each local market. Firm productivity is derived from estimates of permanent unobserved heterogeneity in sales across firms, after controlling for differences in the use of factors of production and aggregate shocks. To estimate the volatility of profitability shocks, we rely on the residuals from the regression of sales on the quantity of inputs used in production, firm dummies, and year dummies.

Our empirical strategy involves estimating production functions separately for each local market. This serves two key purposes. First, it provides greater flexibility, allowing input shares in the production function to vary across local markets (Asker et al., 2014). Second, and most importantly, it enables us to account for heterogeneity in the variance of the error term across local markets. Uncovering this heterogeneity allows us to link volatility to capital misallocation across location-industry pairs.

Thus, for each location-industry pair  $j$ , consider the following revenue

production function:

$$\log y_{it} = \beta^{0j} + \beta^{1j} \log k_{it} + \beta^{2j} \log \ell_{it} + \beta^{3j} \log m_{it} + \mu_i + \mu_t + \epsilon_{it}, \quad (3)$$

where  $y_{it}$  denotes revenues of firm  $i$  in year  $t$ ,  $k_{it}$  is the capital stock,  $\ell_{it}$  is the number of employees,  $m_{it}$  denotes intermediate inputs expenditure,  $\mu_i$  and  $\mu_t$  are firm and year fixed effects, respectively, and  $\epsilon_{it}$  is an error term with mean 0 and variance  $\sigma^j$ .<sup>7</sup>

The estimated firm fixed effects  $\hat{\mu}_i$  capture all firm-level unobserved factors that permanently affect revenues. In our framework, they map into firm productivity  $z_i$ ; i.e.,

$$\log z_i = \hat{\mu}_i.$$

The estimated residuals  $\hat{\epsilon}_{it}$  represent instead a sample of innovations to sales that cannot be explained by variations in input usage, firm-level permanent unobserved heterogeneity, or common factors affecting revenues. In our framework, they correspond to realized profitability shocks. These residuals contain information that can be used to estimate idiosyncratic uncertainty. Our preferred approach consists of first estimating the volatility of residuals at the firm level,  $\hat{\sigma}_i$ , and then averaging these estimates across firms within the same location. Specifically, we begin by computing the within-firm standard deviation of the estimated residuals over time as:

$$\hat{\sigma}_i = \frac{1}{T} \sqrt{\sum_{t=1}^T (\hat{\epsilon}_{it} - \bar{\hat{\epsilon}}_{it})^2},$$

where  $\bar{\hat{\epsilon}}_{it} = \frac{1}{T} \sum_{t=1}^T \hat{\epsilon}_{it}$  is the time-average firm-level residual. Next, we compute the within-local-market average standard deviation of the estimated residuals:

$$\hat{\sigma}^j = \sum_{i \in \mathcal{I}^j} \frac{\hat{\sigma}_i}{|\mathcal{I}^j|},$$

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<sup>7</sup>In equation (3) we include labor and intermediate inputs, in addition to capital, to account for the observed heterogeneity in labor and material usage across firms. In Appendix A.3, we discuss a version of the model where entrepreneurs are allowed to choose multiple factors of production.

where  $\mathcal{I}^j$  denotes the set of firms in local market  $j$ . Figure B.2 and Table B3 in Appendix B.1 present the distribution of estimated firm productivity and shock volatility. Firm productivity exhibits substantial dispersion across firms, with the standard deviation of  $\hat{\mu}_i$  approximately 0.656. The average standard deviation of profitability shocks is around 0.347.

Next, we explore the relationship between shock volatility and capital misallocation across local markets. To measure capital misallocation within each location-industry pair, we employ two distinct strategies. First, we compute the correlation between firm productivity and average (log) capital holdings across firms within each local market. Specifically, for each local market  $j$ , we estimate the following regression:

$$\overline{\log k_{it}} = v^j + \eta^j \hat{\mu}_i + \varepsilon_i,$$

where  $\overline{\log k_{it}} = \frac{1}{T} \sum_{t=1}^T \log k_{it}$ . A higher  $\eta^j$  indicates a higher correlation between firm productivity and capital holdings, and hence a lower capital misallocation, in local market  $j$ . Second, leveraging the production function estimates of equation (3), we construct an estimate of the (log) marginal product of capital for each firm  $i$  located in market  $j$  and year  $t$ :

$$\log \text{MPK}_{it}^j = \hat{\beta}^{1j} + \log y_{it} - \log k_{it}.$$

Then, for each firm  $i$ , we compute the average within-firm marginal product of capital as

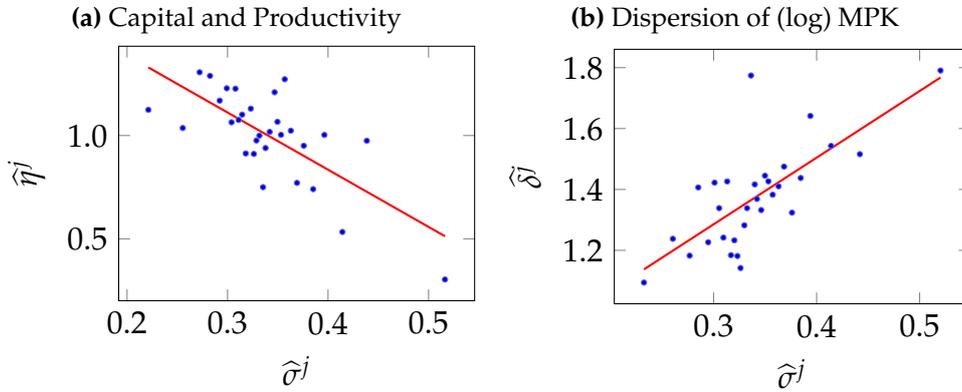
$$\overline{\log \text{MPK}_{it}^j} = \frac{1}{T} \sum_{t=1}^T \log \text{MPK}_{it}^j,$$

and finally we calculate its standard deviation within each local market:

$$\delta^j = \text{s.d.} \left( \overline{\log \text{MPK}_{it}^j} \right).$$

Figure 2 scatters both measures of capital misallocation on the aver-

**Figure 2: Volatility and capital misallocation across local markets**



NOTES:: This figure binscatters the relationship between capital misallocation and profitability shock volatility across local markets, controlling for two-digit CAE industry and NUTS-2 region fixed effects. In Panel A, capital misallocation is measured as the correlation between firm productivity and the average (log) capital holdings of firms within each local market. In Panel B, capital misallocation is measured as the standard deviation of the average within-firm marginal product of capital across firms in each local market. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations.

age firm-level volatility across local markets.<sup>8</sup> In the figure, each dot represents an average local market within a certain bin in the distribution of volatility. The patterns that emerge are striking: local markets with higher firm-level volatility are also those with lower correlation between firm-level capital and productivity (Panel A). This pattern implies that the average within-firm marginal product of capital is not equalized across firms located in high-volatility markets as it would be the case in efficient allocation (Panel B).<sup>9</sup>

Figure 3 zooms into local markets with different levels of average id-

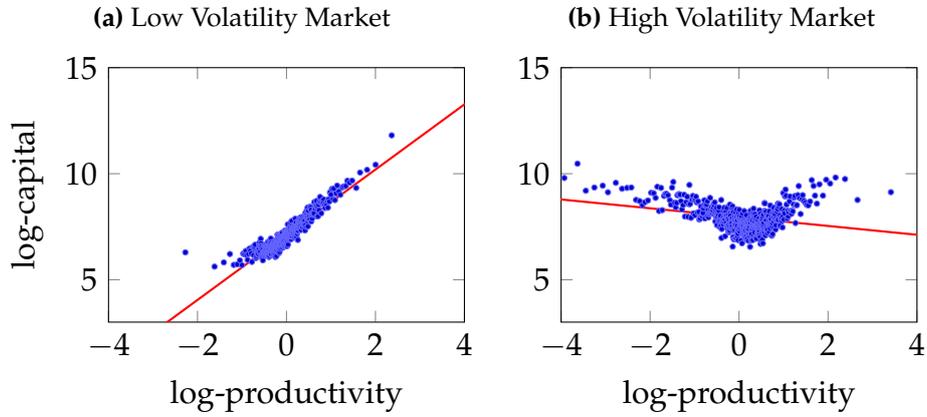
<sup>8</sup>In Table B4 Appendix B, we report point estimates (and standard errors) from regressing our measures of capital misallocation on the average firm-level volatility across local markets, conditional on 2-digit CAE industry and NUTS-2 region fixed effects. That is, we estimate,

$$\hat{\omega}^j = \gamma_0 + \gamma_1 \hat{\delta}^j + \mu^{s(j)} + \mu^{\ell(j)} + \varepsilon^j,$$

where  $\hat{\omega}^j$  is either  $\hat{\eta}^j$  or  $\hat{\delta}^j$ ,  $\mu^{s(j)}$  are 2-digit CAE industry fixed effects and  $\mu^{\ell(j)}$  are NUTS-2 region fixed effects, and  $\varepsilon^j$  is an error term.

<sup>9</sup>Figure B.3 and Table B5 in Appendix B show that a similar relationship holds for labor misallocation and shock volatility across local markets, though the magnitudes are approximately half as large.

**Figure 3:** Capital allocation in low- and high-volatility markets



NOTES: This figure binscatters firm-level (log) capital,  $\log k_i$ , against the estimated firm-level (log) productivity,  $\hat{\mu}_i$ , across firms located in low-volatility (Panel A) and high-volatility (Panel B) local markets. Low- (high-) volatility markets are defined as a tuple of 2-digit CAE industry classification and NUTS-2 region with at least 100 firm-year observations and with an estimated average volatility at the bottom (top) 1 percent of the distribution.

iosyncratic volatility. Panel A scatters capital holdings against estimated firm productivity for firms operating in low-volatility markets, while Panel B replicates this analysis for firms in high-volatility markets, where low- (high-) volatility markets are defined as location-industry pairs that fall within the bottom (top) 1% of the estimated distribution of volatility. Consistent with our theory, capital holdings and firm productivity show a strong positive correlation of 0.49 in low-volatility markets. This relationship breaks down in high-volatility markets, with capital allocation deviating from efficiency and becoming essentially uncorrelated with productivity, as reflected in a point estimate of -0.09.

