Advanced Macroeconomics

Misallocation

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- Optimal allocation of production across heterogeneous firms
  - M heterogeneous firms i
  - idiosyncratic productivity A_i
  - decreasing returns to scale in production w.r.t. labor n_i

$$y_i = A_i^{1-\alpha} n_i^{\alpha} \quad \alpha \in (0,1)$$

  - fixed labor supply N

- Planner problem:

$$\max \sum_{i=1}^{n} A_i^{1-\alpha} n_i^{\alpha} \quad \text{s.t.} \sum_{i=1}^{N} n_i \leq N$$

- Planner solution: $\log n_i = \log c + \log A_i$
  where c is a constant

- Aggregate output with efficient allocation:

$$Y = \sum_{i=1}^{M} y_i = \left(\sum_{i=1}^{M} A_i \right)^{1-\alpha} N^{\alpha} = \frac{\frac{1}{M} \sum_{i=1}^{M} A_i}{\left(\frac{1}{M} \sum_{i=1}^{M} A_i \right)^{\alpha}} M^{1-\alpha} N^{\alpha}$$

- Suppose labor allocation deviates from planner solution:

$$\log n_i^O = \log c + \log A_i + \log(1 + \tau_i)$$

where $\sum_{i=1}^{N} A_i \tau_i = 0$ s.t. $\sum_{i=1}^{M} n_i^O = N$

- Aggregate output with non-efficient allocation:

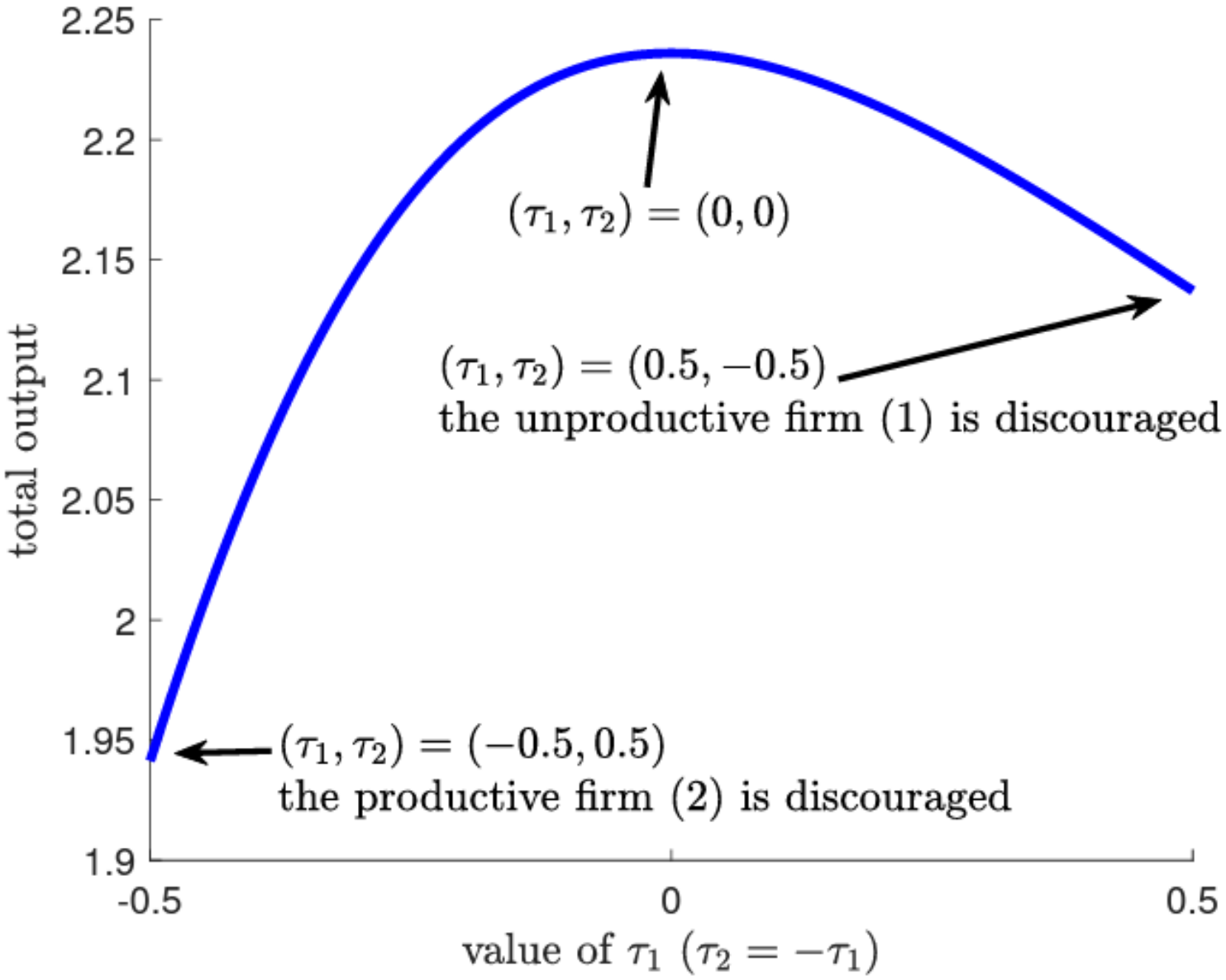
$$Y^O = \sum_{i=1}^{M} y_i = \frac{\sum_{i=1}^{M} A_i(1+\tau_i)^\alpha}{\left(\sum_{i=1}^{M} A_i(1+\tau_i)\right)^\alpha} N^\alpha$$

$$= \frac{\frac{1}{M}\sum_{i=1}^{M} A_i(1+\tau_i)^\alpha}{\left(\frac{1}{M}\sum_{i=1}^{M} A_i(1+\tau_i)\right)^\alpha} M^{1-\alpha} N^\alpha$$

$$= \frac{\frac{1}{M}\sum_{i=1}^{M} A_i(1+\tau_i)^\alpha}{\left(\frac{1}{M}\sum_{i=1}^{M} A_i\right)^\alpha} M^{1-\alpha} N^\alpha$$

- Output loss: $\frac{Y^O}{Y} = \frac{\sum_{i=1}^{M} A_i(1+\tau_i)^\alpha}{\sum_{i=1}^{M} A_i} < 1 \quad \text{if} \quad \alpha \in (0,1)$

Example: economy with two firms



- Misallocation can change aggregate TFP, A, without changing firm-level productivity A_i
- The effect of distorted allocation on aggregate TFP can be sizable:
  - how can we measure distortions in the data?
  - what are the sources of these distortions?
  - to what extent can this explain cross-country differences in income p.c.?
  - what would happen if distortions disappear?

- Restuccia D. and Rogerson R. 2008. "Policy Distortions and Aggregate Productivity with Heterogeneous Plants." Review of Economic Dynamics. Vol 11, No. 4, pp. 707-72
  - General equilibrium version of Hopenhayn (1992)
  - Neoclassical model of growth
  - Firms heterogeneity in productivity
  - Exogenous idiosyncratic distortions
  - Productivity losses from *misallocation*

- Evidence of heterogeneous distortions across countries:
  - credit constraints (e.g., Parente and Prescott 1999)
  - labour regulation (e.g., Lagos 2006)
  - gov. subsidies (e.g., Guner et al. 2008)
- Assess quantitative significance of firm-level distortions on output, productivity and employment
  - misallocation across productive units
- Reduced form representation of idiosyncratic distortions to producer prices: tax/subsidy τ on output
- Results: heterogeneity in prices faced by producers can lead to decrease in TFP and output of up 50%

- Time is discrete, focus on steady-state
- Output price is the model numeraire ($p = 1$)
- Wage w and interest rate r determined endogenously
- Representative household
  - Consumption/savings decision
  - Inelastic labor supply
- Endogenous measure of heterogeneous firms
  - Perfect competition in product and labor markets
  - Innate differences in firm-level productivity
  - Entry-exit dynamics
  - Free-entry

- Utility function of consumption C_t

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \beta \in (0,1)$$

where β is the discount factor

- Budget constraint:

$$C_t + I_t \leq w_t + r_t K_t + \Pi_t - T_t$$

where Π_t are aggregate profits and T_t are net taxes

- Capital depreciates at rate δ

$$K_{t+1} = (1-\delta)K_t + I_t$$

- Problem of the HH:

$$\mathcal{U} = \max_{C_t, K_{t+1}} \quad \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\text{s.t.} \quad C_t + K_{t+1} \leq w_t + (1 + r_t - \delta) K_t + \Pi_t - T_t$$

$$K_0 > 0 \quad \text{given}$$

- First order condition (at the interior) imply:

$$U'(C_t) = \beta U'(C_{t+1})(1 + r_t - \delta)$$

- In steady-state, $C_t = C_{t+1}$ and

$$r^* = \frac{1}{\beta} - 1 + \delta$$

- Firms differ in productivity, z and distortion, τ
- Firm-level output

$$f(z, n, k) = zn^{\alpha}k^{\gamma} \quad \alpha, \gamma \in (0, 1)$$

where $\alpha + \gamma < 1$: decreasing return to scale.

- Static firm-level profits

$$\pi(z, \tau; w, r) = \max_{n \geq 0, k \geq 0} \quad (1 - \tau) f(z, n, k) - wn - rk - c_o$$

where c_o denotes per-period operating costs.

- Let $n(z, \tau; w, r)$ and $k(z, \tau; w, r)$ be the optimal employment and capital demand function

- $V(z, \tau; w, r)$: value function for firm in states (z, τ) and prices (w, r)

$$V(z, \tau; w, r) = \pi(z, \tau; w, r) + \frac{(1-\lambda)}{1+r-\delta} V(z, \tau; w, r)$$

where λ is an exogenous probability of exit

- Expected discounted value of per-period profits:

$$V(z, \tau; w, r) = \frac{\pi(z, \tau; w, r)}{1 - \frac{(1-\lambda)}{1+r-\delta}}$$

- Large measure of identical entrants pay c_e and draw productivity z and distortion τ from a joint distribution

$$\Gamma(z, \tau) = P(\tau|z)H(z)$$

- Entry decision: $V^e(z, \tau; w, r) = \max\{0, V(z, \tau; w, r)\}$. Solution to this problem is a policy for optimal entry: $\mathbf{1}^e(z, \tau; w, r)$
- Measure of entry: $M \geq 0$
- Free entry condition:

$$v^e(w, r) = \sum_z \sum_\tau V^e(z, \tau; w, r)\Gamma(z, \tau) \leq c_e$$

with equality if $M > 0$.

- Let $\mu_t(z, \tau; w_t, r_t)$ be the measure of firms over individual state (z, τ) when wage and interest rate rate (w_t, r_t) at time t
- Evolution of distribution over time:

$$\mu_{t+1}(z, \tau; w_{t+1}, r_{t+1}) = T_t(\mu_t(z, \tau; w_t, r_t), M_t, w_t, r_t)$$

where

$$T_t(\mu_t(z, \tau; w_t, r_t), M_t, w_t, r_t), M_t, p_t) = \\ (1 - \lambda)\mu_t(z, \tau; w_t, r_t) + M\mathbf{1}^e(z, \tau; w_t, r_t)P(\tau|z)H(z)$$

- Endogenous labor and capital demand

$$L^d = \sum_{z,\tau} n(z,\tau;w,r)d\mu(z,\tau;w,r)$$
$$K^d = \sum_{z,\tau} k(z,\tau;w,r)d\mu(z,\tau;w,r)$$

- Aggregate output

$$Y^s = \sum_{z,\tau} [zn(z,\tau;w,r)^\alpha k(z,\tau;w,r)^\gamma - c_o]d\mu(z,\tau;w,r)$$

- Goods demand: $Y^d = C + \delta K + Mc_e$
- Aggregate taxes: $\sum_{z,\tau} \tau f(z, n(z,\tau), k(z,\tau))d\mu(z,\tau;w,r)$

A steady-state competitive equilibrium is a wage rate w, a rental rate r, a lump-sum tax T, a policy function $\mathbf{1}^e(z,\tau)$, a distribution $\mu(z,\tau)$, and a mass of entry M such that:

- **Consumers optimality**: $r = 1/\beta - 1 + \delta$
- **Firms optimality**: $\mathbf{1}^e(z,\tau)$ solves the problem of the entrants
- **Free-entry**: $v^e(w,r) = c_e$
- **Markets clearing**: $L^d = 1 \quad K^d = \bar{K} \quad Y^s = Y^d$
- **Balanced budget**:

$$T = \sum_{z,\tau} \tau f(z, n(z,\tau), k(z,\tau)) d\mu(z,\tau; w, r)$$

- **Time-invariance**: $\mu(z,\tau; w, r) = M \frac{\mathbf{1}^e(z,\tau;w,r) P(\tau|z) H(z)}{\lambda}$

- Distortions: U.S treated as un-distorted benchmark, $\tau = 0$
- Period model: 1 Year $\implies r = 0.04, \beta = 0.96$
- Decreasing return to scale: $\alpha + \gamma = 0.85$. Two-third assigned to labor return $\implies \alpha = 0.57, \gamma = 0.28$
- Depreciation to match capital/output ratio of 2.3 $\implies \delta = 0.08$
- No operating costs, $c_o = 0$.
- Entry cost normalized to 1, $c_e = 1$ (identification issues: changes to c_e isomorphic to changes in establishment-level productivity)

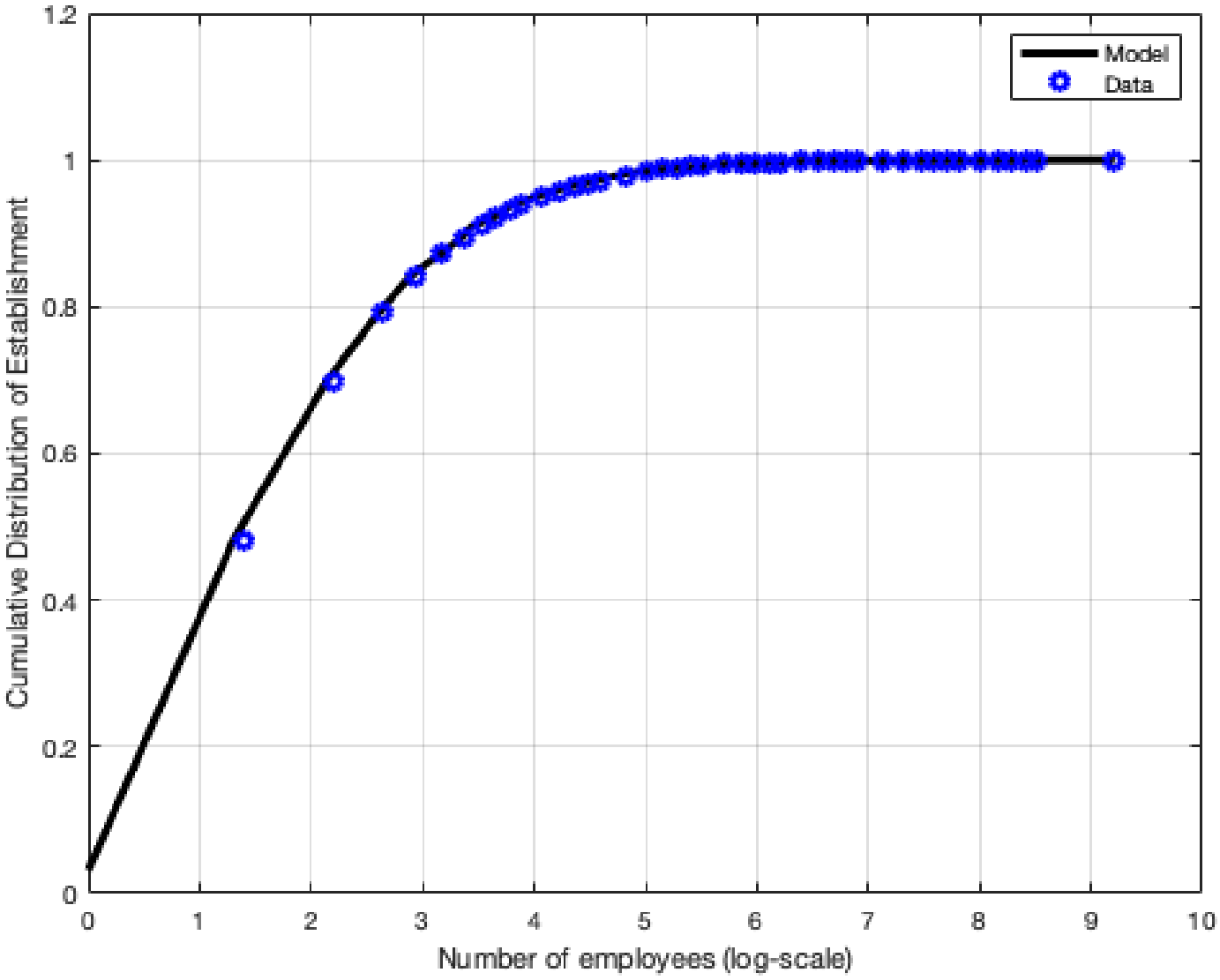
Productivity state-space:

- 100 nodes
- Lowest productivity normalized to 1
- Range of values chosen to match the range of employment across establishments

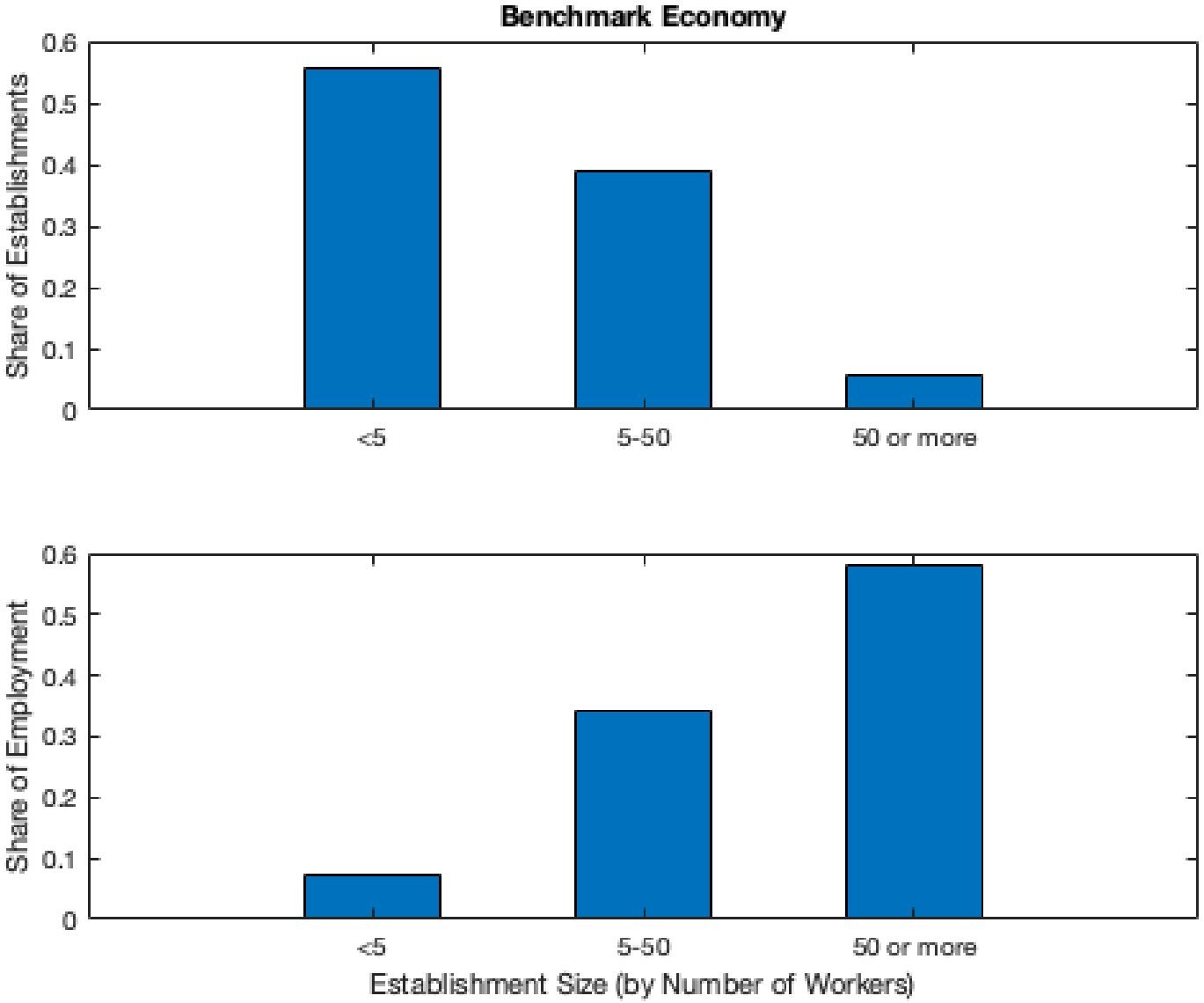
$$\frac{n_i}{n_j} = \left(\frac{z_i}{z_j}\right)^{\frac{1}{1-\gamma-\alpha}}$$

  In US data, biggest firms are 10000 times larger than smallest. Given α and γ, largest productivity equal to 3.98
- Distribution of productivity $H(z)$ to match observed firm-size distribution

Calibration



Benchmark economy



- Two main experiments:
  - Uncorrelated distortions, τ independent of z
    - half producers taxed, half subsidized
    - resources flow from taxed to subsidized, but no systematic effect across productivity classes
  - Correlated distortions, either positively or negatively
    - lowest half producers subsidized, top half taxed
    - systematic reallocation across productivity classes, not just within productivity class
- Size of the subsidy so that the net effect on steady-state capital accumulation is zero.