### 3.3 Addressing endogeneity

**Shock persistence.** In our baseline specification (equation 3), the error terms represent profitability shocks, which our framework assumes to be independently and identically distributed over time. This approach aligns with recent work on the effects of uninsurable idiosyncratic risk on entrepreneurial activity (Di Tella and Hall, 2022; Di Tella et al., 2025). However, we test the robustness of our results to allowing the shocks to be persistent over time. To do it, we extend the production function in

equation (3) as follows:

$$\log y_{it} = \beta^{0j} + \beta^{1j} \log k_{it} + \beta^{2j} \log \ell_{it} + \beta^{3j} \log m_{it} + \mu_i + v_{it},$$

where  $v_{it}$  follows an autoregressive process of order one; i.e.,

$$v_{it} = \rho v_{it-1} + \epsilon_{it},$$

$\rho \in (0,1)$  captures the persistence of  $v_{it}$ , and  $\epsilon_{it}$  is an i.i.d. mean-zero error term. To obtain consistent estimates of  $\beta^{1j}$ ,  $\beta^{2j}$ , and  $\beta^{3j}$ , we follow Blundell and Bond (2000) and employ a dynamic panel data approach while treating capital as a predetermined variable. Further details are provided in Appendix B. This estimation strategy yields estimates of firm-level permanent productivity and sales residuals, which we use to construct measures of capital misallocation and shock volatility at the local market level. Tables B6 and B7 report the conditional correlation between volatility and measures of misallocation across location-industry pairs. Our results are robust to the inclusion of shock persistence. The relationship between uncertainty and capital misallocation remains significant, with magnitudes comparable to or exceeding those in our baseline specification.

**Firm exit.** Each year, approximately 6.5 percent of firms exit the sample. Since exiting firms are likely to be non-randomly selected from the population of operating firms — e.g., they tend to be smaller than surviving firms, ignoring this attrition could introduce bias in our estimates of the production function parameters. To address this concern, we consider the following binary selection equation for firm exit:

$$d_{it}^* = \zeta_t \zeta_i + \epsilon_{it}, \quad d_{it} = \mathbf{1}[d_{it}^* > 0] \quad (4)$$

where  $\mathbf{1}[\cdot]$  is an indicator function equal to one if its argument is true, and zero otherwise,  $\zeta_i$  are unobserved individual-specific effects, with a possibly time-varying effect,  $\zeta_t$ , and  $\epsilon_{it}$  is an unobserved disturbance term. We jointly estimate equations (3) and (4) separately for every period  $t$  following the semi-parametric approach proposed by Wooldridge (1995), and using panel-level average of time-varying covariates in equa-

tion (3) to parametrize  $\zeta_i$ . We report details in Appendix B.

Tables B8 and B9 present the conditional correlation between profitability shock volatility and our measures of misallocation, constructed accounting for self-selection in firm exit. Although accounting for endogenous firm exit slightly reduces the correlation, it remains large and statistically significant.

## 4 The aggregate cost of incomplete markets

In this section, we leverage the structure of our model to answer the following question: What is the aggregate cost of incomplete financial markets? We begin by specifying a production function in which capital is the only factor of production:

$$f(z, k) = zk^\alpha,$$

where  $\alpha \in (0, 1)$ .<sup>10</sup> We assume that productivity  $z$  follows a gamma distribution with shape and scale parameters  $\kappa$  and  $\theta$ , respectively:

$$z \sim \Gamma(\kappa, \theta).$$

Finally, we maintain the assumption that shocks  $s$  follow a log-normal distribution. Since our focus is on aggregate outcomes, we abstract from the heterogeneity in capital share  $\alpha$  and shock volatility  $\sigma$  across local markets, focusing instead on their average values.

Table 1 summarizes the model parameters and their respective sources. The output elasticity of capital,  $\alpha$  is externally calibrated to match an aggregate capital share of 0.416, constructed as one minus the aggregate labor share for the period 2011-2019 (FRED, 2025). The volatility of shocks  $\sigma$  is calibrated at 0.347, a value directly estimated from the data. The shape and scale parameters of the firm productivity distribution,  $\kappa$  and  $\theta$ , are obtained by fitting by maximum likelihood a gamma distribution to the estimated firms' fixed effects and set to 1.404 and 1.060, respec-

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<sup>10</sup>Later in the paper, we consider a model that includes labor as a factor of production, allowing entrepreneurs to adjust it after the shocks realize.

**Table 1: Parameters**

Parameters	Description	Value	Target/Source
<i>A. Parameters calibrated</i>			
$\alpha$	Output elasticity of capital	0.416	Aggregate capital share (FRED)
$\sigma$	Volatility of shocks	0.347	Data
$(\kappa, \theta)$	Distribution of permanent productivity	(1.404, 1.060)	Data
<i>B. Parameters estimated</i>			
$\gamma$	Relative risk aversion	3.361	corr. $[\log k, \log z] = 0.494$

NOTES: This table reports the values and sources of the calibrated and estimated parameters in the benchmark model.

tively. Finally, we estimate the coefficient of relative risk aversion,  $\gamma$ , to match the observed correlation of 0.494 between (log) capital holdings and (log) permanent productivity across firms. The estimation yields  $\hat{\gamma} = 3.361$ , which is close to the estimate of 4.2 found in Chiappori and Paiella (2011).

Equipped with our estimates, we perform two counterfactual exercises. In the first, we eliminate production uncertainty by setting  $\sigma$  to zero; i.e., we simulate the aggregate behavior of a deterministic economy where entrepreneurs face no profitability shocks. In the second, we complete financial markets while maintaining shock volatility at its baseline level.

Table 2 presents the results of these counterfactual exercises. Transitioning from the baseline to a deterministic economy removes the distortions in capital allocation caused by production uncertainty under incomplete financial markets. As a consequence, the correlation between firm productivity and capital holdings increases, leading to greater dispersion of capital holdings across firms. This reallocation enhances aggregate productivity, resulting in aggregate income gains of 20.3%.

A counterfactual economy with complete markets exhibits the same capital allocation across firms as the economy without production uncertainty, but achieves higher income gains, reaching 27.4%, respectively. This difference arises because eliminating production uncertainty, that is, setting  $\sigma$  to zero, affects higher moments of the productivity distribution as well as the expected value of the shock. In contrast, the availability of state-contingent claims when markets are complete enables entrepreneurs to fully insure against consumption uncertainty, ef-

ffectively eliminating the distortionary effects of the higher moments of the productivity distribution without reducing the expected value of profitability shocks. Intuitively, completing financial markets eliminates misallocation without reducing the likelihood of favorable shock realizations.<sup>11</sup>

**Table 2:** The aggregate cost of incomplete markets

	Counterfactual scenarios		
	Baseline (1)	No Uncertainty (2)	Complete markets (3)
Volatility, $\sigma$	0.347	0	0.347
Average distortion, $\overline{\tau(z, k)}$	0.458	0	0
corr[log $z$ , log $k$ ]	0.485	1	1
$\eta$ [log $z$ , log $k$ ]	0.827	1.712	1.712
sd[log $k$ ]	0.776	1.594	1.594
Aggregate Income	1	1.203	1.274

NOTES: This table reports selected outcomes for the baseline economy (column 1), a counterfactual economy with no idiosyncratic production uncertainty (column 2), and a counterfactual economy with complete insurance markets (column 3).

#### 4.1 Accounting for other sources of misallocation

The correlation between firm-level productivity and capital holding is likely to be affected by other sources of misallocation, such as size-dependent markups, regulations or corporate taxes. In Appendix C, we test the robustness of our results to the inclusion of other distortions.

To do so, we introduce generic firm-specific distortions in the form of wedges between the marginal product and the price of capital that are correlated with firm productivity. Specifically, as in Hsieh and Klenow

<sup>11</sup>Effectively, the gains from completing financial markets could be decomposed into the gains from efficiently allocating capital to entrepreneurs and those from allowing them to fully exploit the prospect of favorable shock realizations. In other words, completing markets generates aggregate gains that extend beyond pure allocative efficiency.

(2009), we assume that each firm  $i$  is subject to a tax on capital expenditures,  $\tau_i$ .<sup>12</sup> Moreover, as in Bento and Restuccia (2017), we assume the tax is equal to

$$\tau_i = 1 - z_i^{-\varphi},$$

where  $\varphi \geq 0$  governs the correlation between tax and productivity.

To quantify the gains from full insurance, we proceed by fixing the coefficient of relative risk aversion,  $\gamma$ , to either 1.5 or 3, and calibrate  $\varphi$  in both scenarios to match the correlation between (log) productivity and (log) capital holdings. We obtain values  $\varphi$  of 0.295 and 0.195, respectively, implying a distortion tax of up to 50% for the most productive firms, and a distortion subsidy of up to 100% for the least productive ones (Figure C.1).

Table C2 presents the counterfactual outcomes for both cases. The gains from completing financial markets remain large when other distortions are factored in: aggregate income increases 6.4% when  $\gamma = 1.5$ , and by 23.7% when  $\gamma = 3$ , accounting for between 46 and 98% of the overall gains achievable from moving to a fully-undistorted economy.

## 4.2 Factor shares

In Appendix C we also assess the robustness of our findings to different values of the output elasticity of capital,  $\alpha$ . Table C3, we report the percentage gains in aggregate productivity and income from eliminating production uncertainty or completing financial markets while varying  $\alpha$  from 0.5 to 0.9. In each counterfactual economy, all parameters remain as in the baseline, except for the relative risk aversion coefficient,  $\gamma$ , which is re-calibrated to match the observed correlation between firm productivity and capital holdings. We find that increasing the output elasticity of capital raises the estimated RRA coefficient, from 3.51 when  $\alpha = 0.5$  to 4.72 when  $\alpha = 0.9$ . This occurs because, holding all else constant,

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<sup>12</sup>In Appendix A.5 we show that there is a direct isomorphism between our framework extended to include firm-specific distortions in the form of taxes, and the exact same economy with financial frictions. See also Liu (2019) and Bigio and La’o (2020) for a similar argument.

a higher  $\alpha$  increases the expected marginal product of capital relatively more for high-productivity entrepreneurs. Consequently, a higher value of  $\gamma$  is required to match the correlation between firm productivity and capital holdings to the targeted moment. As before, the counterfactual gains in aggregate productivity from completing insurance markets consistently exceed those from eliminating production uncertainty. Moreover, higher values of the output elasticity of capital  $\alpha$  amplify both types of gains. Eliminating production uncertainty increases aggregate output from 30% when  $\alpha = 0.5$  up to 252% when  $\alpha = 0.9$ . Similarly, the gains from completing markets range from 39% when  $\alpha = 0.5$  to 270% when  $\alpha = 0.9$ .

### 4.3 Technology

Finally, we assess the robustness of our findings to incorporating labor as a factor of production and allowing firms to adjust it after uncertainty unfolds. To do so, we solve and estimate the model introduced in Section C.3. Table C4 reports the values of the calibrated parameters. We set the span of control parameter,  $\eta$ , to 0.9 as in (Erosa et al., 2023), and calibrate the output elasticity of labor,  $\alpha$ , to 0.65 to match the aggregate labor share FRED (2025). As in the benchmark model, we estimate the relative risk aversion coefficient to match the observed correlation between firm productivity and capital holdings, obtaining a value of 5.111.

Table C5 presents the counterfactual outcomes. The gains from completing markets significantly amplify when firms produce using capital and labor: aggregate income increases by 46%, compared to 20% in the benchmark model. Although chosen ex-post, market incompleteness propagates to labor demand, introducing a further source of misallocation.

## 5 Conclusion

We study the allocation of capital to risk-averse entrepreneurs with heterogeneous productivities in an economy where financial markets are incomplete and output requires time to build. We prove that markets' in-

completeness acts like a *correlated distortion*; that is, an output tax that reallocates capital from more productive to less productive entrepreneurs relative to the complete markets benchmark. Using Portuguese administrative data, we show that capital misallocation is greater in locations and industries where output is subject to higher idiosyncratic production uncertainty, consistent with our framework. By closing the model in general equilibrium, we quantify the misallocation cost of incomplete markets. Our findings suggest that completing financial markets would result in substantial aggregate efficiency gains.

Our results speak to the link between financial markets' incompleteness and the aggregate costs of resource misallocation. In doing so, our paper introduces a complementary channel for misallocation, which can operate alongside the well-known drivers examined in the literature, such as policy distortions and factor market frictions.

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# Online appendix (not intended for publication)

## A Theory

### A.1 Definition of competitive equilibrium

**Complete markets.** A competitive equilibrium for an economy with complete markets is a list of capital choices  $\{k_i^*\}_{i \in \mathcal{I}}$ , state-contingent claims  $\{\theta_i^*(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$ , consumption plans  $\{c_i^*(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$ , a rental price of capital  $r^*$ , and a list of prices for contingent claims  $\{q^*(s)\}_{s \in \mathcal{S}}$  such that

- $\{k_i^*\}_{i \in \mathcal{I}}$ ,  $\{\theta_i^*(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$  and  $\{c_i^*(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$  are the solution to the problem of each entrepreneur  $i$ ;
- the price of contingent claims is fair; i.e.,  $q^*(s) = \phi(s)$ ;
- the market for capital clears; i.e.,

$$\int_i k_i^* di = \bar{K}.$$

**Incomplete markets.** A competitive equilibrium for an economy with incomplete markets is a list of capital choices  $\{k_i^o\}_{i \in \mathcal{I}}$ , consumption plans  $\{c_i^o(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$ , and a rental price of capital  $r^o$  such that

- $\{k_i^o\}_{i \in \mathcal{I}}$  and  $\{c_i^o(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$  solve the problem of the entrepreneurs;
- the market for capital clears; i.e.,

$$\int_i k_i^o di = \bar{K}.$$

### A.2 Proofs

*Proof of Proposition 1.* Recall that an equilibrium capital allocation is such that the marginal product of capital is constant across firms and equal to

the rental price of capital; i.e.,

$$\int_{s \in \mathcal{S}} [sf_k(z_i, k_i^*) - r^*] \phi(s) ds = 0, \quad \forall i \in \mathcal{I}.$$

Next, consider the problem of a planner that allocates capital to entrepreneurs to maximize aggregate expected output subject to an aggregate feasibility constraint. The problem can be stated as follows:

$$\begin{aligned} & \max_{\{k_i\}_{i \in \mathcal{I}}} \int_{s \in \mathcal{S}} sf(z_i, k_i) \phi(s) ds \\ \text{s.t.} \quad & \sum_i k_i di \leq \bar{K}, \end{aligned}$$

where  $\bar{K}$  is the economy-wide capital stock. A solution to this problem is characterized by the following first-order conditions:

$$\int_{s \in \mathcal{S}} sf_k(z_i, k_i^*) \phi(s) ds - \lambda = 0, \quad \forall i \in \mathcal{I},$$

where  $\lambda$  is the Lagrange multiplier attached to the feasibility constraint. That is, to maximize output, the planner allocates capital so that each entrepreneur's expected marginal product equals its shadow price,  $\lambda$ .  $\square$

*Proof of Proposition 3.* Let  $\{k_i^*\}_{i \in \mathcal{I}}$  and  $\{k_i^o\}_{i \in \mathcal{I}}$  be the equilibrium capital allocations under complete and incomplete markets. Let us express both allocations as policy functions mapping entrepreneurial productivity to equilibrium capital choices; i.e.,  $k^*(z)$  and  $k^o(z)$ , respectively.

To prove that market incompleteness results in the reallocation of capital from more to less productive firms, we show that

- there exists a unique value for the entrepreneurial productivity such that the optimal capital chosen under complete markets equates the one under incomplete markets; i.e.,

$$\exists! \tilde{z} \in \mathcal{Z} \quad \text{s.t.} \quad k^*(\tilde{z}) = k^o(\tilde{z});$$

- at the crossing point,  $\tilde{z}$ , the slope of the policy function for capi-

tal under complete markets is larger than the slope of the policy function under incomplete markets; i.e.,

$$\left. \frac{\partial k^*(z)}{\partial z} \right|_{z=\tilde{z}} > \left. \frac{\partial k^o(z)}{\partial z} \right|_{z=\tilde{z}}.$$

We start by proving the second bullet point. Let  $\tilde{z}$  be a point in  $\mathcal{Z}$  such that  $k^*(\tilde{z}) = k^o(\tilde{z}) = \tilde{k}$ . We show later that this point exists and is unique.

Let  $H(z, k) = \int_{s \in \mathcal{S}} s f_k(z, k) \Gamma(s) ds - r$ . By the implicit function theorem, under complete market it must be the case that:

$$\left. \frac{\partial k^*(z)}{\partial z} \right|_{z=\tilde{z}} = - \frac{\left. \frac{\partial H(z, k^*(z))}{\partial z} \right|_{z=\tilde{z}}}{\left. \frac{\partial H(z, k^*(z))}{\partial k} \right|_{z=\tilde{z}}} = - \frac{f_{kz}(\tilde{z}, k^*(\tilde{z}))}{f_{kk}(\tilde{z}, k^*(\tilde{z}))} = - \frac{f_{kz}(\tilde{z}, \tilde{k})}{f_{kk}(\tilde{z}, \tilde{k})} > 0$$

Let  $G(z, k) = f_k(z, k)[\mu_s - (\gamma - 1)\sigma_s^2 f(z, k)] - r$ . By the implicit function theorem, under no insurance it must be the case that:

$$\begin{aligned} \left. \frac{\partial k^o(z)}{\partial z} \right|_{z=\tilde{z}} &= - \frac{\left. \frac{\partial G(z, k^o(z))}{\partial z} \right|_{z=\tilde{z}}}{\left. \frac{\partial G(z, k^o(z))}{\partial k} \right|_{z=\tilde{z}}} = - \frac{f_{kz}(\tilde{z}, k^o(\tilde{z}))[\mu_s - (\gamma - 1)\sigma_s^2 f(\tilde{z}, k^o(\tilde{z}))] - f_k(\tilde{z}, k^o(\tilde{z}))[(\gamma - 1)\sigma_s^2 f_z(\tilde{z}, k^o(\tilde{z}))]}{f_{kk}(\tilde{z}, k^o(\tilde{z}))[\mu_s - (\gamma - 1)\sigma_s^2 f(\tilde{z}, k^o(\tilde{z}))] - f_k(\tilde{z}, k^o(\tilde{z}))[(\gamma - 1)\sigma_s^2 f_k(\tilde{z}, k^o(\tilde{z}))]} \\ &= - \frac{\mu_s f_{kz}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 f(\tilde{z}, \tilde{k}) f_{kz}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 f_z(\tilde{z}, \tilde{k}) f_k(\tilde{z}, \tilde{k})}{\mu_s f_{kk}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 f(\tilde{z}, \tilde{k}) f_{kk}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 f_k(\tilde{z}, \tilde{k}) f_k(\tilde{z}, \tilde{k})}. \end{aligned}$$

We need to prove that:

$$\left. \frac{\partial k^*(z)}{\partial z} \right|_{z=\tilde{z}} > \left. \frac{\partial k^o(z)}{\partial z} \right|_{z=\tilde{z}}$$

or, equivalently,

$$-\frac{f_{kz}(\tilde{z}, \tilde{k})}{f_{kk}(\tilde{z}, \tilde{k})} > -\frac{\mu_s f_{kz}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 [f(\tilde{z}, \tilde{k}) f_{kz}(\tilde{z}, \tilde{k}) + f_z(\tilde{z}, \tilde{k}) f_k(\tilde{z}, \tilde{k})]}{\mu_s f_{kk}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 [f(\tilde{z}, \tilde{k}) f_{kk}(\tilde{z}, \tilde{k}) + f_k(\tilde{z}, \tilde{k}) f_k(\tilde{z}, \tilde{k})]}$$

This condition is equivalent to:

$$\begin{aligned} f_{kz}(\tilde{z}, \tilde{k}) \left[ \mu_s f_{kk}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 [f(\tilde{z}, \tilde{k}) f_{kk}(\tilde{z}, \tilde{k}) + f_k(\tilde{z}, \tilde{k}) f_k(\tilde{z}, \tilde{k})] \right] < \\ f_{kk}(\tilde{z}, \tilde{k}) \left[ \mu_s f_{kz}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 [f(\tilde{z}, \tilde{k}) f_{kz}(\tilde{z}, \tilde{k}) + f_z(\tilde{z}, \tilde{k}) f_k(\tilde{z}, \tilde{k})] \right] \end{aligned}$$

which can be re-arranged as follows:

$$f_{kz}(\tilde{z}, \tilde{k})\mu_s f_{kk}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 f_{kz}(\tilde{z}, \tilde{k})[f(\tilde{z}, \tilde{k})f_{kk}(\tilde{z}, \tilde{k}) + f_k(\tilde{z}, \tilde{k})f_k(\tilde{z}, \tilde{k})] < \\ f_{kk}(\tilde{z}, \tilde{k})\mu_s f_{kz}(\tilde{z}, \tilde{k}) - (\gamma - 1)\sigma_s^2 f_{kk}(\tilde{z}, \tilde{k})[f(\tilde{z}, \tilde{k})f_{kz}(\tilde{z}, \tilde{k}) + f_z(\tilde{z}, \tilde{k})f_k(\tilde{z}, \tilde{k})]$$

Simplifying terms, we obtain:

$$-(\gamma - 1)\sigma_s^2 f_{kz}(\tilde{z}, \tilde{k})[f(\tilde{z}, \tilde{k})f_{kk}(\tilde{z}, \tilde{k}) + f_k(\tilde{z}, \tilde{k})f_k(\tilde{z}, \tilde{k})] < \\ -(\gamma - 1)\sigma_s^2 f_{kk}(\tilde{z}, \tilde{k})[f(\tilde{z}, \tilde{k})f_{kz}(\tilde{z}, \tilde{k}) + f_z(\tilde{z}, \tilde{k})f_k(\tilde{z}, \tilde{k})]$$

Because  $\gamma > 1$  and  $\sigma_s^2 > 0$ , then we can further simplify the condition above as follows:

$$f_{kz}(\tilde{z}, \tilde{k})[f(\tilde{z}, \tilde{k})f_{kk}(\tilde{z}, \tilde{k}) + f_k(\tilde{z}, \tilde{k})f_k(\tilde{z}, \tilde{k})] > f_{kk}(\tilde{z}, \tilde{k})[f(\tilde{z}, \tilde{k})f_{kz}(\tilde{z}, \tilde{k}) + f_z(\tilde{z}, \tilde{k})f_k(\tilde{z}, \tilde{k})]$$

This condition is true if and only if

$$\underbrace{f_{kz}(\tilde{z}, \tilde{k})f_k(\tilde{z}, \tilde{k})}_{>0} > \underbrace{f_{kk}(\tilde{z}, \tilde{k})}_{<0} \underbrace{f_z(\tilde{z}, \tilde{k})}_{>0}$$

which is always the case because  $f_{kz}(z, k) > 0$ ,  $f_k(z, k) > 0$ ,  $f_z(z, k) > 0$  and  $f_{kk}(z, k) < 0$ ,  $\forall z$  and  $\forall k$ . This completes the first part of the proof.