Variable	Description	τ_t 0.1	0.2	0.3	0.4
Y	Relative Output	0.98	0.96	0.93	0.92
TFP	Relative TFP	0.98	0.96	0.93	0.92
E	Relative employment	1.00	1.00	1.00	1.00
Y_s/Y	Output of subsidized	0.72	0.85	0.93	0.97
S/Y	Subsidy share of output	0.05	0.08	0.09	0.10
τ_s	Subsidy rate	0.06	0.09	0.10	0.11

- comparatively small effect on TFP and output, no effect on E
- subsidies to undo effects on capital accumulation are smaller
- as tax increases, larger TFP-effect and larger subsidies

Variable	Description	τ_t 0.1	0.2	0.3	0.4
Y	Relative Output	0.90	0.80	0.73	0.69
TFP	Relative TFP	0.90	0.80	0.73	0.69
E	Relative employment	1.00	1.00	1.00	1.00
Y_s/Y	Output of subsidized	0.42	0.67	0.83	0.92
S/Y	Subsidy share of output	0.17	0.32	0.43	0.49
τ_s	Subsidy rate	0.40	0.48	0.52	0.53

- qualitatively similar to uncorrelated case
- larger negative effect on TFP and output
- also more costly to finance (higher subsidies)

- Non-constant capital stock
  - taxing all but some exempt producers at 40% rate and no subsidy
  - lower capital stock, wages and entry rate also fall in proportion
  - amplifies effects on TFP
- Taxes on capital and labor

- **Step 0**: fix interest rate to steady-state value: r^*
- **Step 1**: guess a wage rate w_0
- **Step 2**: solve for the value of the incumbent, $v(z, \tau; w_0, r^*)$
- **Step 3**: solve the problem of the potential entrant, $\mathbf{1}^e(z, \tau; w_0, r^*)$
- **Step 4**: compute the value of entry, $v^e(w_0, r^*)$ and check if free-entry condition is satisfied:
  - if no, make a new guess w_1 and go back to step 2 till convergence
  - if yes, store $w^* = w_0$
- **Step 5**: compute the invariant distribution of plants (normalized $M = 1$)
- **Step 6**: find mass of firms such that the labor market clears

- Hsieh, Chang-Tai, and Peter J. Klenow. 2009. "Misallocation and manufacturing TFP in China and India." The Quarterly journal of economics. Vol.124, N.4, pp. 1403-1448
  - Distortions hinder optimal allocations of factors across plants
    - static misallocation
  - Inferring distortions from measures gaps in marginal products
  - Hypothetical gains from reallocating capital and labor
  - Explain differences in TFP and GDP p.c. across countries

- Large aggregate TFP differences across countries
  - TFP in US manufacturing sector is 2.3 $\times$ China, 2.6 $\times$ India
- What explains differences in TFP?
  - Traditional view: barriers to technology diffusion
  - This paper: inefficient allocation of resources (labor and capital) due to distortions/regulations
- This paper
  - Technical contribution: build a model to estimate distortions from data
  - Quantitative contribution: quantify TFP gains from improving resource allocation
    - 43% in US, 87% in China, 127% in India

- One final producer
- Industry $j = 1, .., J$
- Individual firm $i = 1, .., M_j$ in each industry
- Two production factors: capital K and labor L

- Markets structure
  - firms employ labor and capital to produces a differential variety
    - monopolistic competition in the product market
    - perfect competition in the factor markets
  - industries aggregate varieties
  - final producer aggregate industrial outputs

- Firm-level (idiosyncratic) distortions:
  - distortions to capital and labor allocations, τ_{ij}^{Y}
  - distortions to capital allocation relative to labor allocation, τ_{ij}^{K}

- Final output Y is a cobb-douglas aggregate of J industrial output, Y_j

$$\log Y = \sum_{j=1}^{J} \theta_j \log Y_j \qquad \theta_j \in (0,1), \sum_{j=1}^{J} \theta_j = 1$$

- Cost minimization of final producer

$$\min \sum_{j=1}^{J} P_j Y_j \quad \text{s.t} \quad \sum_{j=1}^{J} \theta_j \log Y_j \leq Y$$

gives a constant expenditure share of each industry$-j$ output

$$P_j Y_j = \theta_j Y \underbrace{P}_{=1} \quad \text{(final output is the numeraire)}$$

Problem of the producer in industry j

- Industrial output Y_j is CES aggregate of M_j products, Y_{ij}

$$Y_j = \left(\sum_{i=1}^{M_j} Y_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \qquad \sigma > 1$$

- Cost minimization of the representative producer in industry j

$$\min \sum_{i=1}^{M_j} P_{ij} Y_{ij} \quad \text{s.t} \qquad \left(\sum_{i=1}^{M_j} Y_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \leq Y_j$$

gives the inverse demand function for each variety i

$$Y_{ij} = \left(\frac{P_j}{P_{ij}} \right)^{-\sigma} Y_j$$

Problem of firm i in industry j

- Firm i produces with a cobb-douglas function in capital and labor

$$Y_{ij} = A_{ij} K_{ij}^{\alpha_j} L_{ij}^{1-\alpha_j} \qquad \alpha_j \in (0,1), \forall i = 1, ...M_j$$

- Profit maximization problem of firm i

$$\pi_{ij} = \max_{L_{ij}, K_{ij}, P_{ij}} \quad (1-\tau_{ij}^{Y}) P_{ij} Y_{ij} - w L_{ij} - (1+\tau_{ij}^{K}) r K_{ij}$$

$$\text{s.t.} \quad Y_{ij} = A_{ij} K_{ij}^{\alpha_j} L_{ij}^{1-\alpha_j}$$

$$\left(\frac{P_j}{P_{ij}}\right)^{-\sigma} Y_j \leq Y_{ij}$$

- Assumption: all firms face the same wage

- Let: $c(w, r, \alpha_j) = \left(\frac{R}{\alpha_j}\right)^{\alpha_j} \left(\frac{w}{1-\alpha_j}\right)^{1-\alpha_j}$
- Pricing decision: constant markup over marginal costs

$$P_{ij} = \frac{\sigma}{\sigma - 1} \frac{c(w, r, \alpha_j)}{A_{ij}} \frac{(1 + \tau_{ij}^K)^\alpha}{(1 - \tau_{ij}^Y)} \tag{1}$$

- Factor demands

$$(1 + \tau_{ij}^K) r K_{ij} = \alpha_j \frac{c(w, r, \alpha_j)}{A_{ij}} (1 + \tau_{ij}^K)^\alpha Y_{ij} \tag{2}$$

$$w L_{ij} = (1 - \alpha_j) \frac{c(w, r, \alpha_j)}{A_{ij}} (1 + \tau_{ij}^K)^\alpha Y_{ij} \tag{3}$$

- Output demand: $Y_{ij} = \left(\frac{P_j}{P_{ij}}\right)^{-\sigma} \theta_j \frac{Y}{P_{ij}}$

- United States: Census of Manufactures
  - 1977, 1982, 1987, 1992, 1997
  - census of manufacturing plants
  - sample of approx 160k plants per year, 400 industries (4 digit)
- India: Annual Survey of Industries
  - annual, 1988-1995
  - census of large manufacturing plants, sample of small plants
  - sample of approx 40k plants per year, 400 industries (4 digit)
- China: Annual Surveys of Industrial Production
  - annual 1998-2005
  - census of large non-state firms, census of all state firms
  - sample increases up to 200k in 2005, 400 industries (4 digit)

- Interest rate r=10%
  - inclusive of depreciation rate ($\approx$ 5%)
- Elasticity of substitution σ=3
  - assumed fixed across sectors
  - gains from removing distortions increasing in σ
- Elasticity of output with respect to capital α_k= 1 minus the labor share in the corresponding industry in the US
  - source: NBER Productivity Database
  - re-scaled to match labor share reported in the Census of Manufactures

- Firm-level labor-capital expenditure ratio (from eq. 2 and 3)

$$\frac{wL_{ij}}{(1+\tau_{ij}^{K})rK_{ij}} = \frac{1-\alpha_j}{\alpha_j}$$

$$\implies (1+\tau_{ij}^{K}) = \frac{\alpha_j}{1-\alpha_j}\frac{wL_{ij}}{rK_{ij}}$$

- Firm-level labor expenditure share of total revenues (from eq. 1 and 3)

$$wL_{ij} = (1-\alpha_j)\frac{\sigma-1}{\sigma}(1-\tau_{ij}^{Y})P_{ij}Y_{ij}$$

$$\implies (1-\tau_{ij}^{Y}) = \frac{\sigma}{\sigma-1}\frac{1}{(1-\alpha_j)}\frac{wL_{ij}}{P_{ij}Y_{ij}}$$

- Physical productivity: $\text{TFPQ}_{ij} := \frac{Y_{ij}}{K_{ij}^{\alpha_j} L_{ij}^{1-\alpha_k}} = A_{ij}$
  - reflects differences in technology/quality of product
  - varies across firms

- Revenue productivity: $\text{TFPR}_{ij} := \frac{P_{ij} Y_{ij}}{K_{ij}^{\alpha_j} L_{ij}^{1-\alpha_k}} = P_{ij} A_{ij}$
  - reflects differences in technologies and pricing
    - w/o distortion, no price dispersion across firms

$$P_{ij} = \frac{\sigma}{\sigma - 1} \frac{c(w, r, \alpha_j)}{A_{ij}} \implies \text{TFPR}_{ij} = \frac{\sigma}{\sigma - 1} c(w, r, \alpha_j)$$

    - in distorted economy price dispersion across firms

$$P_{ij} = \frac{\sigma}{\sigma - 1} \frac{c(w, r, \alpha_j)}{A_{ij}} \frac{(1 + \tau_{ij}^K)^\alpha}{(1 - \tau_{ij}^Y)}$$

$$\implies \text{TFPR}_{ij} = \frac{\sigma}{\sigma - 1} c(w, r, \alpha_j) \frac{(1 + \tau_{ij}^K)^\alpha}{(1 - \tau_{ij}^Y)}$$

Inferring productivity from revenues

- Without information about prices P_{ij}, physical productivity A_{ij} cannot be measured
- Infer A_{ij} from revenues $P_{ij}Y_{ij}$
- Recall the demand function:

$$Y_{ij} = \left(\frac{P_{ij}}{P_j}\right)^{-\sigma} Y_j \implies \left(\frac{Y_{ij}}{Y_j}\right)^{\frac{\sigma-1}{\sigma}} = \frac{P_{ij}Y_{ij}}{P_jY_j}$$

$$Y_{ij} = \left(\frac{P_{ij}Y_{ij}}{P_jY_j}\right)^{\frac{\sigma}{\sigma-1}} Y_j$$

which implies

$$A_{ij} = \frac{Y_{ij}}{K_{ij}^{\alpha_j} L_{ij}^{1-\alpha_k}} = \underbrace{\left(w^{1-\alpha_j} Y_j (P_jY_j)^{\frac{\sigma}{1-\sigma}}\right)}_{\kappa_j,\text{normalized to 1}} \frac{(P_{ij}Y_{ij})^{\frac{\sigma}{\sigma-1}}}{{K_{ij}}^{\alpha_j}(wL_{ij})^{1-\alpha_j}}$$

Distribution of TFPQ

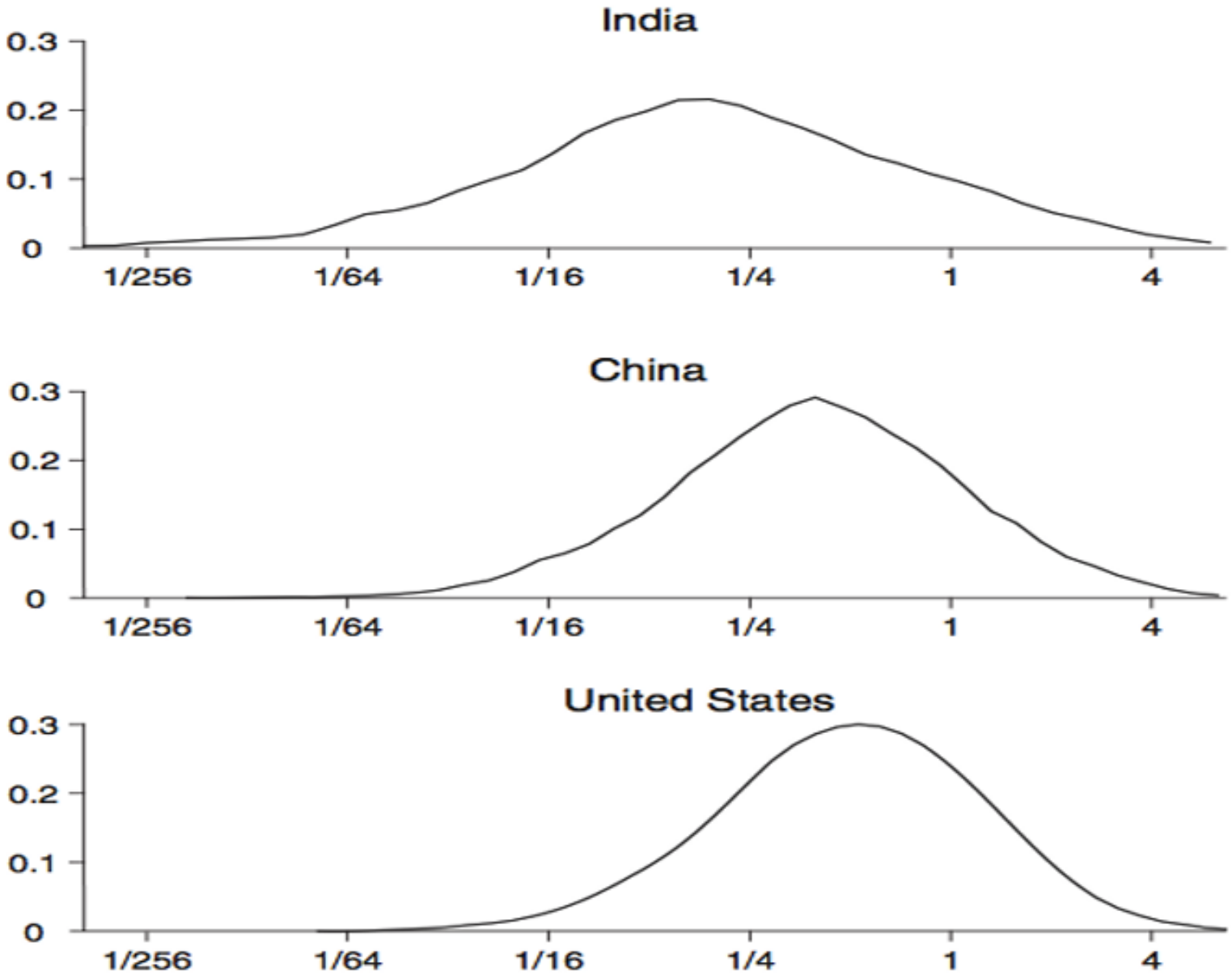


TABLE I
DISPERSION OF TFPQ

China	1998	2001	2005
S.D.	1.06	0.99	0.95
75 − 25	1.41	1.34	1.28
90 − 10	2.72	2.54	2.44
N	95,980	108,702	211,304
India	**1987**	**1991**	**1994**
S.D.	1.16	1.17	1.23
75 − 25	1.55	1.53	1.60
90 − 10	2.97	3.01	3.11
N	31,602	37,520	41,006
United States	**1977**	**1987**	**1997**
S.D.	0.85	0.79	0.84
75 − 25	1.22	1.09	1.17
90 − 10	2.22	2.05	2.18
N	164,971	173,651	194,669

Notes. For plant i in industry s, $\text{TFPQ}_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s}(w_{si}L_{si})^{1-\alpha_s}}$. Statistics are for deviations of log(TFPQ) from industry means. S.D. = standard deviation, 75 − 25 is the difference between the 75th and 25th percentiles, and 90 − 10 the 90th vs. 10th percentiles. Industries are weighted by their value-added shares. N = the number of plants.

Distribution of TFPR

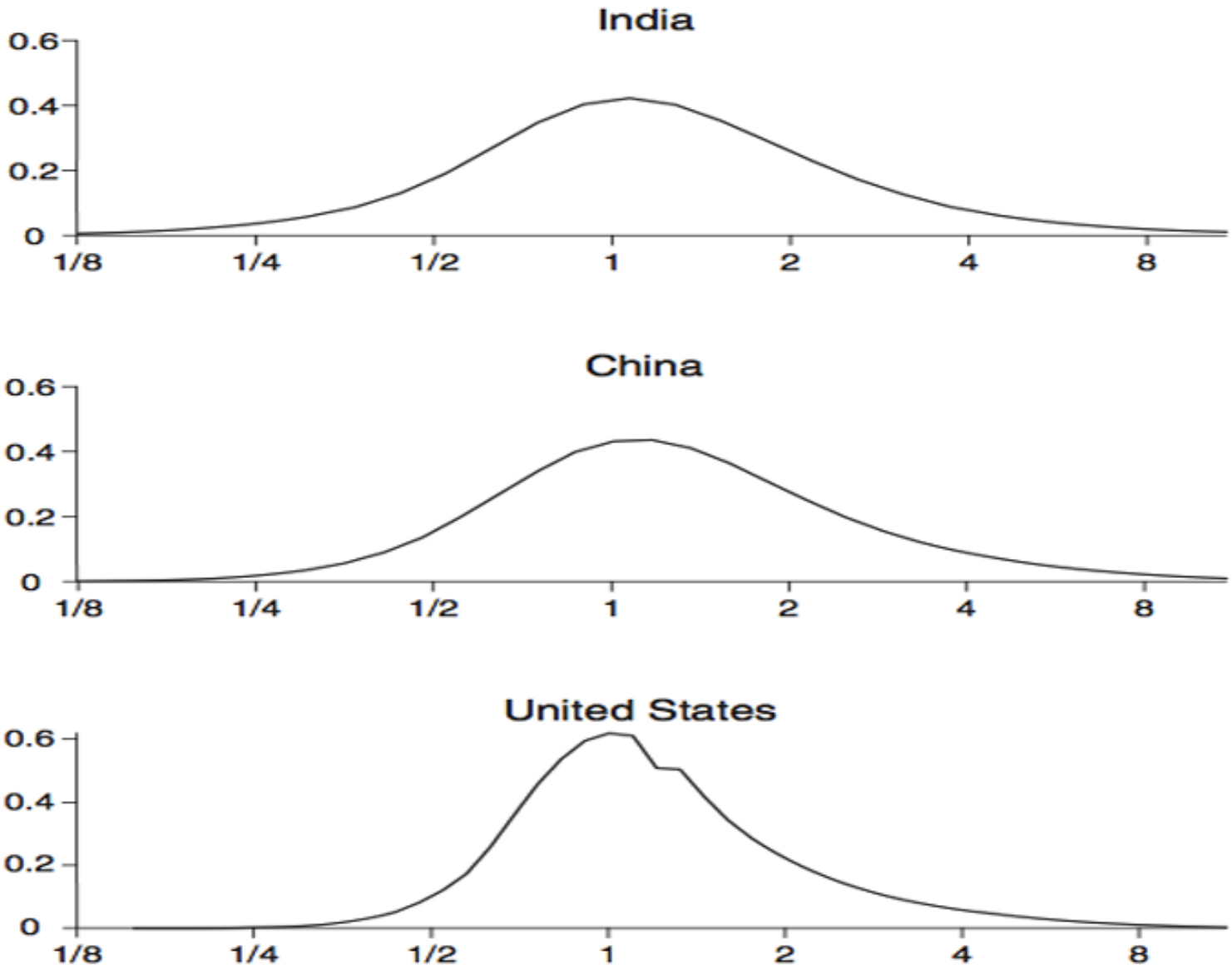


TABLE II

DISPERSION OF TFPR

China	1998	2001	2005
S.D.	0.74	0.68	0.63
75 − 25	0.97	0.88	0.82
90 − 10	1.87	1.71	1.59
India	**1987**	**1991**	**1994**
S.D.	0.69	0.67	0.67
75 − 25	0.79	0.81	0.81
90 − 10	1.73	1.64	1.60
United States	**1977**	**1987**	**1997**
S.D.	0.45	0.41	0.49
75 − 25	0.46	0.41	0.53
90 − 10	1.04	1.01	1.19

Notes. For plant i in industry s, $\text{TFPR}_{si} = \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} (u_{si} L_{si})^{1-\alpha_s}}$. Statistics are for deviations of log(TFPR) from industry means. S.D. = standard deviation, 75 − 25 is the difference between the 75th and 25th percentiles, and 90 − 10 the 90th vs. 10th percentiles. Industries are weighted by their value-added shares. Number of plants is the same as in Table I.