We now move to prove existence and uniqueness of a crossing point between the capital policy function under complete market and the one under incomplete markets.

Let us start by assuming there were no crossing points. Without loss of generality, let us assume that  $k^*(z) > k^o(z), \forall z \in \mathcal{Z}$ . If this is the case, then it must be true that:

$$\sum_{i \in \mathcal{I}} k_i^* di > \sum_{i \in \mathcal{I}} k_i^o di$$

On the other hand, in equilibrium the aggregate demand for capital under both scenarios (i.e., complete and incomplete markets) has to equal

to an exogenously given supply  $\bar{K}$ ; i.e.,

$$\sum_{i \in \mathcal{I}} k_i^* di = \sum_{i \in \mathcal{I}} k_i^o di = \bar{K}$$

which contradicts the previous condition. By contradiction, this implies that there must exist at least one point  $\tilde{z}$  such that  $k^*(\tilde{z}) = k^o(\tilde{z})$ .

The crossing point must also be unique. Suppose not; i.e.,  $\exists \tilde{z}_1, \dots, \tilde{z}_n \in \mathcal{Z}$  such that  $k^*(\tilde{z}_j) = k^o(\tilde{z}_j), \forall j = 1, \dots, n$ . Following what we proved in the first part of the proof, it must be the case that

$$\left. \frac{\partial k^*(z)}{\partial z} \right|_{z=\tilde{z}_j} > \left. \frac{\partial k^o(z)}{\partial z} \right|_{z=\tilde{z}_j} \quad \forall j = 1, \dots, n.$$

On the other hand, because both policy functions  $k^*(z)$  and  $k^o(z)$  are continuous in  $\mathcal{Z}$ , and because  $k^*(z)$  is monotonically increasing in  $z$ —i.e.,  $\frac{\partial k^*(z)}{\partial z} > 0$ —the only possibility where two (or more) crossing points exist is a scenario where

$$\left. \frac{\partial k^*(z)}{\partial z} \right|_{z=\tilde{z}_{\hat{j}}} \leq \left. \frac{\partial k^o(z)}{\partial z} \right|_{z=\tilde{z}_{\hat{j}}}$$

for some  $\hat{j}$ , contradicting what we proved in the first part of the proof. This completes the second part of the proof.

*Proof of Proposition 4.* Consider a decentralized equilibrium under incomplete markets. We need to prove that a higher volatility of the shocks  $\sigma_s^2$  reduces the correlation between capital holdings  $k^o$  and productivity  $z$ . To do so, notice that the derivative of  $\partial k^o (\partial z)^{-1}$  with respect to  $\sigma^2$  is negative if and only if

$$\left( \frac{\partial \mu_s}{\partial \sigma^2} \sigma_s^2 - \mu_s \frac{\partial \sigma_s^2}{\partial \sigma^2} \right) (\gamma - 1) f_k(z, k^o) [f_{kz}(z, k^o) f_k(z, k^o) - f_{kk}(z, k^o) f_z(z, k^o)] < 0.$$

Since  $f_{kk}(z, k^o) < 0$ ,  $f_k(z, k^o) > 0$ ,  $f_{kz}(z, k^o) > 0$  and  $f_z(z, k^o) > 0$ ,

$$f_k(z, k^o) [f_{kz}(z, k^o) f_k(z, k^o) - f_{kk}(z, k^o) f_z(z, k^o)] > 0.$$

Thus, if  $\gamma > 1$ , the derivative of  $(\partial z)^{-1} / \partial k^o$  with respect to  $\sigma^2$  is negative

if and only if

$$\left( \frac{\partial \mu_s}{\partial \sigma^2} \sigma_s^2 - \mu_s \frac{\partial \sigma_s^2}{\partial \sigma^2} \right) < 0. \quad (5)$$

Recall that  $s \sim \log \mathcal{N}(\underline{s}, \sigma^2)$ ; thus,

$$\mu_s = \exp \left( \underline{s} + \frac{\sigma^2}{2} \right)$$

and

$$\sigma_s^2 = (\exp(\sigma^2) - 1) \exp(2\underline{s} + \sigma^2).$$

Substituting these expressions into equation (5), we have that

$$\left( \frac{\partial \left( \exp \left( \underline{s} + \frac{\sigma^2}{2} \right) \right)}{\partial \sigma^2} [(\exp(\sigma^2) - 1) \exp(2\underline{s} + \sigma^2)] \right. \\ \left. - \exp \left( \underline{s} + \frac{\sigma^2}{2} \right) \frac{\partial [(\exp(\sigma^2) - 1) \exp(2\underline{s} + \sigma^2)]}{\partial \sigma^2} \right) < 0;$$

which is always the case because  $\sigma^2 > 0$ . □

### A.3 An extension of the model that allows for other factors of production, monopolistic competition in the product market, and demand shocks

We consider a version of the model outlined in Section 2.2 that allows for other factors of production, monopolistic competition in the product market, and demand shocks. For simplicity, we focus on a model where labor is the only additional input besides capital; however, it is clear from the discussion that there is no conceptual difficulty in including other factors of production as well. Entrepreneurs have access to a production technology  $f(z, k, \ell)$  that depends on productivity  $z$ , capital  $k$ , and labor  $\ell$ . As before, output produced is subject to idiosyncratic, multiplicative shocks  $s$  distributed according to a probability density function  $\phi(s)$ . Differently from the previous model, where entrepreneurs produce a homogeneous commodity, each entrepreneur now produces a

differentiated variety of final goods or services, and faces a downward-sloping demand curve given by

$$y_i(v, \omega_i, p_i) = \Delta(v\omega_i)^{\eta-1} p_i^{-\eta},$$

where  $\eta$  denotes the price elasticity of demand,  $\Delta$  is a parameter governing aggregate demand for final goods and services,  $\omega_i \in \mathcal{W}$  is a parameter capturing permanent taste heterogeneity for firm-specific products, and  $v \in \mathcal{V}$  is an idiosyncratic demand shock, distributed with a pdf  $\Lambda(v)$ .

Entrepreneurs choose capital before the realization of productivity and taste shocks, but decide how much labor to employ after these shocks are realized. Let  $w$  be the wage rate (normalized to 1),  $r$  be the rental rate for capital, and  $\bar{K}$  be the exogenous supply of capital. For simplicity, we adopt a partial equilibrium framework for the labor market, taking wages as given.<sup>13</sup> Entrepreneur  $i$ 's problem can be written as follows:

$$\begin{aligned} \max_{k_i, \{c_i(s, v)\}_{s \in \mathcal{S}}} & \int_{s \in \mathcal{S}} \int_{v \in \mathcal{V}} u(c_i(s, v)) \phi(s) \Lambda(v) ds dv \\ \text{s.t.} & c_i(s, v) \leq \pi(s, v, z_i, \omega_i, k_i) - rk_i + T \quad \forall s \in \mathcal{S}, \forall v \in \mathcal{V}, \end{aligned}$$

where  $T = r\bar{K}$  are revenues from renting capital, assumed to be rebated lump-sum to each entrepreneur, while  $\pi(s, v, z_i, \omega_i, k_i)$  denotes per-period maximized revenues net of labor costs (henceforth, the profit function); i.e.,

$$\begin{aligned} \pi(s, v, z_i, \omega_i, k_i) &= \max_{\ell_i, p_i} y_i(s, z_i, k_i, \ell_i) p_i - \ell_i \\ \text{s.t.} & y_i(s, z_i, k_i, \ell_i) = sf(z_i, k_i, \ell_i) \\ & \text{and } y_i(v, \omega_i, p_i) = \Delta(v\omega_i)^{\eta-1} p_i^{-\eta}. \end{aligned} \tag{6}$$

Substituting price  $p_i$  from the demand function into the profit function, we can rewrite the problem in equation (6) as follows:

$$\pi(s, v, z_i, \omega_i, k_i) = \max_{\ell_i} \Delta^{1-\zeta} (v\omega_i)^\zeta (sf(z_i, k_i, \ell_i))^\zeta - \ell_i,$$

---

<sup>13</sup>Alternatively, we may assume that labor is supplied at no utility cost by a unit mass of identical workers.

where  $\zeta = 1 - 1/\eta$ .