Sources of TFPR variation within industry

	Ownership	Age	Size	Region
India	0.58	1.33	3.85	4.71
China	5.25	6.23	8.44	10.01

Notes. Entries are the cumulative percent of within-industry TFPR variance explained by dummies for ownership (state ownership categories), age (quartiles), size (quartiles), and region (provinces or states). The results are cumulative in that "age" includes dummies for both ownership and age, and so on.

- Differences across ownership, firm age, firm size and region account for only 5% of dispersion in TFPR in India, 10% in China

- Aggregate capital and labor demand across firms

$$K_j = \sum_{i=1}^{M_j} K_{ij} = \frac{\alpha_j c(w, r, \alpha_j)}{r} \sum_{i=1}^{M_j} \frac{Y_{ij}}{A_{ij}} (1 + \tau_{ij}^K)^{\alpha - 1}$$

$$L_j = \sum_{i=1}^{M_j} L_{ij} = \frac{(1 - \alpha_j) c(w, r, \alpha_j)}{w} \sum_{i=1}^{M_j} \frac{Y_{ij}}{A_{ij}} (1 + \tau_{ij}^K)^{\alpha}$$

- Factor shares (using $P_{ij} A_{ij}$)

$$\Theta_{K,j} = \frac{r K_j}{P_j Y_j} = \alpha_j \frac{\sigma - 1}{\sigma} \sum_{i=1}^{M_j} \frac{(1 - \tau_{ij}^Y)}{(1 + \tau_{ij}^K)} \frac{Y_{ij} P_{ij}}{P_j Y_j}$$

$$\Theta_{L,j} = \frac{w L_j}{P_j Y_j} = (1 - \alpha_j) \frac{\sigma - 1}{\sigma} \sum_{i=1}^{M_j} (1 - \tau_{ij}^Y) \frac{Y_{ij} P_{ij}}{P_j Y_j}$$

- Re-arranging terms:

$$P_j Y_j = \underbrace{\left(\frac{r}{\Theta_{K,j}}\right)^{\alpha_j} \left(\frac{w}{\Theta_{L,j}}\right)^{\alpha_j}}_{\mathrm{TFTR}_j = P_j A_j} K_j^{\alpha_j} L_j^{1-\alpha_j} \implies Y_j = A_j K_j^{\alpha_j} L_j^{1-\alpha_j}$$

$$\text{where} \quad A_j = \left[\sum_{i=1}^{M_j} \left(A_{ij} \frac{\mathrm{TFPR}_j}{\mathrm{TFPR}_{ij}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}$$

- When A_{ij} and TFPR_{ij} are jointly log-normal:

$$\log A_j = \frac{1}{\sigma - 1} \log \left(\sum_{i=1}^{M_j} A_{ij}^{\sigma-1}\right) - \frac{\sigma}{2} \mathrm{var}(\log \mathrm{TFPR}_{ij})$$

TFP gains across countries

TABLE IV
TFP GAINS FROM EQUALIZING TFPR WITHIN INDUSTRIES

China	1998	2001	2005
%	115.1	95.8	86.6
India	1987	1991	1994
%	100.4	102.1	127.5
United States	1977	1987	1997
%	36.1	30.7	42.9

Notes. Entries are $100(Y_{\text{efficient}}/Y - 1)$ where $Y/Y_{\text{efficient}} = \prod_{s=1}^{S}[\sum_{i=1}^{M_s}(\frac{A_{si}}{A_s}\frac{\overline{\text{TFPR}_s}}{\text{TFPR}_{si}})^{\sigma-1}]^{\theta_s/(\sigma-1)}$ and $\text{TFPR}_{si} \equiv \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s}(w_{si}L_{si})^{1-\alpha_s}}$.

TFP gains across countries, relative to US

TABLE VI

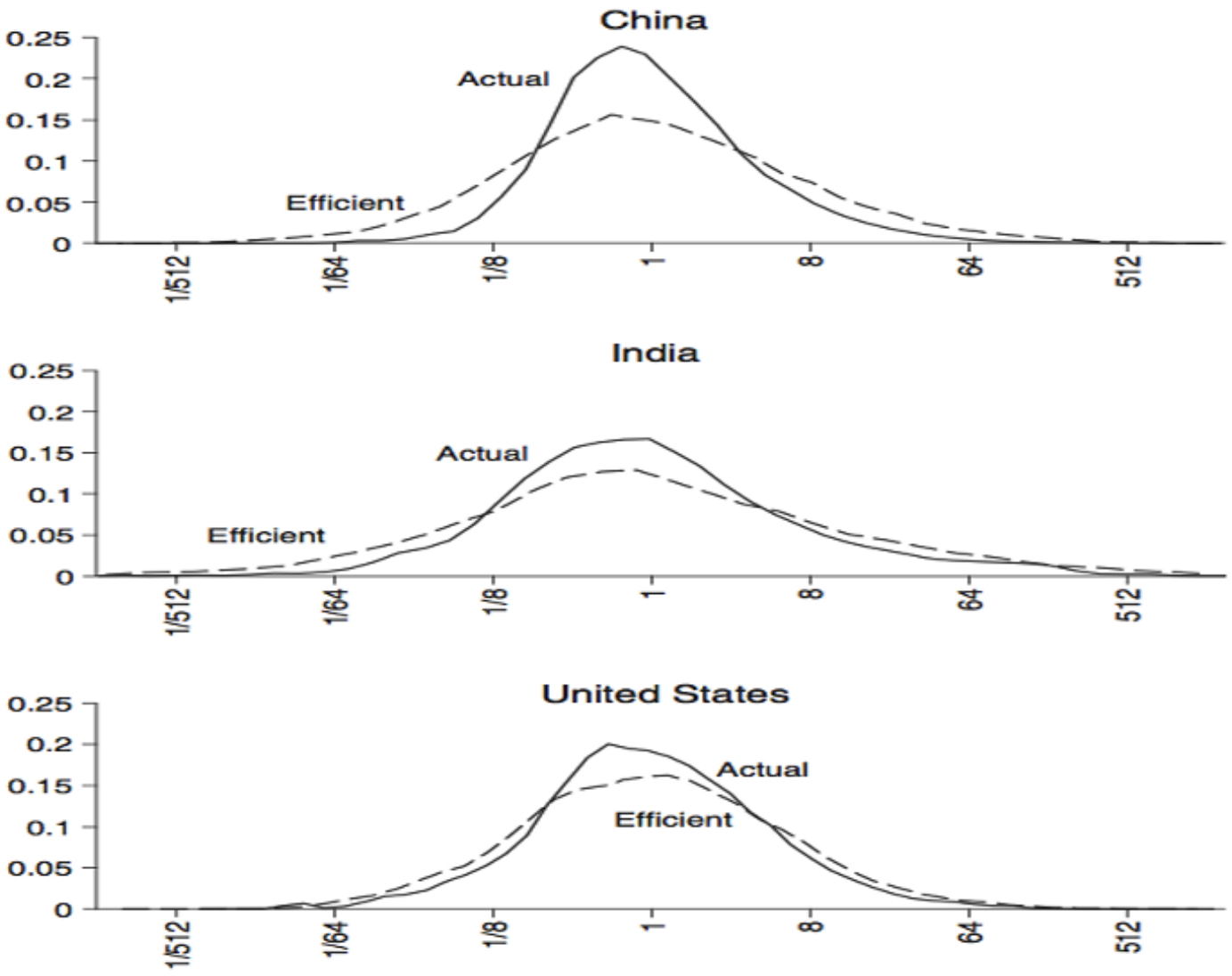
TFP GAINS FROM EQUALIZING TFPR RELATIVE TO 1997 U.S. GAINS

China	1998	2001	2005
%	50.5	37.0	30.5
India	1987	1991	1994
%	40.2	41.4	59.2

Notes. For each country-year, we calculated $Y_{\text{efficient}}/Y$ using $Y/Y_{\text{efficient}} = \prod_{s=1}^{S} \left[\sum_{i=1}^{M_s} \left(\frac{A_{si}}{A_s} \frac{\overline{\text{TFPR}}_s}{\text{TFPR}_{si}} \right)^{\sigma-1} \right]^{\theta_s/(\sigma-1)}$ and $\text{TFPR}_{si} \equiv \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} (w_{si} L_{si})^{1-\alpha_s}}$.

We then took the ratio of $Y_{\text{efficient}}/Y$ to the U.S. ratio in 1997, subtracted 1, and multiplied by 100 to yield the entries above.

Actual vs Efficient size distribution



- Efficient firm size distribution is more dispersed

- Large dispersion in TFPR across plants, suggestive of distortions
- Large TFP gains from reallocating resources across plants to eliminate dispersion in TFPR
- Alternative explanations
  - measurement errors
  - within-industry variations in markups
  - frictions and adjustment costs
  - unobserved investment (i.e. ntangible capital)
  - within-industry variation in technology (heterogeneous capital intensity)

- Peters, M. 2020. Heterogeneous markups, growth, and endogenous misallocation. Econometrica. Vol.88 N.5 pp.2037-2073.
  - Quality ladder with endogeneous misallocation
    - Hsieh and Klenow (2009) takes marginal product gaps etc as exogenous
  - Firm-level markups depend on productivity gap between incumbent and rivals
    - Bertrand competition on prices
  - Static (and dynamics) implications of markups dispersion

- Unit measure of differentiated products indexed by i
- S_i firms indexed by j competing in market i
  - firms are heterogeneous in productivity $A_{ji} \sim h(z)$
- Market structure:
  - different products i and i' are imperfect substitutes
  - perfect substitutability between different firms j within a product line i
- Firms are price takers, and operate with CRS technology
  - competition a la Bertrand with other producers within the same product line
  - only the most productive firm will be active in equilibrium
  - the presence of competitors alter optimal firm pricing

- Final good composite of a continuum of products

$$\ln Y = \int_{i=0}^{1} \ln \left(\sum_{j \in S_i} y_{ji} \right) di$$

- Firm-level technology linear in labor

$$y_{ji} = A_{ji} \ell$$

- Marginal costs: w/A_{ji}, where w is the equilibrium wage
- Bertrand competition of firms within same product line
  - highest productivity firms takes the market and changes a price p_i equals the marginal cost of the second most productive firm

- Equilibrium markup for leading firm in product-line i equal to

$$\mu_i = \frac{p_i}{w/A_i} = \frac{w/A_{Ci}}{w/A_i} = \frac{A_i}{A_{Ci}}$$

  where A_{Ci} is productivity of second most productive firms
- Expenditure equalized across products: $p_i y_i = Y$, $\forall i$
- Labor demand inverse of markup:

$$\ell_i = \frac{1}{A_i} y_i = \frac{1}{A_i} \frac{Y}{p_i} = \mu_i^{-1} \frac{Y}{w}$$

- Profits independent of productivity:

$$\pi_i = p_i y_i - w\ell_i = (1 - \mu_i^{-1}) PY$$

- Physical productivity (TFPQ$_i$): A_i
- Revenue productivity (TFPR$_i$): $p_i A_i = \mu_i w$
  - Since w is common, all cross-sectional variation in TFPR$_i$ comes from markup variation (i.e. variation in relative productivity)
  - In Hsieh and Klenow (2009) all cross-sectional variation in TFPR$_i$ comes from exogeneous variation in distortions, (τ^K, τ^Y), and high TFPR$_i$ indicates more distorted firms
- Aggregating labor demands over intermediate producers

$$L = \int_0^1 \ell_i di = \frac{Y}{w} \int_0^1 \mu_i^{-1} di$$

- Aggregate price index is the numeraire ($P = 1$) which implies

$$w = \exp\left(\int_0^1 \ln A_i di\right) \exp\left(\int_0^1 \ln \mu_i^{-1} di\right)$$

- Aggregate output (from aggregate labor demand)

$$Y = \frac{w}{\int_0^1 \mu_i^{-1} di} L = A\mathcal{M}L$$

where

  - $A = \exp\left(\int_0^1 \ln A_i di\right)$: aggregate TFPQ
  - $\mathcal{M} = \frac{\exp\left(\int_0^1 \ln \mu_i^{-1} di\right)}{\left(\int_0^1 \mu_i^{-1} di\right)}$: index of distortions

- Notice that $\mathcal{M} \leq 1$ and $= 1$ only when $\mu_i = \mu, \forall i$ (Jensen's inequality)

- Index of distortions $\mathcal{M}$ is
  - homogeneous degree zero in $\mu_i \implies$ common proportional increase in markups leaves the degree of misallocation unchanged
  - decreasing in a mean-preserving-spread of $\ln \mu_i \implies$ higher markup dispersion increases distortions, reduces allocative efficiency and aggregate TFP

- Monopoly power affects also factor prices

$$\frac{wL}{Y} = \int_0^1 \mu_i^{-1} di$$

- The labor share depends on the level of markups
  - it is homogeneous degree one in $1/\mu_i$: a pure level shift in markups reduces wages and increase aggregate profits
  - it is invariant to a mean-preserving-spread of $1/\mu_i$

- Firm-level information
  - annual census 1991–2000
  - manufacturing plants $\geq$ 20 employees
  - revenue, wage bill, productions and non-production workers, capital stock, entry, region
- Geographical information
  - 240 regencies aggregated to 33 provinces
  - other geographic/regional controls from Village Potential Statistics aggregated to province

- Firm labor share sufficient statistics for markup: $\frac{w\ell_i}{p_i y_i} = \mu_i^{-1}$

- Production data y_i, or value added, $\text{VA}_{it} = p_i y_i$, might contain both unanticipated shocks and measurement error

- De Loecker and Warzinsky (2013)
  - Step 1: production function estimation using control function approach (Olley and Pakes 1994)

    $$\text{VA}_{it} = \phi(\ell_{it}, k_{it}, m_{it}) + \epsilon_{it}$$

    where m_{it}: material, k_{it}: capital, ℓ_{it}: labor, ϕ: second-order polynomial in all (log) inputs and interactions

  - Step 2: recover measurement error $\hat{\epsilon}$

  - Step 3: construct corrected firms labor share, i.e. $\hat{s}_{it} = \frac{w_t \ell_{it}}{\frac{\text{VA}_{it}}{\hat{\epsilon}}}$

Distribution of markups

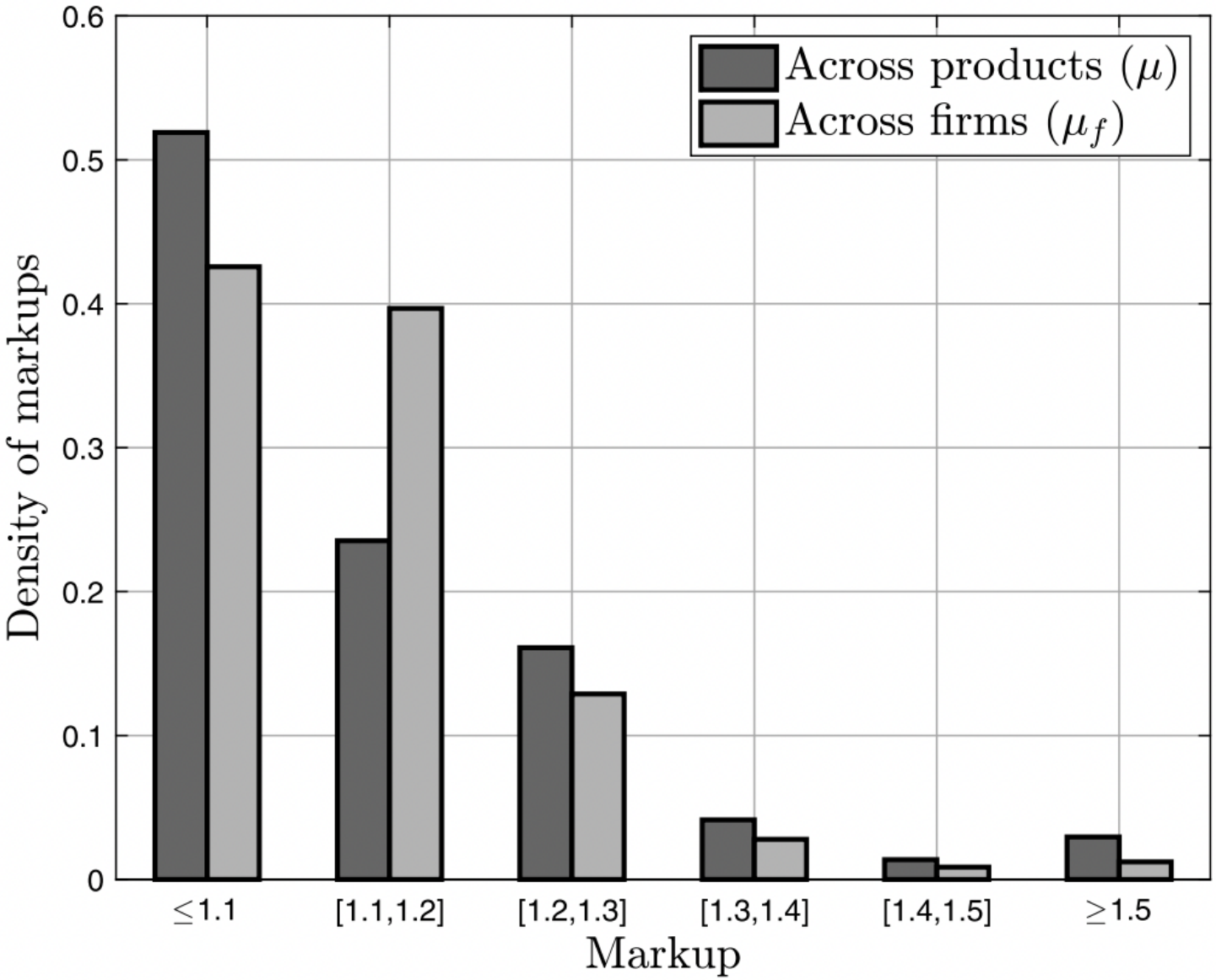


TABLE V

Markups and Misallocation in Indonesia[a]

Markups and Misallocation					
θ	$E[\mu]$	$\sigma(\ln \mu)$	$\sigma(\ln \mu_f)$	$\mathcal{M}$	Λ
9.5	11.8%	0.103	0.079	0.995	0.9

[a]The table reports the endogenous tail parameter of the markup distribution θ, the average markup ($E[\mu]$), the dispersion of log markups across products ($\sigma(\ln \mu)$) and firms ($\sigma(\ln \mu_f)$), and the two misallocation wedges $\mathcal{M}$ and Λ (see (3) and (4)).