For a given  $\Delta$ , a competitive equilibrium for this economy is a list of capital choices,  $\{k_i^o\}_{i \in \mathcal{I}}$ , labor choices,  $\{\ell_i^o(s, \nu)\}_{s \in \mathcal{S}, \nu \in \mathcal{V}, i \in \mathcal{I}}$ , product prices  $\{p_i^o(s, \nu)\}_{s \in \mathcal{S}, \nu \in \mathcal{V}, i \in \mathcal{I}}$ , consumption plans,  $\{c_i^o(s, \nu)\}_{s \in \mathcal{S}, \nu \in \mathcal{V}, i \in \mathcal{I}}$ , and a rental price of capital  $r^o$  such that

- $\{k_i^o\}_{i \in \mathcal{I}}$ ,  $\{\ell_i^o(s, \nu)\}_{s \in \mathcal{S}, \nu \in \mathcal{V}, i \in \mathcal{I}}$ ,  $\{p_i^o\}_{s \in \mathcal{S}, \nu \in \mathcal{V}, i \in \mathcal{I}}$  and  $\{c_i^o(s, \nu)\}_{s \in \mathcal{S}, \nu \in \mathcal{V}, i \in \mathcal{I}}$  solve entrepreneur  $i$ 's problem;
- the market for capital clears; i.e.,

$$\int_i k_i^o di = \bar{K}.$$

To characterize the competitive equilibrium, we make the following assumptions:

1. production displays constant return to scale in capital and labor; i.e.,

$$f(z, k, \ell) = zk^{1-\alpha}\ell^\alpha, \quad \alpha \in (0, 1);$$

2. the utility function exhibits constant degree of relative risk aversion; i.e.,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1;$$

3. the shocks  $s$  and  $\nu$  are two independent log-normal random variables; i.e.,

$$s \sim \log \mathcal{N}(0, \sigma_s^2) \quad \text{and} \quad \nu \sim \log \mathcal{N}(0, \sigma_\nu^2).$$

We start by characterizing the solution to the problem in equation (6). Dropping index  $i$  for convenience, and taking the first-order condition with respect to labor, we obtain

$$\alpha \zeta \Delta^{1-\zeta} (s\nu z \omega k^{1-\alpha} \ell^\alpha)^\zeta = \ell,$$

which implies the following labor demand:

$$\ell = \left( \alpha \zeta \Delta^{1-\zeta} [svz\omega k^{1-\alpha}]^\zeta \right)^{\frac{1}{1-\zeta\alpha}}.$$

Plugging the demand for labor into the profit function, we obtain:

$$\pi(s, v, z, \omega, k) = (1 - \alpha\zeta) \Delta^{1-\zeta} [svz\omega k^{1-\alpha}]^\zeta \left( \alpha \zeta \Delta^{1-\zeta} [svz\omega k^{1-\alpha}]^\zeta \right)^{\frac{\zeta\alpha}{1-\zeta\alpha}},$$

or, equivalently,

$$\pi(s, v, z, \omega, k) = (1 - \alpha\zeta) (\alpha\zeta)^{\frac{\zeta\alpha}{1-\zeta\alpha}} \left( \Delta^{1-\zeta} [svz\omega k^{1-\alpha}]^\zeta \right)^{\frac{1}{1-\zeta\alpha}}.$$

Notice the profit function exhibits decreasing returns to scale in capital; i.e.,

$$\frac{\zeta(1-\alpha)}{1-\zeta\alpha} < 1.$$

This is because  $\alpha \in (0, 1)$  and  $\eta > 1$ , which implies that  $\zeta < 1$ .

Let

$$\Omega = (1 - \alpha\zeta) (\alpha\zeta)^{\frac{\zeta\alpha}{1-\zeta\alpha}} \Delta^{\frac{1-\zeta}{1-\zeta\alpha}}.$$

Denote by  $x$  be the product of firm-idiosyncratic permanent productivity and firm-idiosyncratic permanent taste; i.e.,  $x = z\omega$ . Let  $\chi$  be the product of firm-idiosyncratic profitability shocks and firm-idiosyncratic taste shocks; i.e.,  $\chi = sv$ . Then, we can rewrite the profit function as follows:

$$\pi(\chi, x, k) = \Omega \left( \chi^\zeta x^\zeta k^{1-\alpha} \right)^{\frac{1}{1-\zeta\alpha}}$$

Because  $s$  and  $v$  are log-normally distributed,  $\chi$  is also log-normally distributed; i.e.,

$$\chi \sim \log \mathcal{N}(0, \sigma^2),$$

where  $\sigma^2 = \sigma_s^2 + \sigma_v^2$ . Given the log-normality of  $\chi$ , consumption is a three-parameter log-normal random variable with expected

value and variance equal to

$$\bar{c} = \Omega \left( x^{\zeta} k^{\zeta(1-\alpha)} \right)^{\frac{1}{1-\zeta\alpha}} \mathbb{E}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] - rk + T$$

and

$$\sigma_c^2 = \left( \Omega \left( x^{\zeta} k^{\zeta(1-\alpha)} \right)^{\frac{1}{1-\zeta\alpha}} \right)^2 \text{var}[\chi^{\frac{\zeta}{1-\zeta\alpha}}],$$

respectively (and threshold equal to  $-rk + T$ ). Since  $\chi \sim \log \mathcal{N}(0, \sigma^2)$ ,  $\log \chi \sim \mathcal{N}(0, \sigma^2)$ . Thus,

$$\frac{\zeta}{1-\zeta\alpha} \log \chi \sim \mathcal{N} \left( 0, \left( \frac{\zeta}{1-\zeta\alpha} \right)^2 \sigma^2 \right),$$

and

$$\chi^{\frac{\zeta}{1-\zeta\alpha}} \sim \log \mathcal{N} \left( 0, \left( \frac{\zeta}{1-\zeta\alpha} \right)^2 \sigma^2 \right),$$

which implies that

$$\mathbb{E}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] = \exp \left( \frac{\left( \frac{\zeta}{1-\zeta\alpha} \right)^2 \sigma^2}{2} \right)$$

and

$$\text{var}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] = \left( \exp \left( \left( \frac{\zeta}{1-\zeta\alpha} \right)^2 \sigma^2 \right) - 1 \right) \exp \left( \left( \frac{\zeta}{1-\zeta\alpha} \right)^2 \sigma^2 \right).$$

Therefore, an entrepreneur's problem can be rewritten in terms of consumption equivalent as follows:

$$\max_k \quad \Omega x^{\frac{\zeta}{1-\zeta\alpha}} k^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}} \mathbb{E}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] - rk + T - \frac{(\gamma-1)}{2} \left( \Omega x^{\frac{\zeta}{1-\zeta\alpha}} k^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}} \right)^2 \text{var}[\chi^{\frac{\zeta}{1-\zeta\alpha}}].$$

Let  $g(x, k) = \Omega x^{\frac{\zeta}{1-\zeta\alpha}} k^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}}$ . Taking the first-order condition with respect

to capital, we obtain

$$g_k(x, k^o) \left[ \mathbb{E}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] - (\gamma - 1) \text{var}[\chi^{\frac{\zeta}{1-\zeta\alpha}}] g(x, k^o) \right] - r^o = 0,$$

which resembles equation (2) in the main text.

#### A.4 A two-period model with a risk-free asset

We consider a two-period version of the model in Section 2.2, which allows entrepreneurs to self-insure through borrowing and saving in a risk-free asset. Unlike before, entrepreneurs are born with different initial wealth levels  $\omega \in \mathcal{W} \subseteq \mathbb{R}_+$ . Moreover, they can transfer resources over time using a risk-free asset  $a$  that costs  $q$  units of consumption. Entrepreneur  $i$ 's problem can be written as follows:

$$\begin{aligned} & \max_{k_i, a_i, c_i, \{c'_i(s)\}_{s \in \mathcal{S}}} u(c_i) + \int_{s \in \mathcal{S}} u(c'_i(s)) \phi(s) ds \\ \text{s.t. } & c_i \leq \omega_i - qa_i \\ & \text{and } c'_i(s) \leq sf(z_i, k_i) - rk_i + a_i + T, \quad \forall s \in \mathcal{S}. \end{aligned}$$

A competitive equilibrium for this economy is a list of capital choices,  $\{k_i^o\}_{i \in \mathcal{I}}$ , asset choices  $\{a_i^o\}_{i \in \mathcal{I}}$ , consumption plans,  $\{c_i^o(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$ , a rental price of capital  $r^o$ , and a price of the asset  $q^o$  such that:

- $\{k_i^o\}_{i \in \mathcal{I}}$ ,  $\{a_i^o\}_{i \in \mathcal{I}}$  and  $\{c_i^o(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$  solve entrepreneur  $i$ 's problem;
- the market for capital clears; i.e.,

$$\int_i k_i^o di = \bar{K}.$$

- the asset market clears; i.e.,

$$\int_i a_i^o di = 0.$$

Drop index  $i$  for convenience. Notice that, given the log-normality of  $s$ , consumption is a three-parameter log-normal random variable with

expected value and variance equal to

$$\bar{c}' = \mu_s f(z, k) - rk + a + T$$

and

$$\sigma_c^2 = f(z, k)^2 \sigma_s^2,$$

respectively (and threshold equal to  $-rk + a + T$ ). Therefore, an entrepreneur's problem can be rewritten in terms of consumption equivalent as follows:

$$\max_{k, a} u(\omega - qa) + u\left(\bar{c}' - \frac{\gamma - 1}{2} \sigma_c^2\right).$$

Taking the first-order conditions with respect to  $k$  and  $a$ , we obtain

$$0 = u_c \left( \mu_s f(z, k^0) - r^0 k^0 + a^0 + T - \frac{\gamma - 1}{2} f(z, k^0)^2 \sigma_s^2 \right) \left( f_k(z, k^0) [\mu_s - (\gamma - 1) f(z, k^0)^2 \sigma_s^2] - r^0 \right)$$

and

$$q^0 u_c(\omega - q^0 a^0) = u_c \left( \mu_s f(z, k^0) - r^0 k^0 + a^0 + T - \frac{\gamma - 1}{2} f(z, k^0)^2 \sigma_s^2 \right),$$

Because  $u_c > 0$ , the first order conditions can be rewritten as follows:

$$f_k(z, k^0) \left[ \mu_s - \frac{\gamma - 1}{2} f(z, k^0)^2 \sigma_s^2 \right] = r^0$$

and

$$[1 + (q^0)^{(1-\frac{1}{\gamma})}] a^0 = (q^0)^{-\frac{1}{\gamma}} \omega - \left( \mu_s f(z, k^0) - r^0 k^0 + T - \frac{\gamma - 1}{2} f(z, k^0)^2 \sigma_s^2 \right).$$

Notice that the first-order condition for capital allocation mirrors the one derived in a Section 2.2. While the presence of a risk-free asset enables entrepreneurs to smooth consumption over time, it is not sufficient to complete insurance markets, leaving second-period utilities uncertain.

## A.5 A model with financial frictions

We consider a version of the model in Section 2.2, extended to account for financial frictions. Unlike before, the amount of capital that agents can invest is constrained by their production capability. That is, we assume that capital expenditure,  $rk$ , must be less than or equal to a multiple  $\iota \geq 1$ , of expected firm-level production (Bigio and La'io, 2020). Entrepreneur  $i$ 's problem can be written as follows:

$$\begin{aligned} & \max_{k_i, \{c_i(s)\}_{s \in \mathcal{S}}} \int_{s \in \mathcal{S}} u(c_i(s)) \phi(s) ds \\ & \text{s.t. } c_i(s) \leq sf(z_i, k_i) - rk_i + T, \quad \forall s \in \mathcal{S} \\ & \text{and } rk_i \leq \iota \int_{s \in \mathcal{S}} sf(z_i, k_i) \psi(s) ds \end{aligned}$$

where, again,  $T = r\bar{K}$  are revenues from renting capital, assumed to be rebated lump sum to each entrepreneur.

A competitive equilibrium for an economy with incomplete markets and financial frictions is a list of capital choices  $\{k_i^o\}_{i \in \mathcal{I}}$ , consumption plans  $\{c_i^o(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$ , and a rental price of capital  $r^o$  such that

- $\{k_i^o\}_{i \in \mathcal{I}}$  and  $\{c_i^o(s)\}_{s \in \mathcal{S}, i \in \mathcal{I}}$  solve the problem of the entrepreneurs;
- the market for capital clears; i.e.,

$$\int_i k_i^o di = \bar{K}.$$

To characterize the solution to this problem, let us drop subscript  $i$ . Given the log-normality of the shock  $s$ , consumption follows a three-parameter log-normal distribution, with an expected value of  $\bar{c} = \mu_s f(z, k) - rk + T$ , variance  $\sigma_c^2 = f(z, k)^2 \sigma_s^2$ , and a threshold of  $-rk + T$ . Therefore, an entrepreneur's problem can be rewritten in terms of consumption equivalent as follows:

$$\max_k \mu_s f(z, k) - rk + T - \frac{(\gamma - 1)}{2} f(z, k)^2 \sigma_s^2 \quad \text{s.t. } rk \leq \iota \int_{s \in \mathcal{S}} sf(z, k) \phi(s) ds.$$

If the financial constraint is binding, we have that:

$$r^o k = \iota \mu_s f(z, k)$$

whereas, if the financial constraint is not binding, firms' optimality implies:

$$r^o = f_k(z, k) [\mu_s - (\gamma - 1) \sigma_s^2 f(z, k)]$$

Let  $f(z, k)$  be homogeneous of degree  $\alpha \in (0, 1]$ . Then, we can re-write the last equation as:

$$r^o k = \alpha f(z, k) [\mu_s - (\gamma - 1) \sigma_s^2 f(z, k)].$$

Therefore, it follows that:

$$r^o = f_k(z, k) \frac{\phi(z, k)}{\alpha}$$

where  $\phi(z, k) = \min \{ \iota \mu_s, \alpha [\mu_s - (\gamma - 1) \sigma_s^2 f(z, k)] \}$ , represents the firm-specific wedge that arises between the firms' marginal cost and marginal revenue.

In this version of the model, there are two potential sources of misallocation. First, when binding, financial frictions generate a wedge between the marginal product of capital and its rental rate. By preventing entrepreneurs to reach their utility maximizing scale, they distort capital allocation across firms. Second, incomplete markets coupled with time-to-build constraint amplify the existing wedge, i) if entrepreneurs are enough risk averse, i.e., if  $\gamma > 1$ ; and 2) the larger the volatility of the idiosyncratic profitability shock,  $\sigma_s^2$ .

## B Data

### B.1 Summary statistics

Table B1 presents selected summary statistics for our sample. On average, firms are 15 years old and employ about 12 workers. Their average annual sales amount to roughly 1 million euros, corresponding to an average of 56 thousand euros per employee. The value of their fixed assets is approximately twice this amount. On average, firms have an annual payroll of about 230 thousand euros, implying an average salary of 14 thousand euros per employee. The vast majority of these firms are small and privately owned.

**Table B1: Summary Statistics**

	Mean (1)	S.d. (2)	p10 (3)	Median (4)	p90 (5)	Observations (6)
Age	15.3	13.3	2	12	32	2195287
N. of Employees	12.5	123.1	1	4	18	2195287
Fixed Assets	2188449.6	67542046.5	23074.4	168697.4	1772260.8	2195287
Sales	1032526.8	22591464.4	21683.9	115972	981823.7	2195287
Sales per employee	56227.1	780720.9	10878.5	30355.9	92847.9	2195287
Material expenditure	9462.3	194044.9	190	1816	13597	2195287
Payroll	233563.6	2615366.9	10041.9	44258.2	307049.9	2195287
Wage per employee	14031.1	12311.8	6941.6	11667.1	22786.5	2195287
Value added	1023064.5	22561743.2	20782.8	113089.6	966400.6	2195287
Value added per employee	55256.6	780490.3	10479.2	29600.5	91097.4	2195287

NOTES: Nominal variables are deflated using the national CPI and expressed in 2010 price levels. The number of employees includes both full-time and part-time workers. Fixed assets encompass both current and non-current assets.

Table B2 shows the share of firms by type of ownership. Over 90% of firms are held privately help (“Sociedade por quotas”).

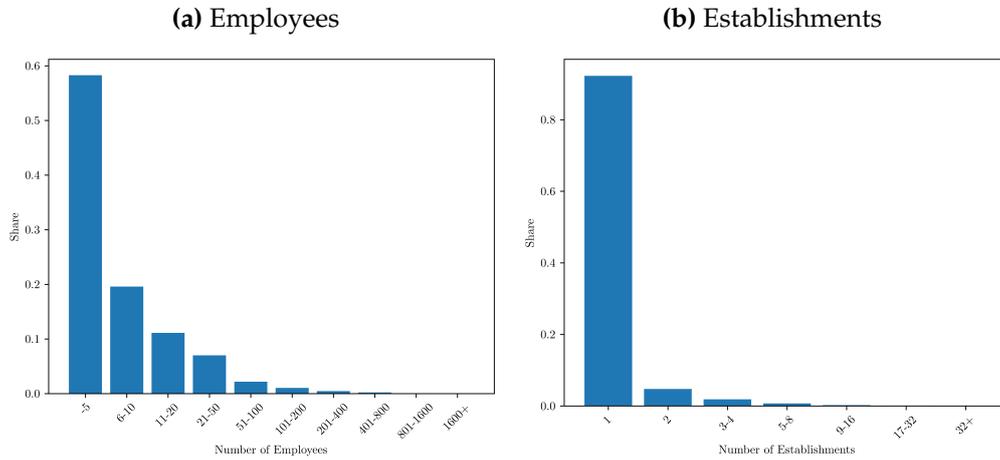
**Table B2: Firm distribution by ownership**

Type	Share
Sociedade por quotas	0.642
Sociedade unipessoal por quotas	0.289
Sociedade anonima	0.052
Other	0.016

Figure B.1 shows the distributions of firm size and establishments per

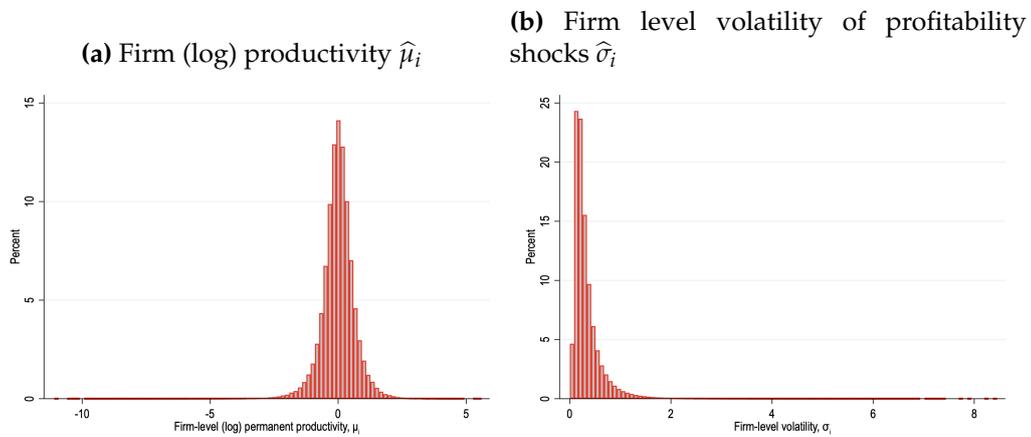
firm. Approximately 80% of firms have 10 or fewer employees and that 90% operate a single establishment.

**Figure B.1: Firm size distributions**



NOTES: This figure reports the empirical distribution of firm size by employees (Panel A), and number of establishments (Panel B).

**Figure B.2: Production function estimates - Distributions**



NOTES: This figure reports the empirical distribution of firm productivity estimates  $\hat{\mu}_i$  (Panel A), and the estimate volatility of profitability shocks at the firm level  $\hat{\sigma}_i$  (Panel B) across firms in the sample.

Figure B.2 shows the distribution of firm productivity  $\mu_i$  and firm-level volatility of shocks  $\sigma_i$ , while Table B3 reports selected statistics for both variables. (Log) permanent productivity is largely dispersed across firms: the standard deviation is 65.6%. Profitability shocks are volatile: the average firm-level volatility in the sample is about 34.7%.

**Table B3:** Production function estimates—summary statistics

	Mean (1)	S.d. (2)	p10 (3)	Median (4)	p90 (5)	Observations (6)
Firm-level (log) permanent productivity $\mu_i$	0	0.656	-0.691	0.011	0.706	2,170,852
Firm-level volatility of profitability shocks $\sigma_i$	0.347	0.327	0.108	0.243	0.646	2,173,951

NOTES: Nominal variables used in the production function estimation are deflated using the national CPI and expressed in 2010 price levels. The number of employees includes both full-time and part-time workers. Fixed assets encompass both current and non-current assets.

Finally, Table B4 presents the estimated correlation between shock volatility and capital misallocation, controlling for 2-digit CAE industry and NUTS-2 region fixed effects.

**Table B4:** Volatility and capital misallocation across local markets

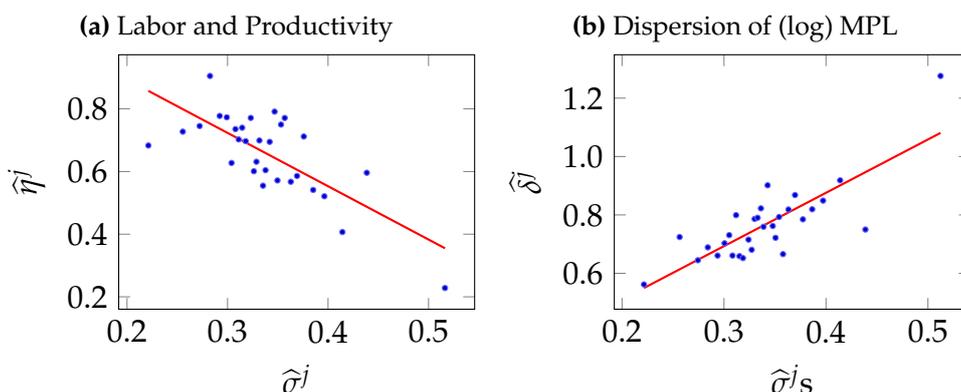
	$\hat{\eta}^j$		$\hat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}^j$	-3.999*** (0.677)	-2.530** (0.730)	2.096*** (0.435)	1.649*** (0.134)
$R^2$	0.433	0.809	0.534	0.902
Observations	397	397	397	397
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of capital misallocation on profitability shock volatility across local markets. In Columns 1 and 2, capital misallocation is measured as the correlation between firm productivity and the average (log) capital holdings of firms within each local market. In Columns 3 and 4, capital misallocation is measured as the standard deviation of the average within-firm marginal product of capital within each local market. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parenthesis) are clustered at the location-industry level. \* $p < 0.1$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

## B.2 Labor misallocation

Figure B.3 scatters two measures of labor misallocation against the volatility of profitability shocks across local markets.