- Markups dispersion lowers TFP by only 0.5%
- Average markups lowers wages by 10%

- Hsieh, Chang-Tai, and Peter J. Klenow. 2014. "The life cycle of plants in India and Mexico." The Quarterly Journal of Economics. Vol.129, N.3, pp. 1035-1084.
    - Differences in establishment life-cycle growth rate and age distribution across countries
    - Distortions hinder establishment-specific intangible capital accumulation over the life cycle
        - *dynamic* misallocation
    - Explain differences in TFP and GDP p.c. across countries

Plant size over the life-cycle

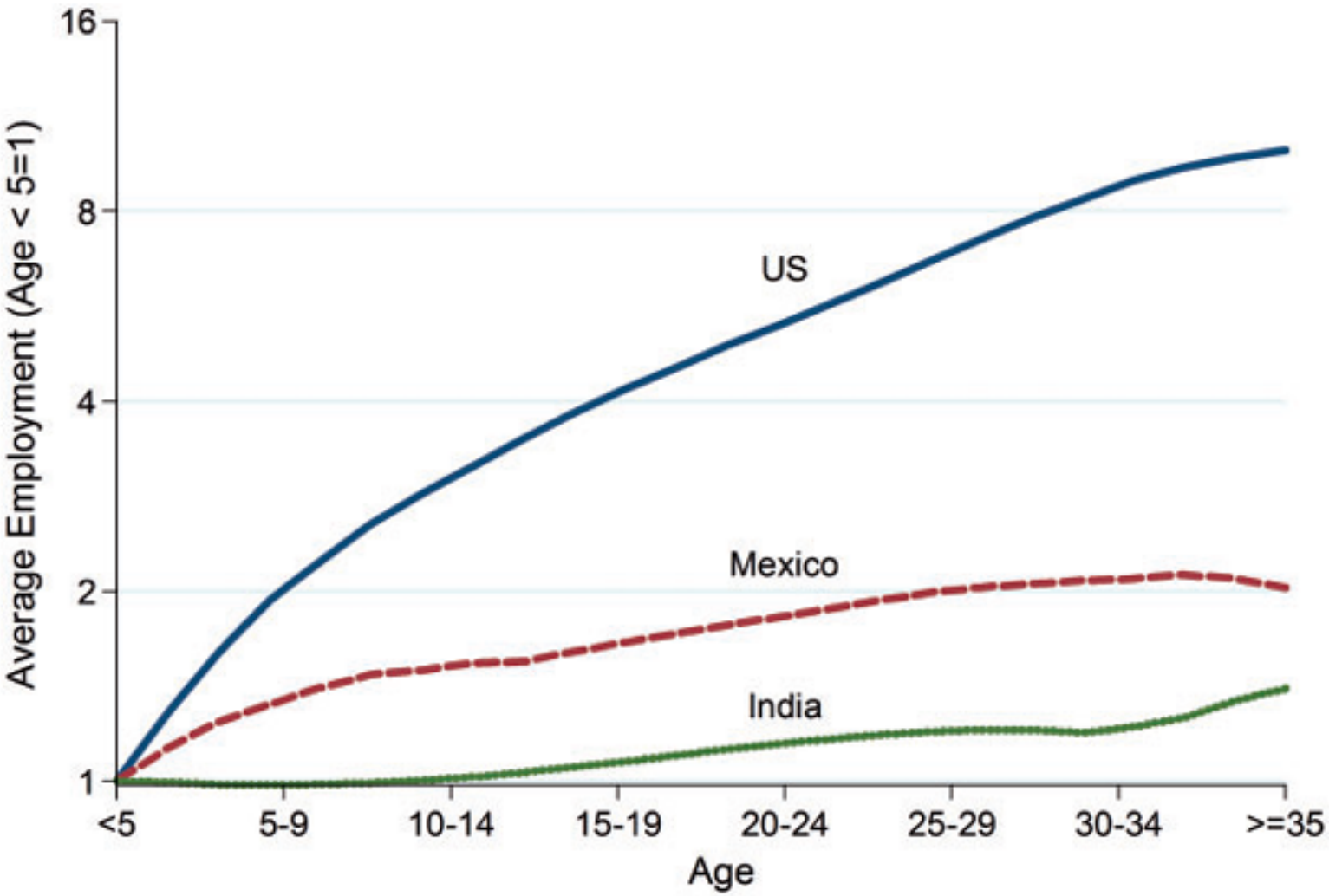


FIGURE IV

Average Employment over the Life Cycle

Employment distribution over firm-age

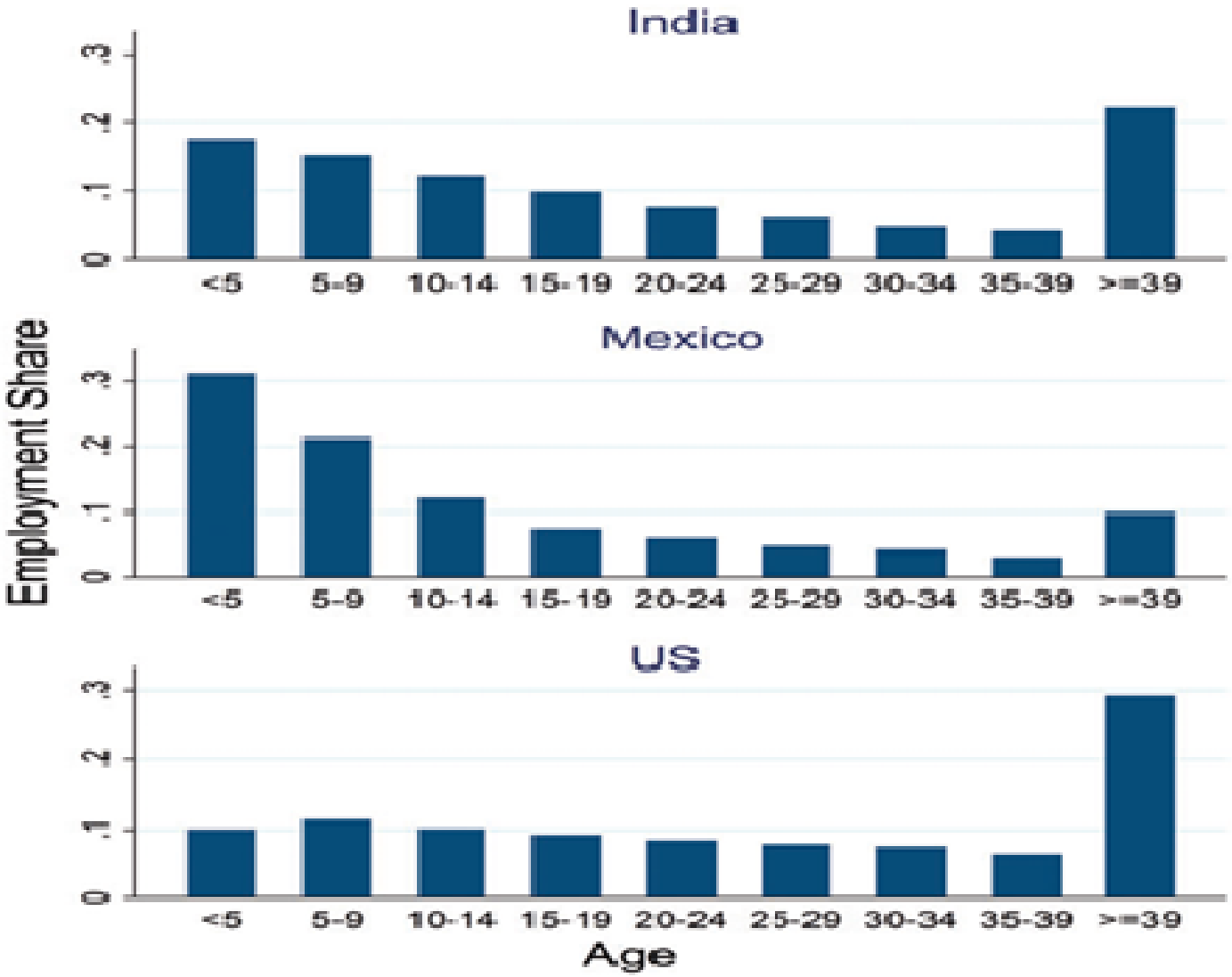


FIGURE VI
Employment Share by Age in Steady State

Firm distribution over firm-age

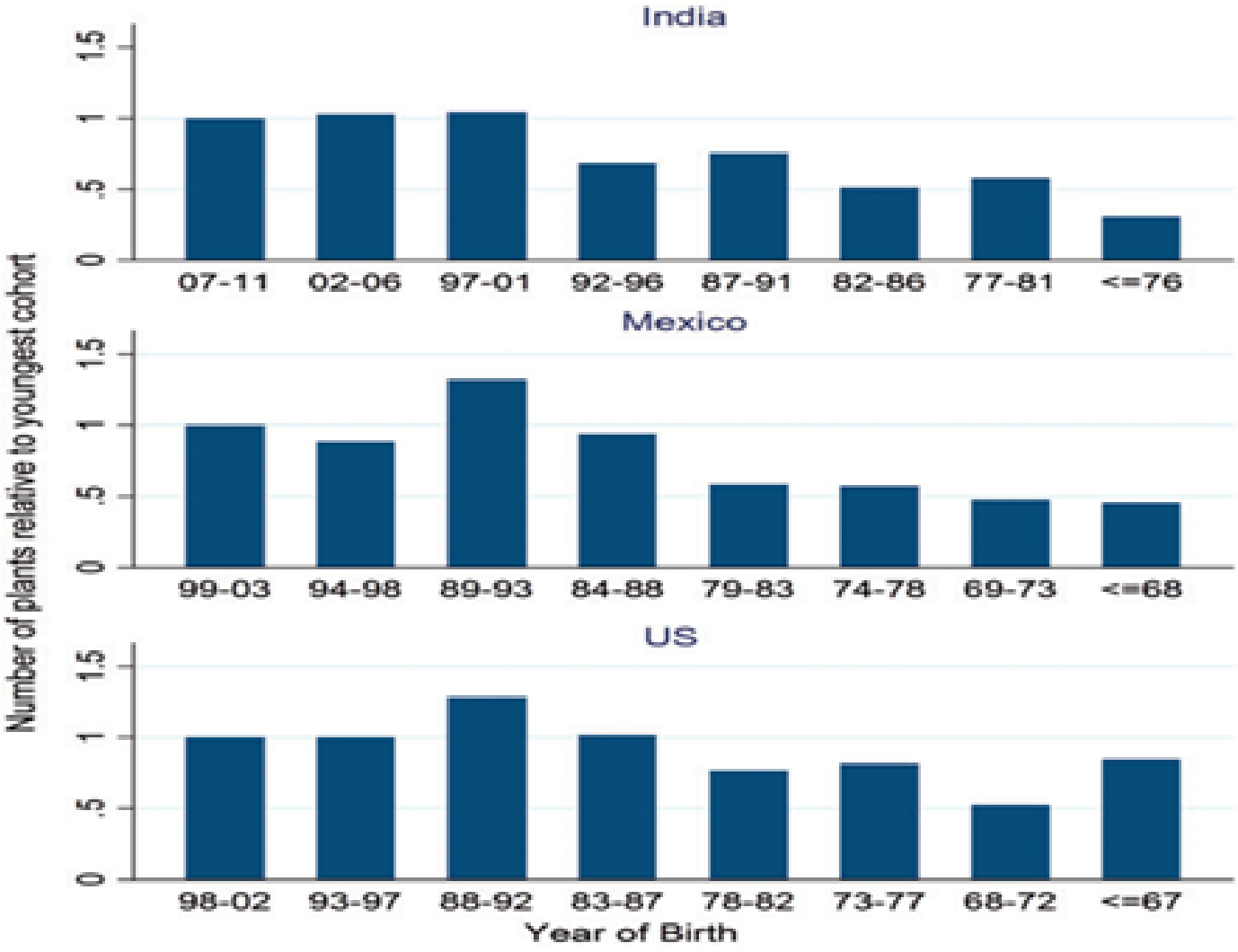


FIGURE V

Number of Plants by Birth Cohort

- Extension of Hsieh and Klenow (2009)
  - firms differ in variety produced, i and age, a
  - age- and variety-dependent productivity, A_{ia}
  - age- and variety-dependent distortions:
    - output distortions, τ_{ia}^{Y}
    - capital distortions, τ_{ia}^{K}