**Figure B.3:** Volatility and labor misallocation across local markets



NOTES: This figure binscatters the relationship between labor misallocation and profitability shock volatility across local markets. In Panel A, labor misallocation is measured as the correlation between firm productivity and the average (log) employment of firms within each local market. In Panel B, labor misallocation is measured as the standard deviation of the average within-firm marginal product of labor across firms in each local market. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations.

Table B5 presents the estimated correlation between the two measures of labor misallocation and the volatility of profitability shocks across local markets, controlling for 2-digit CAE industry and NUTS-2 region fixed effects.

Although firm-level volatility exhibits a stronger correlation with capital misallocation, labor allocation follows similar patterns across local markets. Based on estimates from columns (2) and (4), transitioning from local markets with an average volatility of 20% to those with twice the volatility reduces the correlation between firm productivity and employee count by 0.25, while increasing the dispersion of log marginal product of labor by 0.20.

**Table B5:** Volatility and labor misallocation across local markets

	$\hat{\eta}^j$		$\hat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}^j$	-2.017** (0.586)	-1.251* (0.582)	1.952*** (0.493)	1.304*** (0.244)
$R^2$	0.451	0.744	0.550	0.818
Observations	397	397	397	397
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of labor misallocation on profitability shock volatility across local markets. In Columns 1 and 2, labor misallocation is measured as the correlation between firm productivity and the average (log) employment of firms within each local market. In Columns 3 and 4, labor misallocation is measured as the standard deviation of the average within-firm marginal product of labor within each local market. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the industry and location level. \* $p < 0.1$ ,  $p < 0.05$ , \*\* $p < 0.01$ .

### B.3 Shock persistence

For each local market  $j$ , we estimate the following production function:

$$\log y_{it} = \beta^{0j} + \beta^{1j} \log k_{it} + \beta^{2j} \log \ell_{it} + \beta^{3j} \log m_{it} + \mu_i + v_{it}$$

where  $y_{it}$  represents revenues (sales) for firm  $i$  in year  $t$ ,  $k_{it}$  denotes the capital stock,  $\ell_{it}$  is the number of employees,  $m_{it}$  denotes intermediate inputs, and  $\mu_i$  and  $\mu_t$  represent firm and year fixed effects, respectively. Finally,  $v_{it}$  is an error term that follows an autoregressive process of order 1; i.e.,

$$v_{it} = \rho v_{it-1} + \epsilon_{it}$$

where  $\epsilon_{it}$  is an i.i.d. error term. Following Blundell and Bond (2000), we can express the model using a dynamic (common factor) representation;

i.e.,

$$\begin{aligned}\log y_{it} &= \beta^{0j} + \beta^{1j} \log k_{it} - \rho \beta^{1j} \log k_{it-1} \\ &\quad + \beta^{2j} \log \ell_{it} - \rho \beta^{2j} \log(q_{it-1} \ell_{it-1}) \\ &\quad + \beta^{3j} \log m_{it} - \rho \beta^{3j} \log m_{it-1} \\ &\quad + \rho \log y_{it-1} + \mu_i(1 - \rho) + \epsilon_{it}\end{aligned}$$

Therefore, we can estimate the following unrestricted equation:

$$\begin{aligned}\log y_{it} &= \pi_{0j} + \pi_{1j} \log k_{it} + \pi_{2j} \log k_{it-1} \\ &\quad + \pi_{3j} \log \ell_{it} + \pi_{4j} \log(q_{it-1} \ell_{it-1}) \\ &\quad + \pi_{5j} \log m_{it} + \pi_{6j} \log m_{it-1} \\ &\quad + \pi_{7j} \log y_{it-1} + \mu_i^* + \epsilon_{it},\end{aligned}$$

using suitably lagged levels of the variables as instruments after first differences to control for firm-level permanent unobserved heterogeneity. This allows us to obtain an estimate of our coefficients of interest,  $\beta^{0j}, \beta^{1j}, \beta^{2j}, \beta^{3j}$ , and  $\rho$ , as given by

$$\hat{\beta}^{0j} = \hat{\pi}_{0j}, \quad \hat{\beta}^{1j} = \frac{\hat{\pi}_{1j} + \hat{\pi}_{2j}}{1 - \hat{\pi}_{7j}}, \quad \hat{\beta}^{2j} = \frac{\hat{\pi}_{3j} + \hat{\pi}_{4j}}{1 - \hat{\pi}_{7j}}, \quad \hat{\beta}^{3j} = \frac{\hat{\pi}_{5j} + \hat{\pi}_{6j}}{1 - \hat{\pi}_{7j}}, \quad \hat{\rho} = \hat{\pi}_{7j}.$$

We use  $\hat{\beta}^{0j}, \hat{\beta}^{1j}, \hat{\beta}^{2j}$ , and  $\hat{\beta}^{3j}$  to compute the following residuals:

$$\hat{r}_{it} = \log y_{it} - \hat{\beta}^{0j} - \hat{\beta}^{1j} \log k_{it} - \hat{\beta}^{2j} \log \ell_{it} - \hat{\beta}^{3j} \log m_{it}.$$

We obtain an estimate of firm productivity  $\hat{\mu}_i$  by taking the time average of  $\hat{r}_{it}$  for every firm  $i$ ; i.e.,

$$\hat{\mu}_i = \frac{1}{T} \sum_{t=1}^T \hat{r}_{it}.$$

Finally, notice that  $\hat{r}_{it} - \hat{\mu}_i = \hat{v}_{it}$ . Hence we can estimate the volatility of profitability shocks at the firm level,  $\hat{\sigma}_i$ , as the standard deviation of the demeaned residual, adjusted by the estimated persistence of the error

term process; i.e.,

$$\hat{\sigma}_i = \sqrt{(1 - \hat{\pi}_{7j}^2) \sum_{t=1}^T (\hat{v}_{it} - \bar{\hat{v}}_{it})^2}$$

where  $\bar{\hat{v}}_{it} = \frac{1}{T} \sum_{t=1}^T \hat{v}_{it}$ .

With  $\hat{\beta}^{1j}$ ,  $\hat{\beta}^{2j}$ ,  $\hat{\mu}_i$ , and  $\hat{\sigma}_i$  at hand, we construct the measures of average volatility,  $\hat{\sigma}^j$ , and capital (labor) misallocation,  $\hat{\eta}^j$  and  $\hat{\delta}^j$ , for each local market  $j$ , as described in the main text, and proceed to estimate the following regression:

$$\hat{\omega}^j = \gamma_0 + \gamma_1 \hat{\sigma}^j + \epsilon^j,$$

where  $\hat{\omega}^j$  is either  $\hat{\eta}^j$  or  $\hat{\delta}^j$ .

Tables B6 and B7 present the estimated correlation between capital misallocation and the volatility of profitability shocks across local markets, controlling for 2-digit CAE industry and NUTS-2 region fixed effects. Using estimates from columns (2) and (4), an increase in local market volatility from an average of 20% to twice that level reduces the correlation between firm productivity and capital holdings by 0.53, while the dispersion of log marginal product of capital rises by 1.00. Similarly, the correlation between firm productivity and number of employees decreases by 0.31, while the dispersion of log marginal product of labor increases by 0.35. Consistent with the main specification, variations in profitability shock volatility across local markets exhibit a stronger correlation with capital misallocation than with labor misallocation.

**Table B6:** Volatility and capital misallocation across local markets with shock persistence

	$\hat{\eta}^j$		$\hat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}^j$	-2.826*** (0.728)	-2.672* (1.113)	1.974*** (0.369)	4.978** (1.717)
$R^2$	0.410	0.575	0.397	0.611
Observations	397	397	389	389
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of capital misallocation on profitability shock volatility across local markets. In Columns 1 and 2, capital misallocation is measured as the correlation between firm productivity and the average (log) capital holdings of firms within each local market. In Columns 3 and 4, capital misallocation is measured as the standard deviation of the average within-firm marginal product of capital within each local market. Estimates of firm productivity, the average within-firm marginal product of capital, and firm-level profitability shock volatility—used to construct our measures of capital misallocation and volatility of profitability shocks at the local market level—are obtained from regression models in which the error term follows an autoregressive process of order one. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions in this table include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the location-industry level. \* $p < 0.1$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

**Table B7:** Volatility and labor misallocation across local markets with shock persistence

	$\hat{\eta}^j$		$\hat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}^j$	-1.705** (0.578)	-1.530** (0.538)	2.079*** (0.380)	1.726*** (0.368)
$R^2$	0.443	0.675	0.627	0.812
Observations	397	397	395	395
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of labor misallocation on profitability shock volatility across local markets. In Columns 1 and 2, labor misallocation is measured as the correlation between firm productivity and the average (log) employment of firms within each local market. In Columns 3 and 4, labor misallocation is measured as the standard deviation of the average within-firm marginal product of labor within each local market. Estimates of firm productivity, the average within-firm marginal product of labor, and firm-level profitability shock volatility—used to construct our measures of labor misallocation and volatility of profitability shocks at the local market level—are obtained from regression models in which the error term follows an autoregressive process of order one. Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions in this table include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the location-industry level. \* $p < 0.1$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

## B.4 Firm exit

We correct for firm-level exit selection following the approach proposed by Mundlak (1978). We rely on a full parametrization of the sample selection mechanism, and assume the conditional expectation of the individual effects in the selection equation,  $\zeta_i$  to be a function of the firm-level average of the time-varying covariates used in equation (3), i.e.

$$\zeta_i = f(\overline{\log k_{it}}, \overline{\log \ell_{it}}, \overline{\log m_{it}}) \quad (7)$$

We choose a fully interacted third-order polynomial for  $f(\cdot)$ . Therefore we follow Wooldridge (1995) and implement a two-step procedure. In the first step, we estimate a probit on equations (4) and (7) separately for every year  $t$ , and obtain a selection correction terms  $\lambda_{it}$ , which is equal to the inverse Mills ratio. In the second step we estimate equation (3) including  $\lambda_{it}$  as a control.

Tables B8 and B9 report the estimated correlation of firm-level volatility and capital misallocation conditional on 2-digit CAE industry and NUTS-2 region fixed effects, and accounting for self-selection in firm exit.

Using estimates from columns (2) and (4), as we move from local markets with an average volatility of 20% to those with volatility twice as large, the correlation between firm-level capital and productivity reduces by 0.45, while the dispersion of log MPK increases by 0.30.

Correcting for exit selection of firms only marginally affects the magnitude of our estimates: while it amplifies the estimated correlations in the unweighted specifications (columns 1 and 3), it hampers them in the weighted ones (columns 2 and 4), suggesting the selection introduces a larger bias as we move towards markets with a higher number of firms.

**Table B8:** Volatility and capital misallocation across local markets — Heckman correction for firm exit

	$\hat{\eta}^j$		$\hat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}^j$	-3.891*** (0.707)	-2.288** (0.646)	2.496*** (0.501)	1.547*** (0.103)
$R^2$	0.434	0.786	0.533	0.883
Observations	397	397	389	389
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of capital misallocation on profitability shock volatility across local markets. In Columns 1 and 2, capital misallocation is measured as the correlation between firm productivity and the average (log) capital holdings of firms within each local market. In Columns 3 and 4, capital misallocation is measured as the standard deviation of the average within-firm marginal product of capital within each local market. Estimates of firm productivity, the average within-firm marginal product of capital, and firm-level profitability shock volatility—used to construct our measures of capital misallocation and volatility of profitability shocks at the local market level—are obtained from regression models that account for firm exit as in Wooldridge (1995). Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions in this table include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the location-industry level. \* $p < 0.1$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

**Table B9:** Volatility and labor misallocation across local markets — Heckman correction for exit

	$\hat{\eta}^j$		$\hat{\delta}^j$	
	(1)	(2)	(3)	(4)
$\hat{\sigma}^j$	-2.089** (0.614)	-1.279* (0.529)	2.066*** (0.490)	1.263*** (0.226)
$R^2$	0.455	0.721	0.557	0.822
Observations	397	397	395	395
Weighted	No	Yes	No	Yes

NOTES: This table reports OLS estimates from regressions of our two measures of labor misallocation on profitability shock volatility across local markets. In Columns 1 and 2, labor misallocation is measured as the correlation between firm productivity and the average (log) employment of firms within each local market. In Columns 3 and 4, labor misallocation is measured as the standard deviation of the average within-firm marginal product of labor within each local market. Estimates of firm productivity, the average within-firm marginal product of labor, and firm-level profitability shock volatility—used to construct our measures of labor misallocation and volatility of profitability shocks at the local market level—are obtained from regression models that account for firm exit as in Wooldridge (1995). Local markets are defined as the combination of a two-digit CAE industry classification and a NUTS-2 region with at least 100 firm-year observations. All regressions in this table include location and industry fixed effects. In Columns (2) and (4), observations are weighted by the number of firms in their local markets. Standard errors (in parentheses) are clustered at the location-industry level. \* $p < 0.1$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

## C Estimation and counterfactual

### C.1 Alternative distortions

Table C1 reports the list of model parameters, their calibrated values, and the source or target used to discipline them in the version of the model that includes exogenous correlated distortions.

**Table C1:** Parameters - Model with exogenous correlated distortions

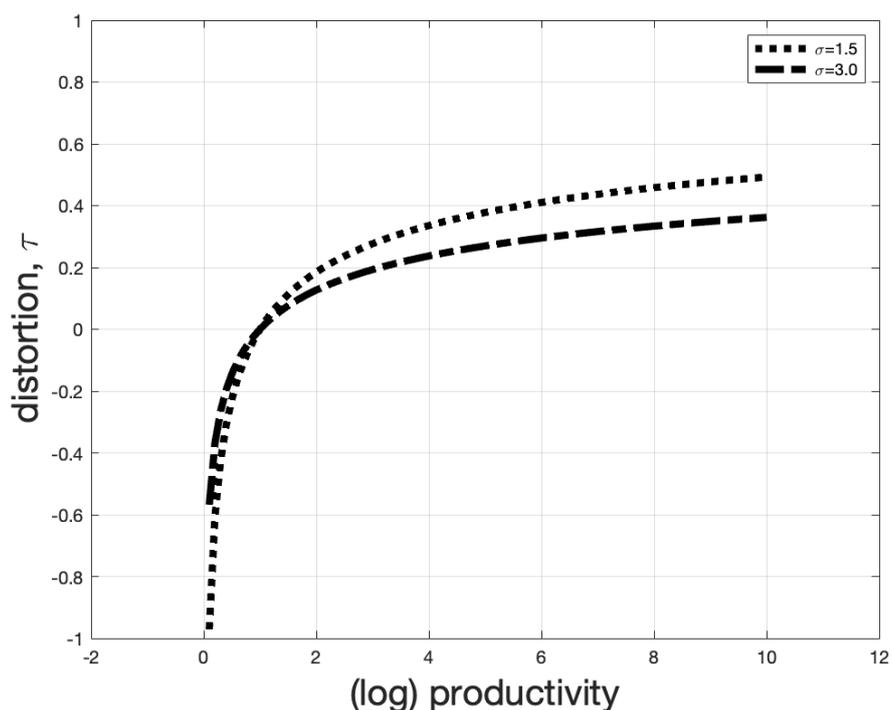
Parameters	Description	Value	Source/Target
<i>A. Parameters calibrated</i>			
$\alpha$	Output elasticity of capital	0.416	Aggregate capital share (FRED)
$\sigma$	Volatility of shocks	0.347	Data
$(\kappa, \theta)$	Distribution of permanent productivity	(1.404, 1.060)	Data
<i>B. Parameters estimated</i>			
<i>Case 1.</i>			
$\gamma$	Relative risk aversion	1.5	Cagetti and De Nardi (2006)
$\varphi$	Correlation distortions	0.295	corr. $[\log k, \log z] = 0.494$
<i>Case 2.</i>			
$\gamma$	Relative risk aversion	3	Fuchs et al. (2024)
$\varphi$	Correlation distortions	0.195	corr. $[\log k, \log z] = 0.494$

NOTES: This table reports the values and sources of the calibrated and estimated parameters in the version of the model that includes exogenous correlated distortions.

Case 1 refers to a version of the model with a constant relative risk aversion of 1.5. Case 2 refers to a version of the model with a constant relative risk aversion of 3. The correlation of distortions,  $\varphi$  is estimated to be 0.295 in Case 1 and 0.195 in Case 2.

Figure C.1 scatters the estimates of correlated distortions,  $\tau_i$  against (log) firm productivity level,  $z_i$ , when the relative risk aversion coefficient,  $\gamma$  is either 1.5 or 3. A positive value of the wedge implies a tax on capital expenditure. A negative value implies a subsidy on capital expenditure.

**Figure C.1:** Exogenous correlated distortions — Estimates



NOTES: This figure reports the estimated correlated distortions  $\tau_i$  for firms of different productivity level,  $z_i$  when the coefficient of relative risk aversion is either 1.5 or 3.

Table C2 shows several counterfactual results in a calibrated model that includes exogenous correlated distortions.

**Table C2:** The aggregate cost of incomplete markets -  
Model with exogenous correlated distortions

	Counterfactual scenarios				Explained % (5)
	Baseline (1)	No uncertainty (2)	Complete markets (3)	Complete markets & no distortions (4)	
Volatility, $\sigma$	0.347	0	0.347	0.347	-
<i>Case 1. (<math>\gamma = 1.5</math>)</i>					
Correlated distortions, $\varphi$	0.295	0.295	0.295	0	-
corr[log z, log k]	0.503	0.615	0.615	1.000	22.54%
$\omega$ [log z, log k]	0.310	0.424	0.424	1.712	8.13%
sd[log k]	0.810	0.989	0.989	1.594	22.83%
Aggregate Income	1.000	1.010	1.064	1.139	46.04%
<i>Case 2. (<math>\gamma = 3</math>)</i>					
Correlated distortions, $\varphi$	0.195	0.195	0.195	0	-
corr[log z, log k]	0.489	0.996	0.996	1.000	99.22%
$\omega$ [log z, log k]	0.583	1.250	1.250	1.712	59.08%
sd[log k]	0.538	1.216	1.216	1.594	64.20%
Aggregate Income	1.000	1.169	1.237	1.242	97.93%

NOTES: This table reports selected outcomes for the baseline economy (column 1), a counterfactual economy with complete insurance markets and correlated distortions (column 2), and a counterfactual economy with complete insurance markets and no correlated distortions (column 3). Values in column (4) refer to the % of each outcomes in column (3) explained by market incompleteness, and are computed as  $100 \times \frac{(3)-(1)}{(4)-(1)}$ . Case 1 refers to a version of the model with a constant relative risk aversion of 1.5. Case 2 refers to a version of the model with a constant relative risk aversion of 3.

## C.2 Alternative elasticity of capital

Table C3 reports the estimated coefficient of relative risk aversion (RRA) for different values of capital elasticity, along with the corresponding output per unit of capital and income gains from the counterfactual exercises under different capital share values.

**Table C3:** Sensitivity to alternative capital shares

Capital share $\alpha$	0.5	0.6	0.7	0.8	0.9
Estimated $\gamma$	3.507	3.715	4.024	4.235	4.717
Income gains %					
from no uncertainty	29.94	47.05	85.37	126.0	251.9
from complete markets	38.86	57.22	96.85	138.3	270.6

NOTES: This table reports output per unit of capital and aggregate income gains (in %) from two counterfactual scenarios: one with no idiosyncratic production uncertainty and another with complete markets, for alternative economies that differ in the capital share  $\alpha$ . In each of these economies, the coefficient of relative risk aversion  $\gamma$  is re-estimated to match the observed correlation between productivity and capital holdings across firms.

### C.3 Alternative technology

Table C4 shows the calibrated parameters for the model that includes labor as a factor of production.

**Table C4:** Parameters - Model with capital and labor inputs

Parameters	Description	Value	Source/Target
<i>A. Parameters calibrated</i>			
$\alpha$	Output elasticity of labor	0.649	Aggregate labor share (FRED)
$\eta$	Span on control	0.9	Erosa et al. (2023)
$\sigma$	Volatility of shocks	0.347	Data
$(\kappa, \theta)$	Distribution of permanent productivity	(1.404, 1.060)	Data
<i>B. Parameters estimated</i>			
$\gamma$	Relative risk aversion	5.111	corr. $[\log k, \log z] = 0.494$

NOTES: This table reports the values and source of the calibrated and estimated parameters for a version of the model that includes labor as a factor of production.

Table C5 shows the counterfactual results when using the calibrated model that includes labor as a factor of production.

**Table C5:** The aggregate cost of incomplete markets - Model with capital and labor inputs

	Baseline (1)	Counterfactual scenarios	
		No uncertainty (2)	Complete markets (3)
Volatility, $\sigma$	0.347	0	0.347
corr $[\log z, \log k]$	0.502	1	1
$\omega[\log z, \log k]$	1.052	4.159	4.159
sd $[\log k]$	2.534	7.010	7.010
Aggregate Income	1.000	1.461	1.632

NOTES: This table reports selected outcomes for the baseline economy (column 1), a counterfactual economy with no idiosyncratic production uncertainty (column 2), and a counterfactual economy with complete insurance markets (column 3), obtained from a version of the model that includes labor as a factor of production.