- Aggregate output Y is CES aggregates of firm-level output, Y_{ia}

$$Y = \left(\sum_{a} \sum_{i=1}^{M_a} Y_{ia}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \qquad \sigma > 1$$

- Production function: $Y_{ia} = A_{ia} K_{ia}^{\alpha} L_{ia}^{1-\alpha}$

- Firm-level profits:

$$\pi_{ia} = \max_{p_{ia}, K_{ia}, L_{ia}} \quad (1-\tau_{ia}^{Y}) p_{ia} Y_{ia} + (1+\tau_{ia}^{K}) r K_{ia} + w L_{ia}$$
$$\text{s.t.} \quad Y_{ia} = \left(\frac{p_{ia}}{P}\right)^{-\sigma} \frac{Y}{P}$$

- Physical productivity: $A_{ia} = \frac{Y_{ia}}{K_{ia}^{\alpha} L_{ia}^{1-\alpha}} = \frac{(P_{ia} Y_{ia})^{\frac{\sigma}{\sigma-1}}}{K_{ia}^{\alpha} L_{ia}^{1-\alpha}}$
- Revenue productivity: $\text{TFPR}_{ia} \propto \frac{(1+\tau_{ia}^{K})}{(1-\tau_{ia}^{Y})}$

Plant productivity over the life-cycle

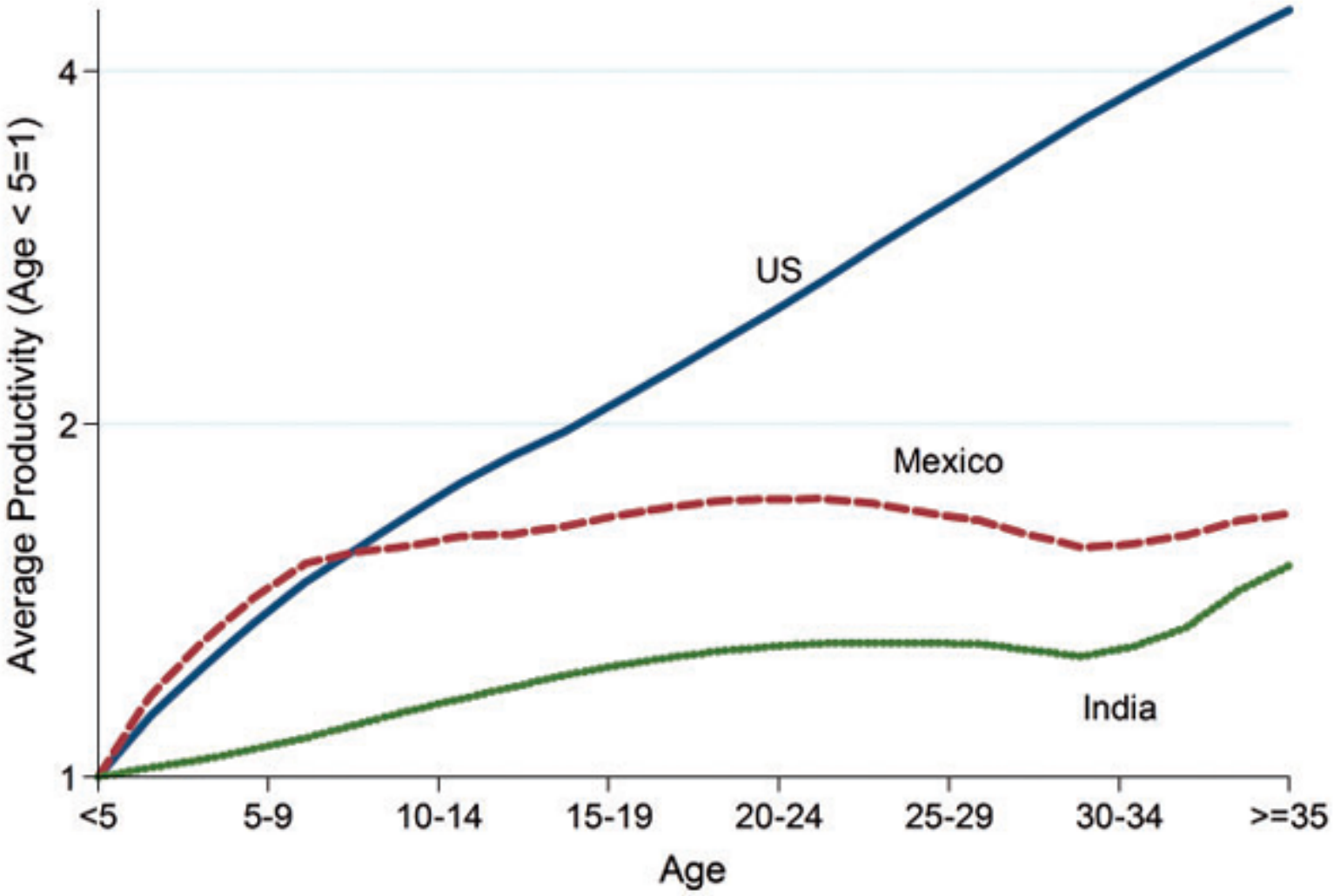


FIGURE X

Productivity over the Life Cycle

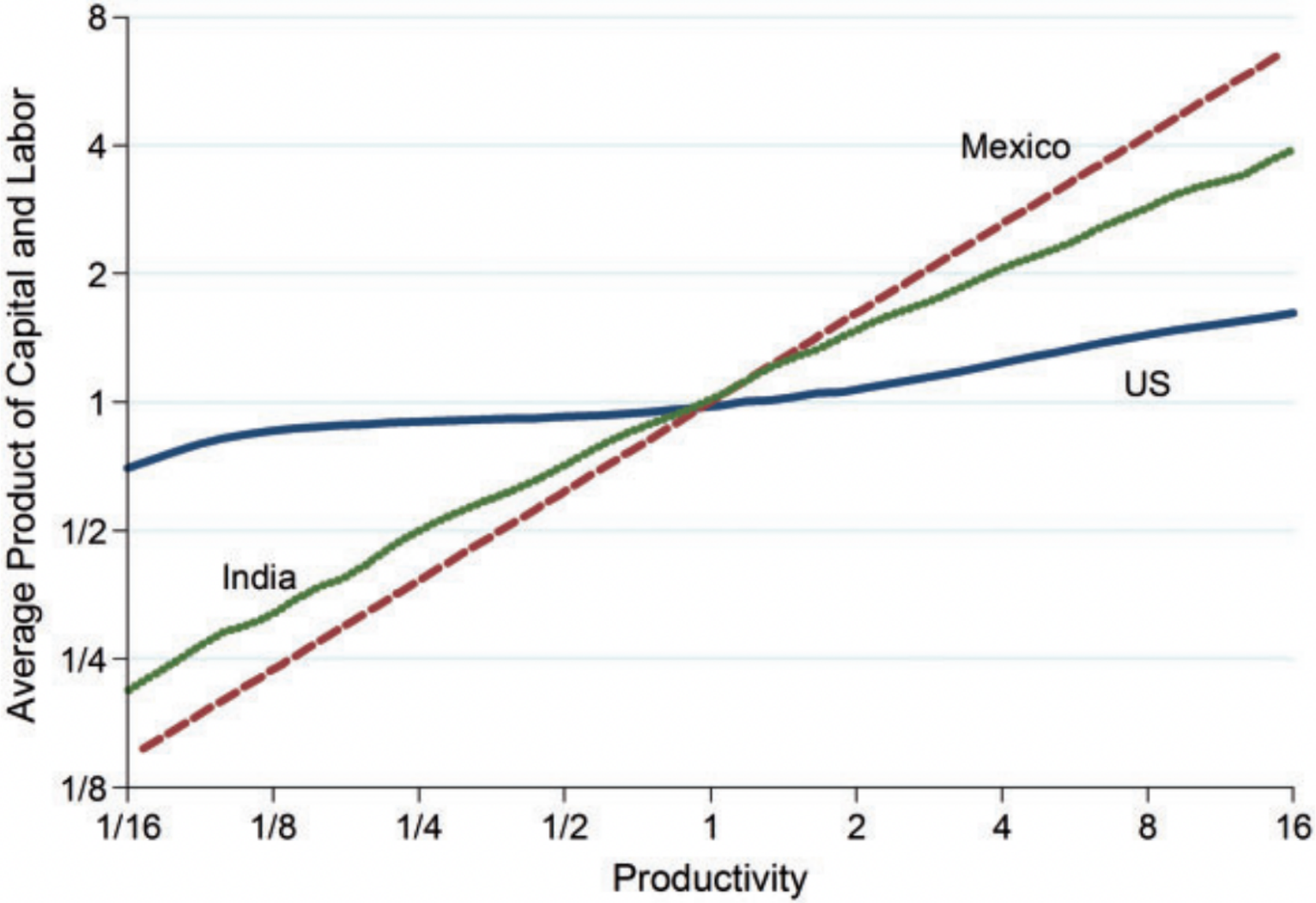


FIGURE XII

Revenue Productivity versus Productivity in the Cross-Section

TFP gains across countries

- Aggregate TFP: $A = \left[\sum_a \sum_{i=1}^{M_a} \left(A_{ia} \frac{\text{TFPR}}{\text{TFPR}_{ia}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}$

TABLE IV

PERCENT CHANGE FROM U.S. TO INDIAN LIFE CYCLE IN MODELS WITH EXOGENOUS LIFE CYCLE PRODUCTIVITY

Cases	Aggregate TFP
Baseline	−25.1
Free entry	−25.1
Overhead costs	−24.8
Adjustment costs	−24.6
Revenue taxes	−25.1

PERCENT CHANGE FROM U.S. TO MEXICAN LIFE CYCLE IN MODELS WITH EXOGENOUS LIFE CYCLE PRODUCTIVITY

Cases	Aggregate TFP
Baseline	−18.2
Free entry	−18.2
Overhead costs	−17.9
Adjustment costs	−18.0
Revenue taxes	−18.2